

## Section 8

1. Out of 1000 computer science students, 400 belong to a club (and may work part time), 500 work part time (and may belong to a club), and 50 belong to a club and work part time.
  - (a) Suppose we choose a random student. Let  $C$  be the event that the student belongs to a club and  $P$  the event that the student works part time. Draw a picture of the sample space  $\Omega$  and the events  $C$  and  $P$ .
  - (b) What is the probability that the student belongs to a club?
  - (c) What is the probability that the student works part time?
  - (d) What is the probability that the student belongs to a club AND works part time?
  - (e) What is the probability that the student belongs to a club given that the student works part time?
  - (f) What is the probability that the student belongs to a club OR works part time?
2. Monty Hall revisited

In this variant of the Monty Hall problem, after the contestant has chosen a door, Monty asks another contestant to open one of the other two doors. That contestant, who also has no idea where the prize is, opens one of those two remaining doors at random, and (as it happens) you both see that there is no prize there. Monty now asks you if you wish to switch or stick with your original choice. What is your best strategy? Why? What is the probability you win if you stick, given that the other contestant's door didn't contain the prize? What is the probability you win if you switch, given that the other contestant's door didn't contain the prize?
3.
  - (a) What is the probability of getting at least one six in 4 throws of a fair die?
  - (b) What is the probability of getting at least one double six in 24 throws of a pair of fair dice?
4. Six people get into an elevator at the ground floor of a hotel which has 10 upper floors. Assuming each person gets off at a randomly chosen floor, what is the probability that no two people get off at the same floor?
5. Birthdays
  - (a) Suppose you record the birthdays of a large group of people, one at a time until you have found a match, i.e., a birthday that has already been recorded.
    - i. What is the probability that it takes more than 20 people for this to occur?
    - ii. What is the probability that it takes exactly 20 people for this to occur?
  - (b) Suppose you record the birthdays of a large group of people, one at a time, until you have found a person whose birthday matches your own birthday. What is the probability that it takes exactly 20 people for this to occur?