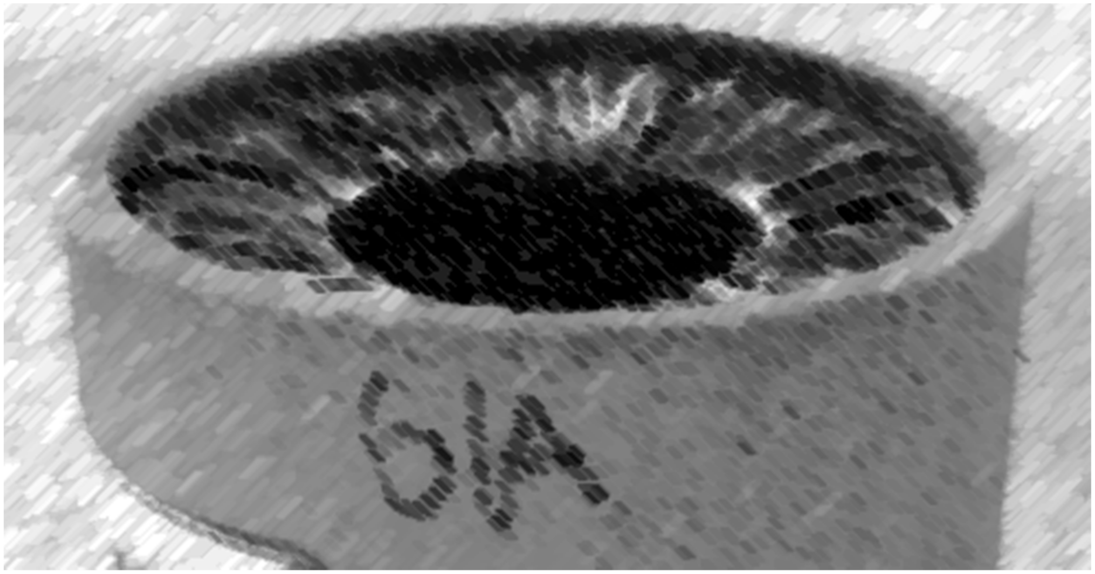


CS61A Lecture 25

Amir Kamil

UC Berkeley

March 20, 2013

Announcements

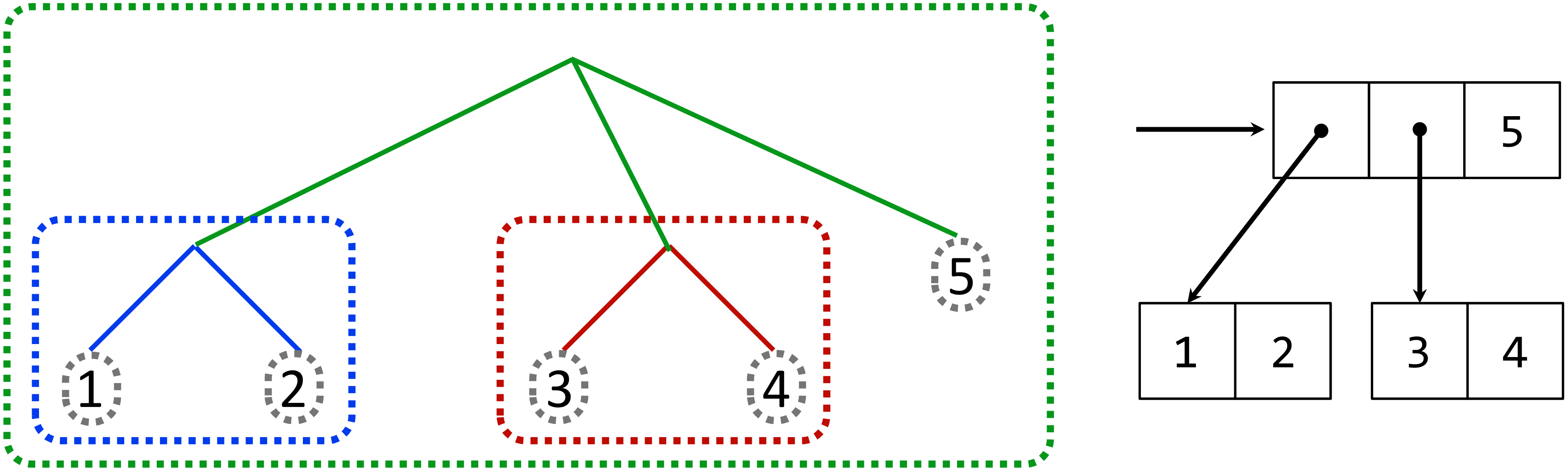
- HW8 due tonight at 7pm

- Midterm 2 Thursday at 7pm
  - See course website for more information

Tree Structured Data

Nested Sequences are Hierarchical Structures.

`((1, 2), (3, 4), 5)`



In every tree, a vast forest

Example: http://goo.gl/0h6n5

Recursive Tree Processing

Tree operations typically make recursive calls on branches

```
def count_leaves(tree):
    if type(tree) != tuple:
        return 1
    return sum(map(count_leaves, tree))
```

```
def map_tree(tree, fn):
    if type(tree) != tuple:
        return fn(tree)
    return tuple(map_tree(branch, fn)
                 for branch in tree)
```

Trees with Internal Node Values

Trees with Internal Node Values

Trees can have values at internal nodes as well as their leaves.

