Section 9

1. You toss a coin n times which has bias p for showing Heads.

(a) Give a probability space to model this experiment.

(b) What is the probability that Heads comes up k times in the n tosses?

2. You have k balls and n bins labelled $1, 2, \ldots, n$, where $n \geq 2$. You drop each ball uniformly at random into the bins.

(a) What is the probability that bin 1 is non-empty?

(b) What is the probability that bin n is empty?

(c) What is the probability that both bin 1 and bin n are empty?

(d) What is the probability that bin 1 is non-empty and bin n is empty?

(e) What is the probability that bin 1 is non-empty given that bin n is empty?

3. A doctor assumes that a patient has one of three diseases d_1, d_2, or d_3. Before any test, he assumes an equal probability for each disease. He carries out a test that will be positive with probability 0.8 if the patient has d_1, 0.6 if he has disease d_2, and 0.4 if he has disease d_3. Given that the outcome of the test was positive, what probabilities should the doctor now assign to the three possible diseases?

4. (a) Mr. Smith has two children, at least one of whom is a boy. What is the probability that both children are boys?

(b) Mr. Smith has two children, one of whom is a boy born on a Tuesday. What is the probability that both children are boys?

NOTE: For both parts, assume that the probability of a boy or girl being born is the same, a child is equally likely to be born on any day of the week, and the genders of all children are independent of each other and independent of the day of the week.

5. The disc containing the only copy of your homework got corrupted, and the disc got mixed up with three other corrupted discs that were lying around. So it is now equally likely that any of the four discs holds the corrupted remains of your homework. Your computer expert friend offers to have a look, and you know from past experience that his probability of finding your homework on any disc is 0.4 (assuming it is actually there). Given that he searches on disc 1 and cannot find your homework, what is the probability that your homework is on disc i, for $i = 1, 2, 3, 4$?