Section 15

1. The length of a carpet produced by a textile mill is uniformly distributed between 20 feet and 25 feet, while the width is uniformly distributed between 15 feet and 18 feet, where the dimensions are independent random variables.

(a) What is the pdf of the length of a carpet?
(b) What is the probability that the length of a carpet is at most x feet, where $x \in \mathbb{R}^+$?
(c) What is the expected length of a carpet?
(d) What is the probability that a carpet has area at least 345 square feet?

2. Suppose X and Y are independent continuous random variables distributed uniformly between 0 and 1. Let $Z = \min(X, Y)$.

(a) What is the probability that $X \geq t$, where $0 \leq t \leq 1$?
(b) What is the probability that $Z \geq t$, where $0 \leq t \leq 1$?
(c) What is the pdf of Z?
(d) What is the expected value of Z?
(e) What is the variance of Z?

3. Mortality is often highly age dependent. Suppose that the age of a person's death in years, Y, is a random variable with pdf $f(y) = c\left(1 - \frac{|y-20|}{80}\right)$, where we assume $0 \leq Y \leq 100$.

(a) Find the constant c.
(b) Find the probability that the person has died by age 20.
(c) Find the probability that the person has died by age 80.
(d) What is the expected lifetime of the person?

4. Your house is close to a bus stop. You arrive at the bus stop at a uniformly distributed random time in the morning.

(a) There is one bus line which comes by this stop exactly every 15 minutes.
  (i) What is the pdf of the time spent waiting for the next bus?
  (ii) What is the expected time spent waiting for the next bus?
  (iii) What is the standard deviation of the time spent waiting for the next bus?
(b) Suppose a new bus line was recently added, which also stops by exactly every 15 minutes. So now you can take either of these buses to your destination and you will take whichever stops by earlier. You can assume that the phase between the two lines is uniformly distributed between 0 and 15.
  (i) What is the probability that you spend more than t minutes waiting for the next bus, where $0 \leq t \leq 15$?
  (ii) What is the pdf of the time spent waiting for the next bus?
  (iii) What is the expected time spent waiting for the next bus?

5. A brand new lightbulb has just been installed in our classroom, and you know the life span of a lightbulb is exponentially distributed with a mean of 50 days.

(a) Suppose an electrician is scheduled to check on the lightbulb in 30 days and replace it if it is broken. What is the probability that the electrician will find the bulb broken?
(b) Suppose the electrician finds the bulb broken and replaces it with a new one. What is the probablity that the new bulb will last at least 30 days?
(c) Suppose the electrician finds the bulb in working condition and leaves. What is the probability that the bulb will last at least another 30 days?