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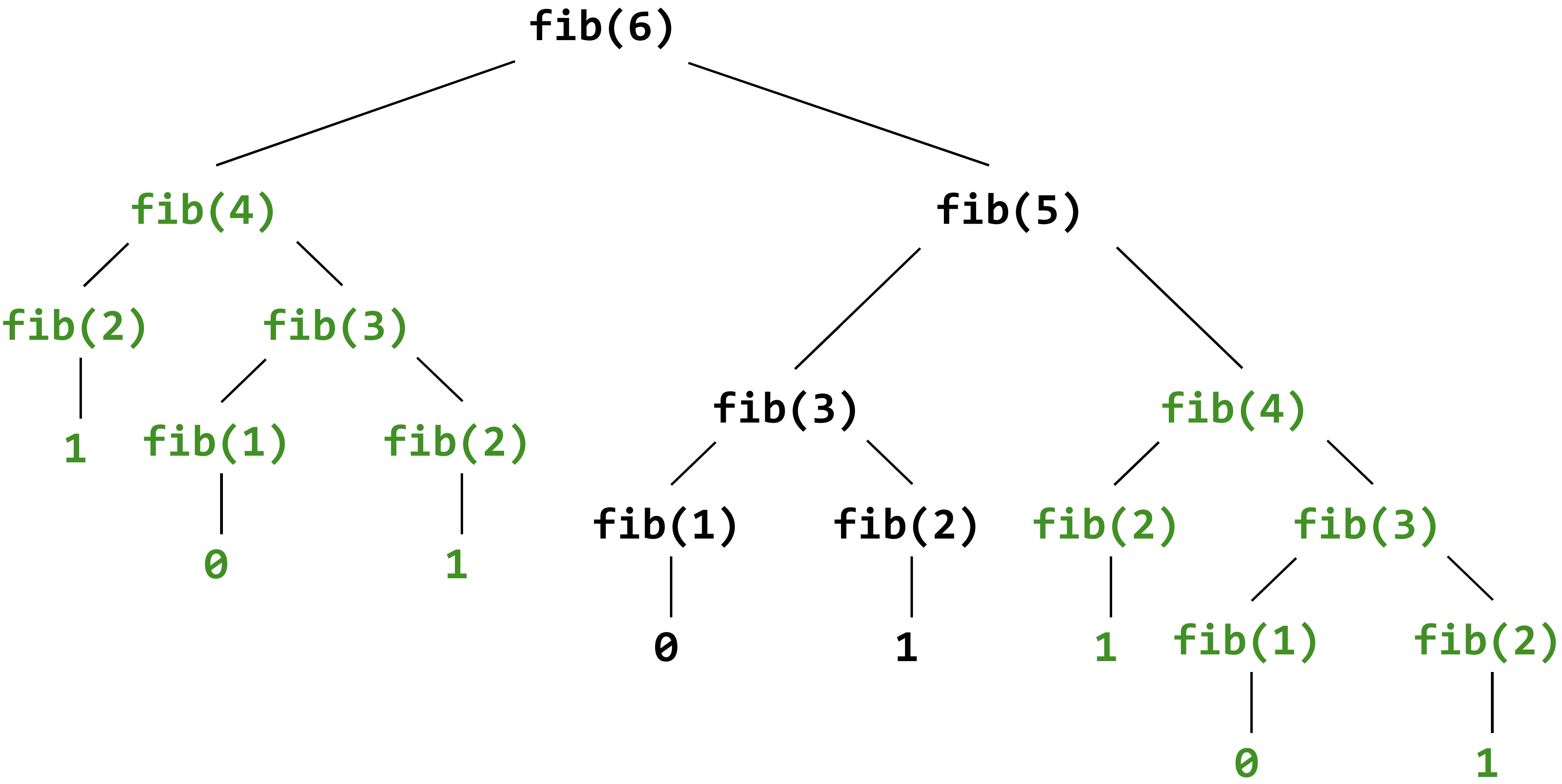

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class Tree(object):
    def __init__(self, entry, left=None, right=None):
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class Tree(object):
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        self.entry = entry
```

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        self.left = left
```

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```
class Tree(object):
    def __init__(self, entry, left=None, right=None):
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        self.right = right
```

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class Tree(object):
    def __init__(self, entry, left=None, right=None):
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def fib_tree(n):
```

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def fib_tree(n):
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class Tree(object):
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def fib_tree(n):
    if n == 1:
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    if n == 2:
```

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def fib_tree(n):
    if n == 1:
        return Tree(0)
    if n == 2:
        return Tree(1)
    left = fib_tree(n - 2)
```

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def fib_tree(n):
    if n == 1:
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    if n == 2:
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    left = fib_tree(n - 2)
    right = fib_tree(n - 1)
    return Tree(left.entry + right.entry, left, right)
```

Memoization

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Tree recursive functions can compute the same thing many times

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Idea: Remember the results that have been computed before

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Keys are arguments that map to return values

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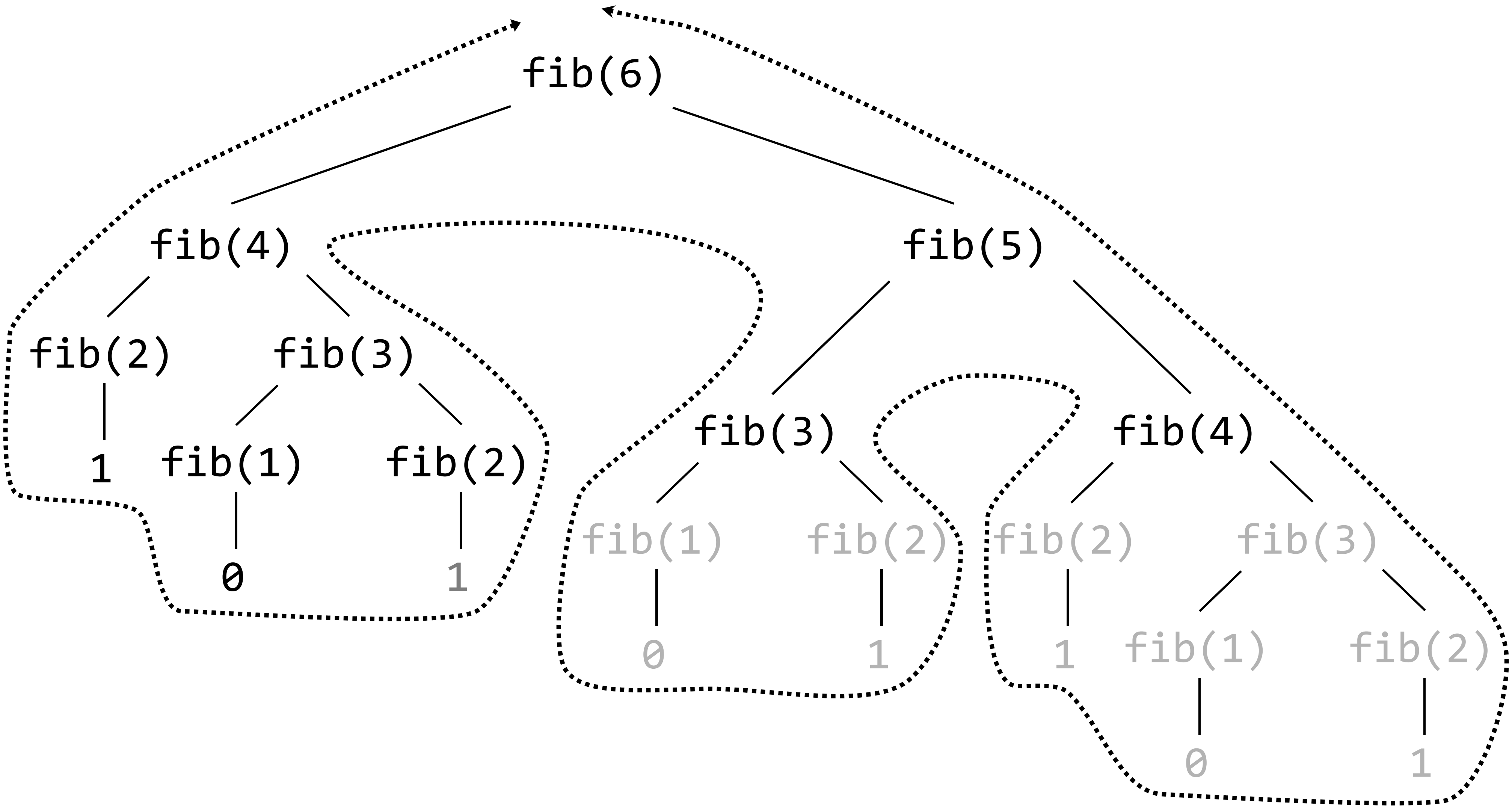
Keys are arguments that map to return values

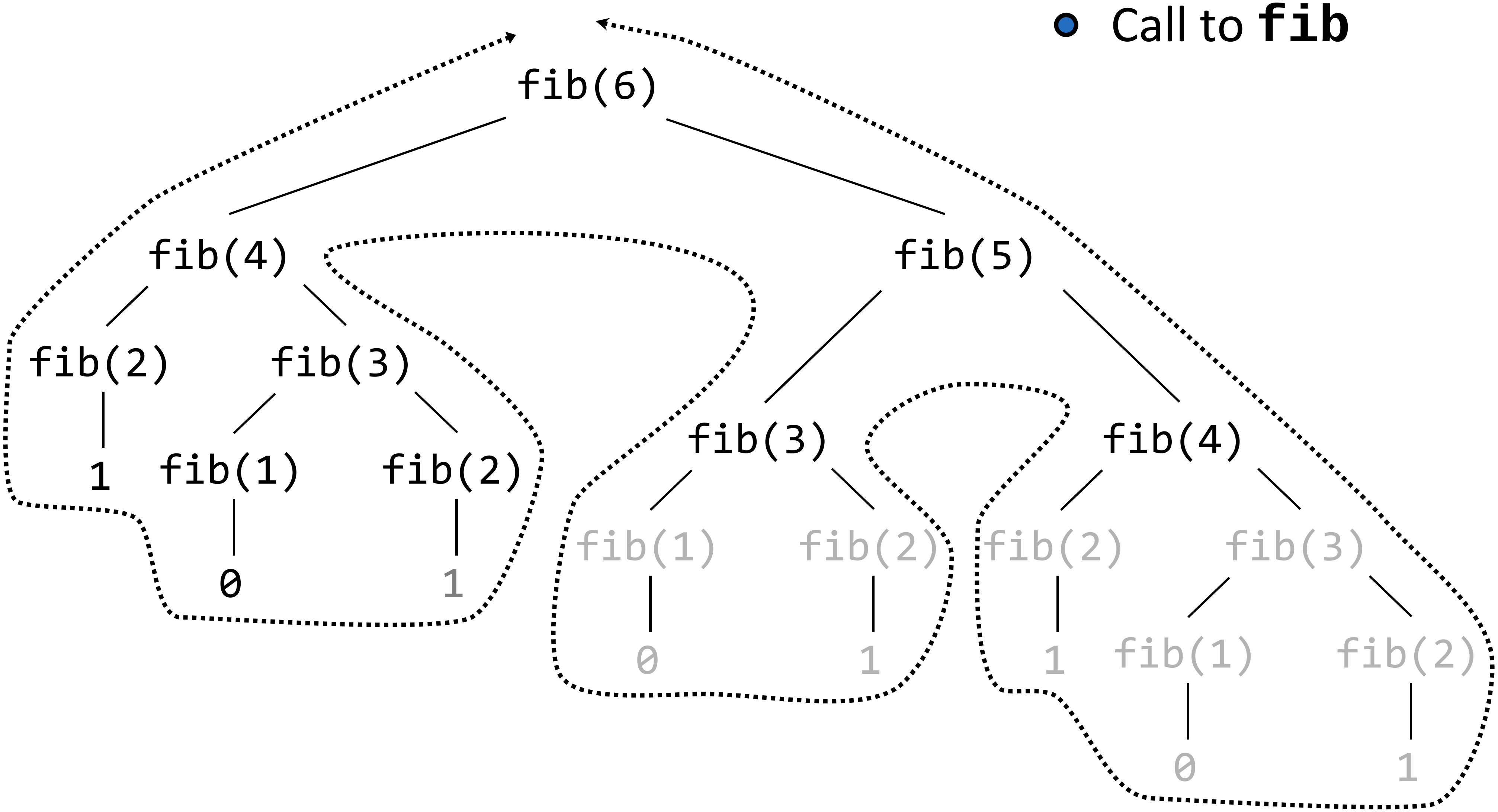
Same behavior as **`f`**, if **`f`** is a pure function

Memoized Tree Recursion



Memoized Tree Recursion

● Call to **`fib`**



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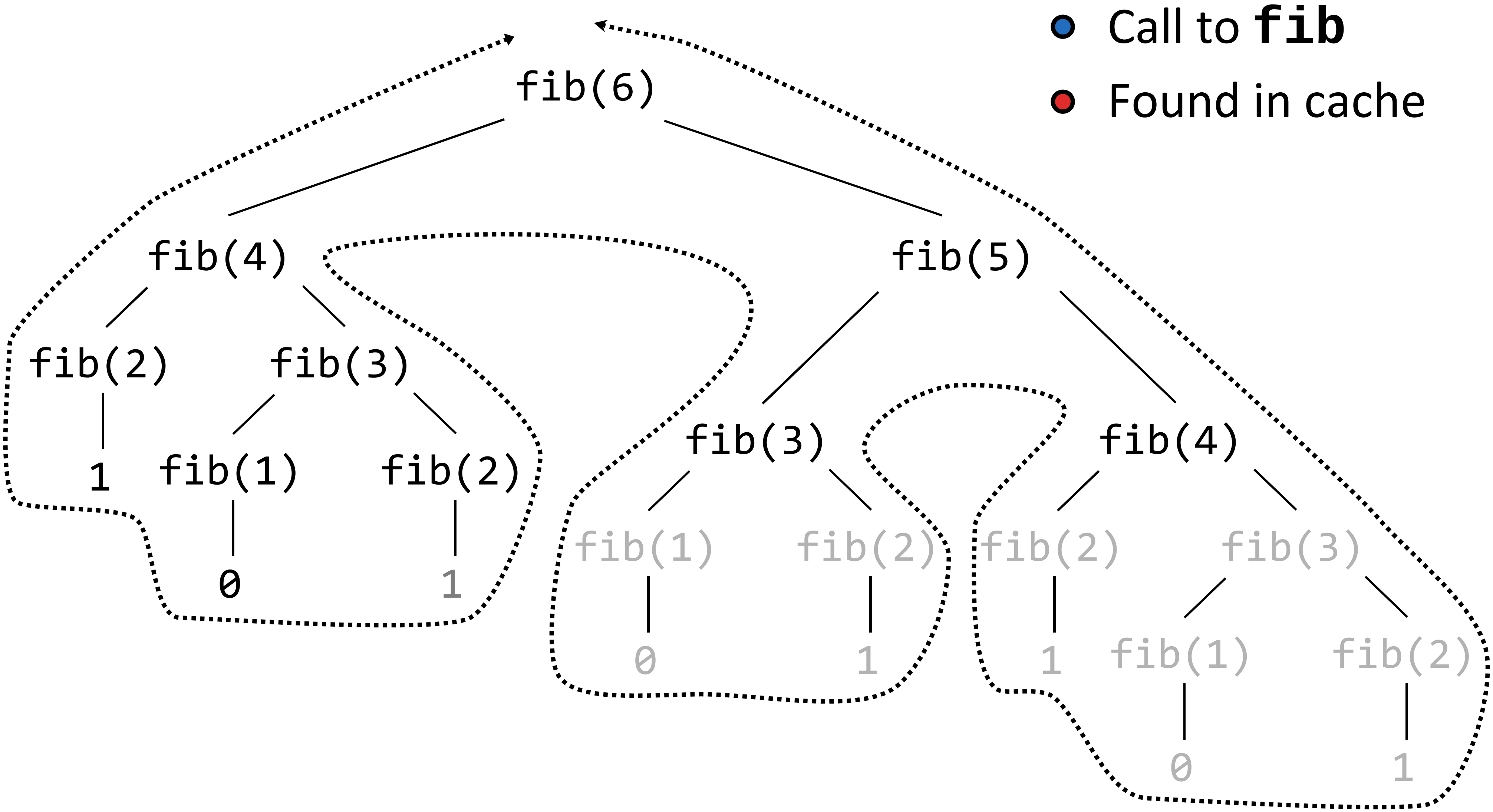
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● Found in cache

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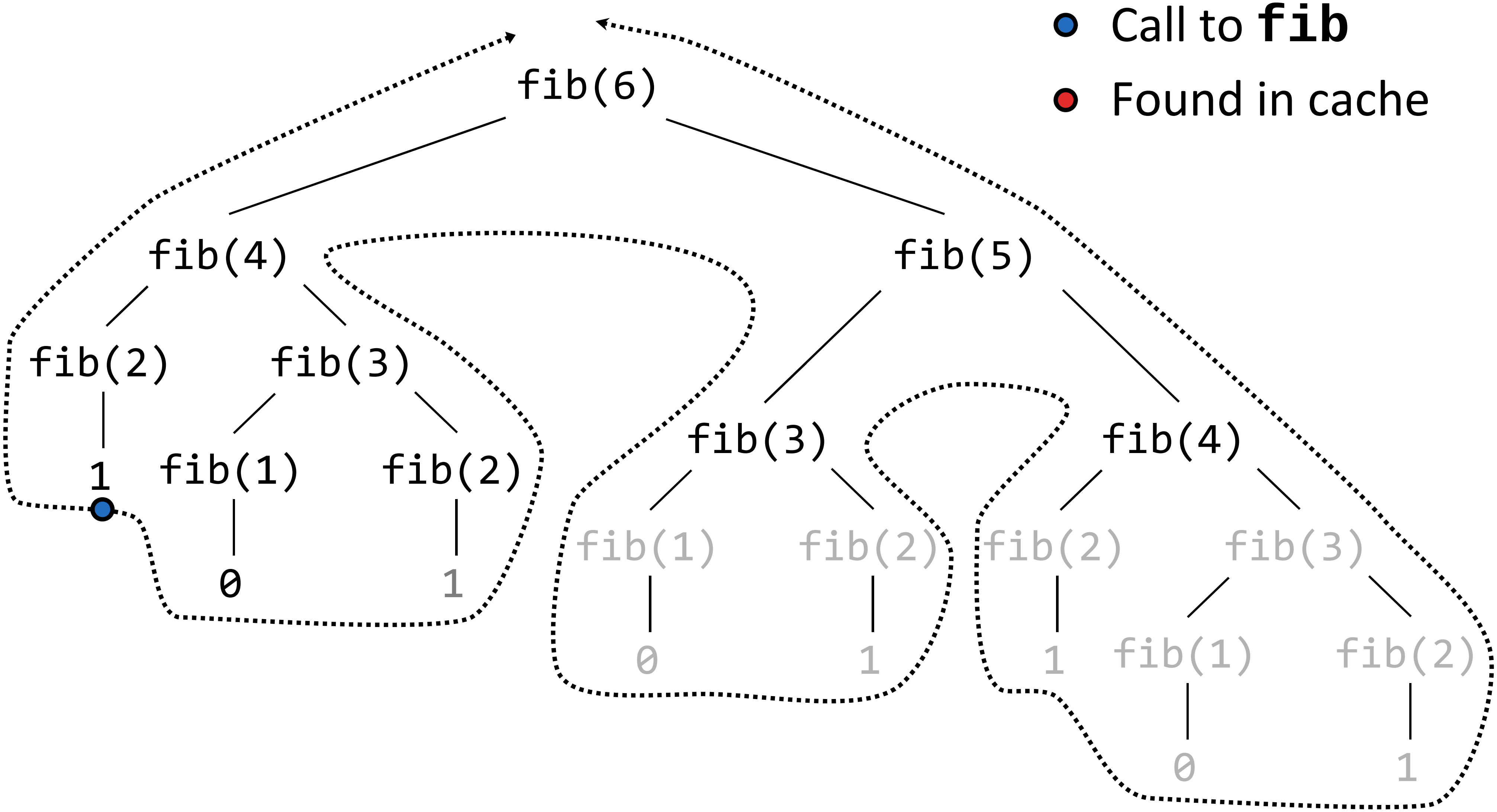

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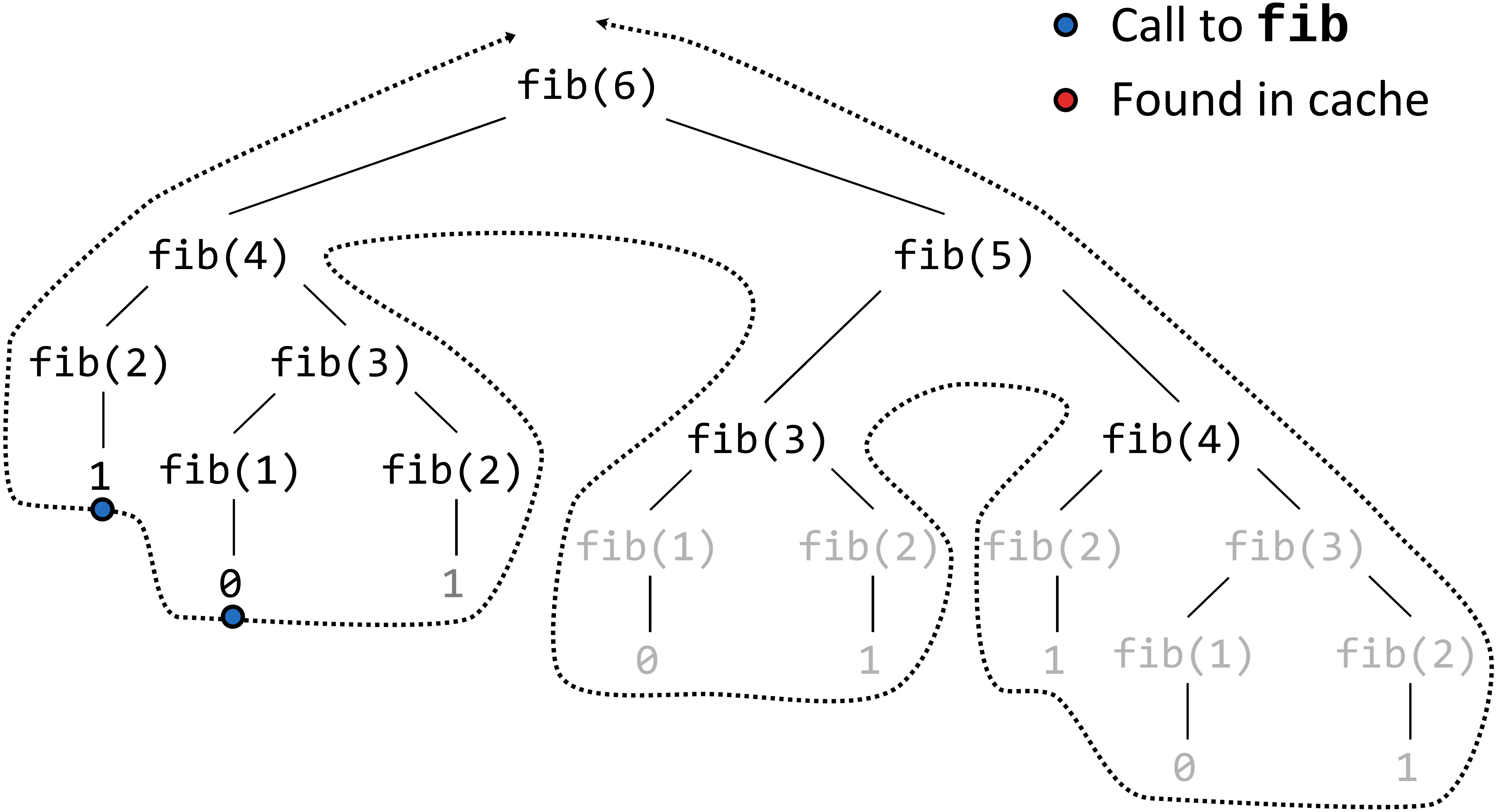

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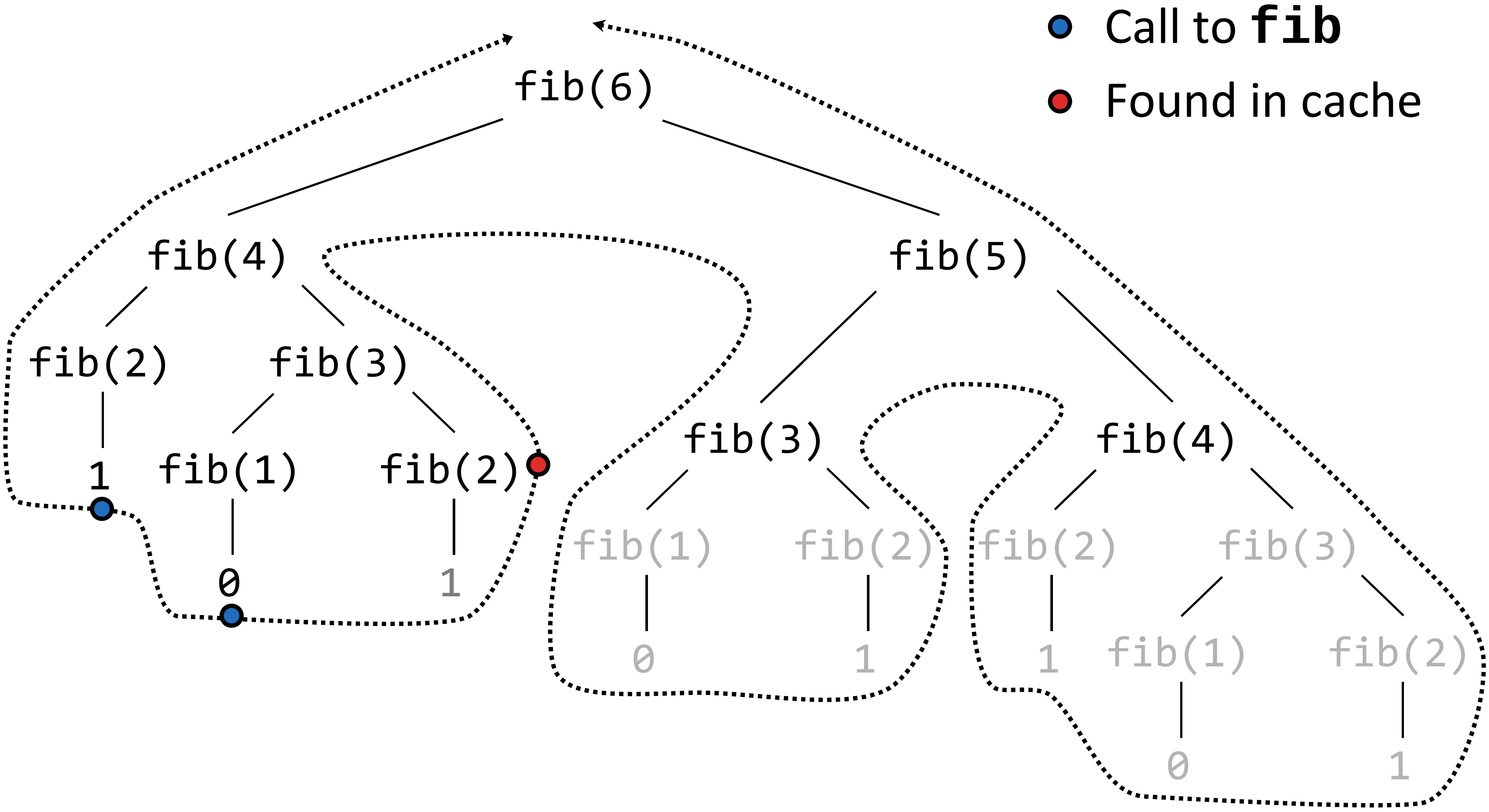

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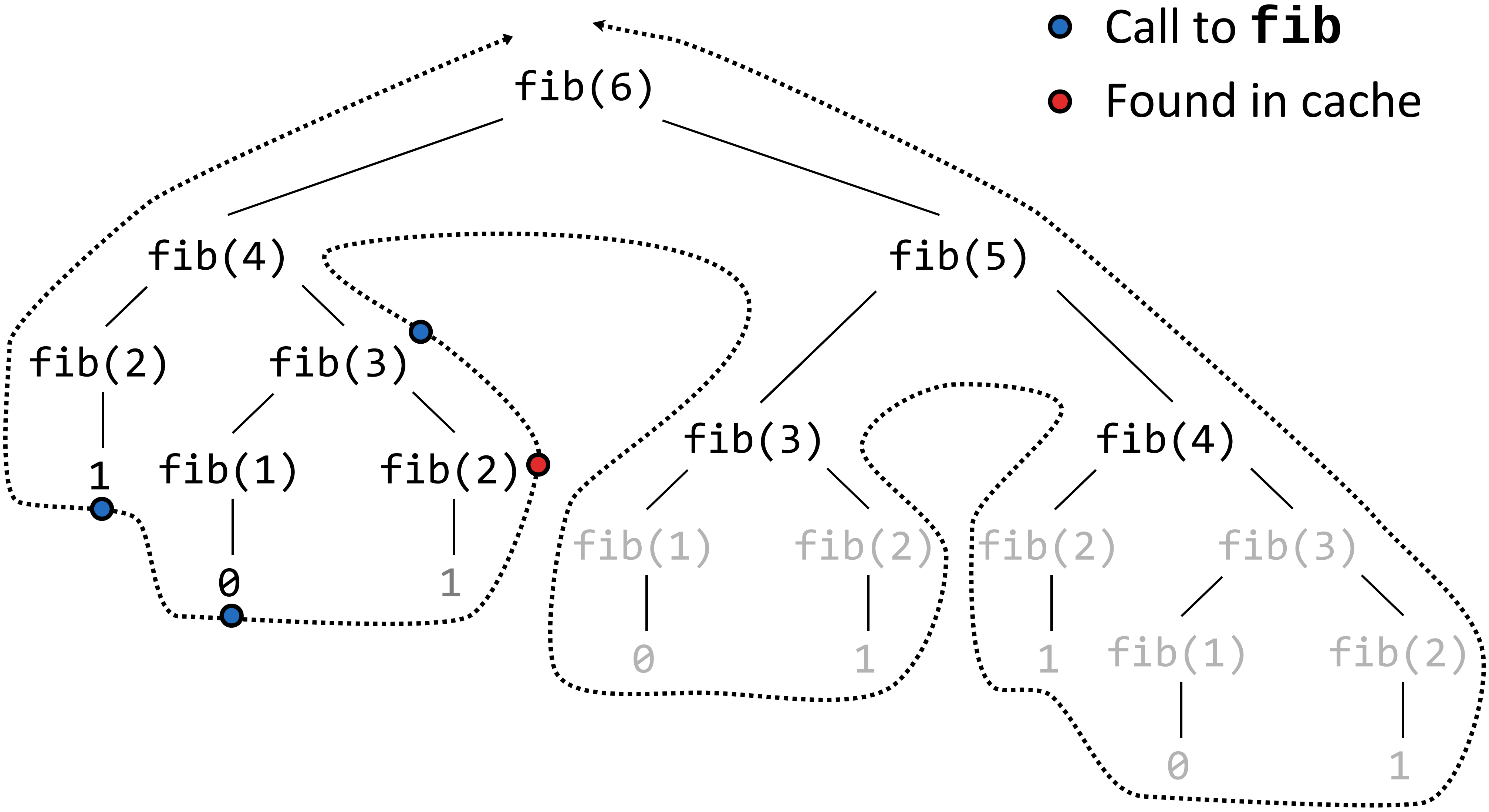

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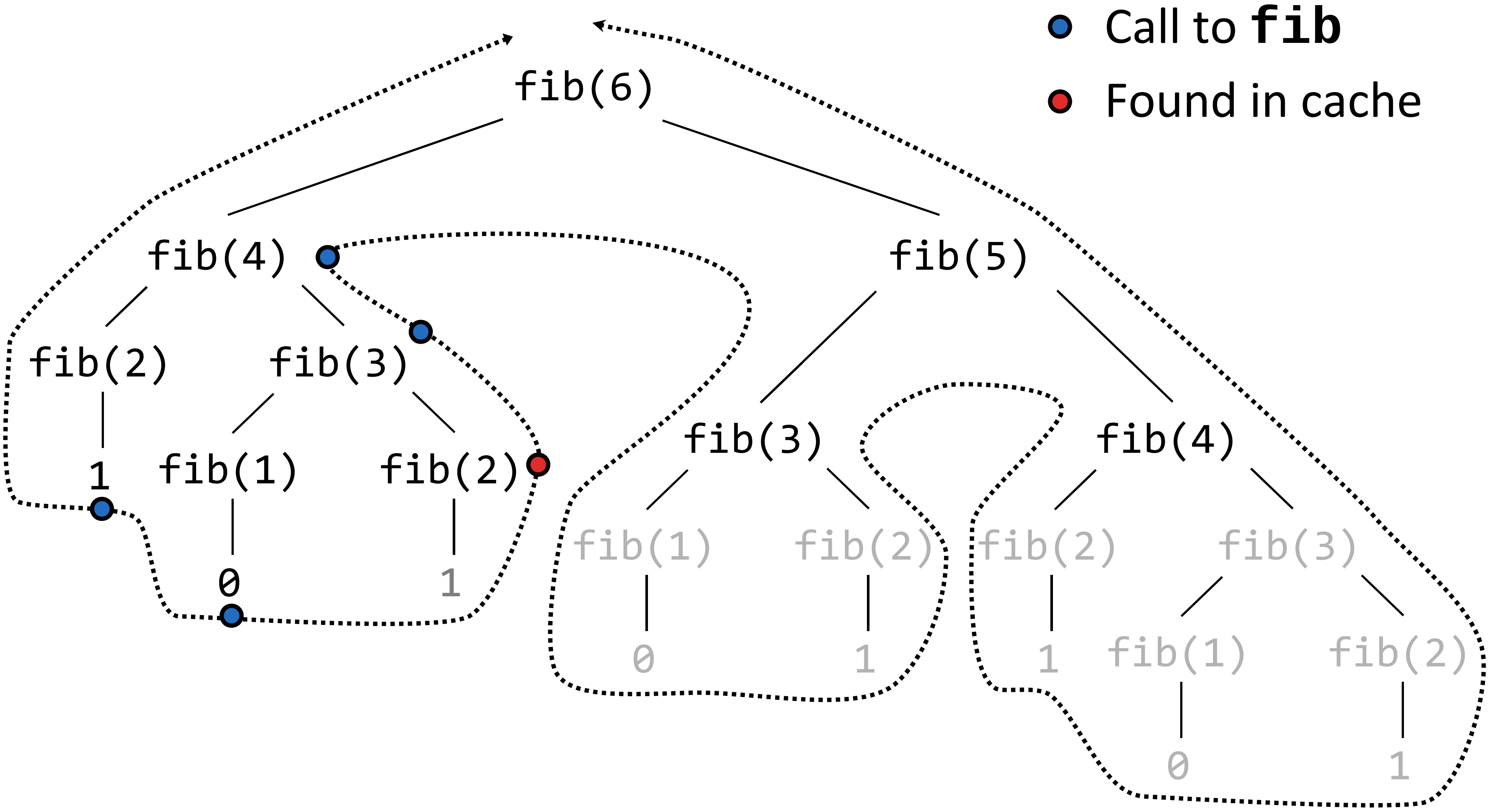

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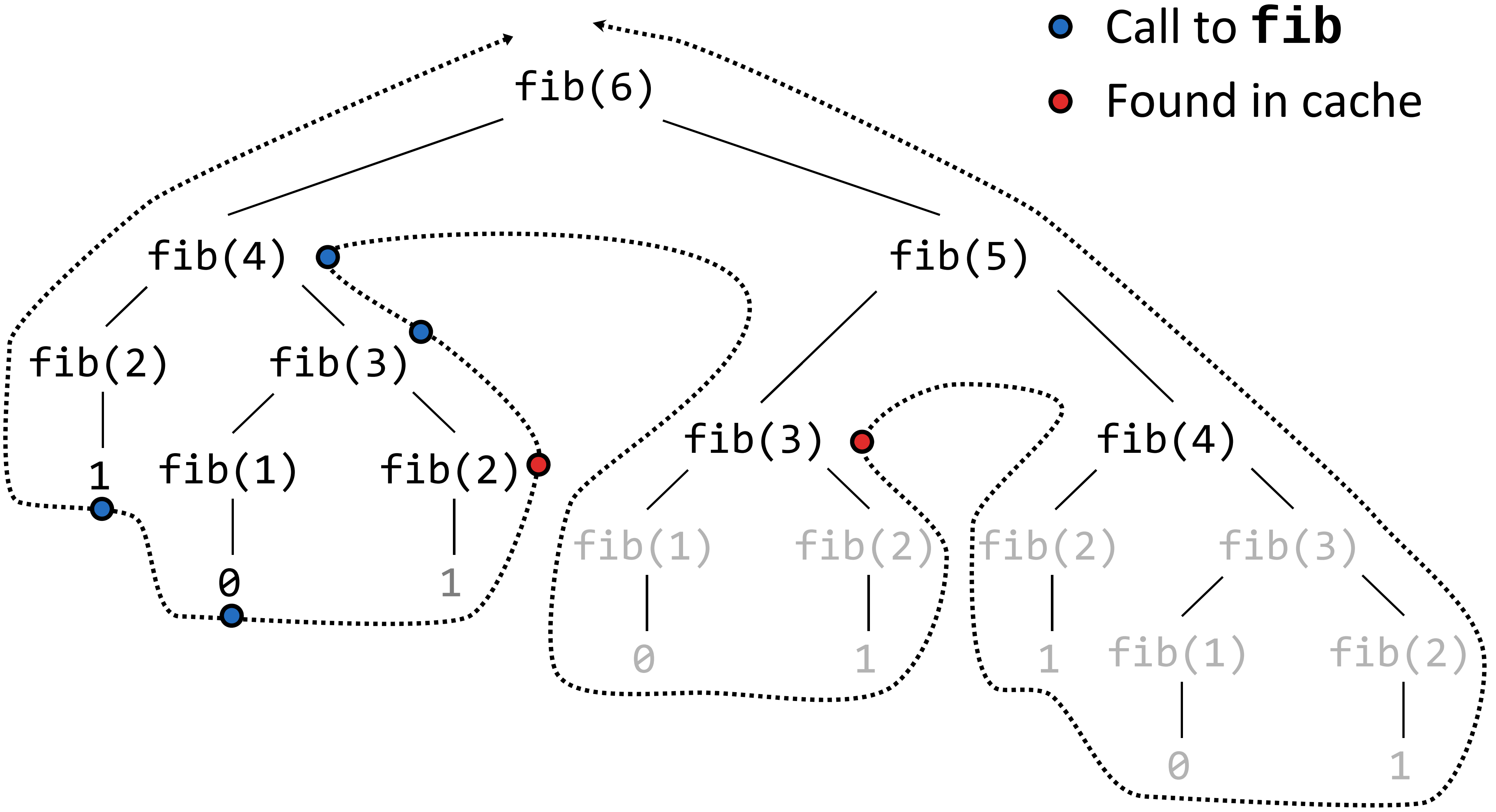

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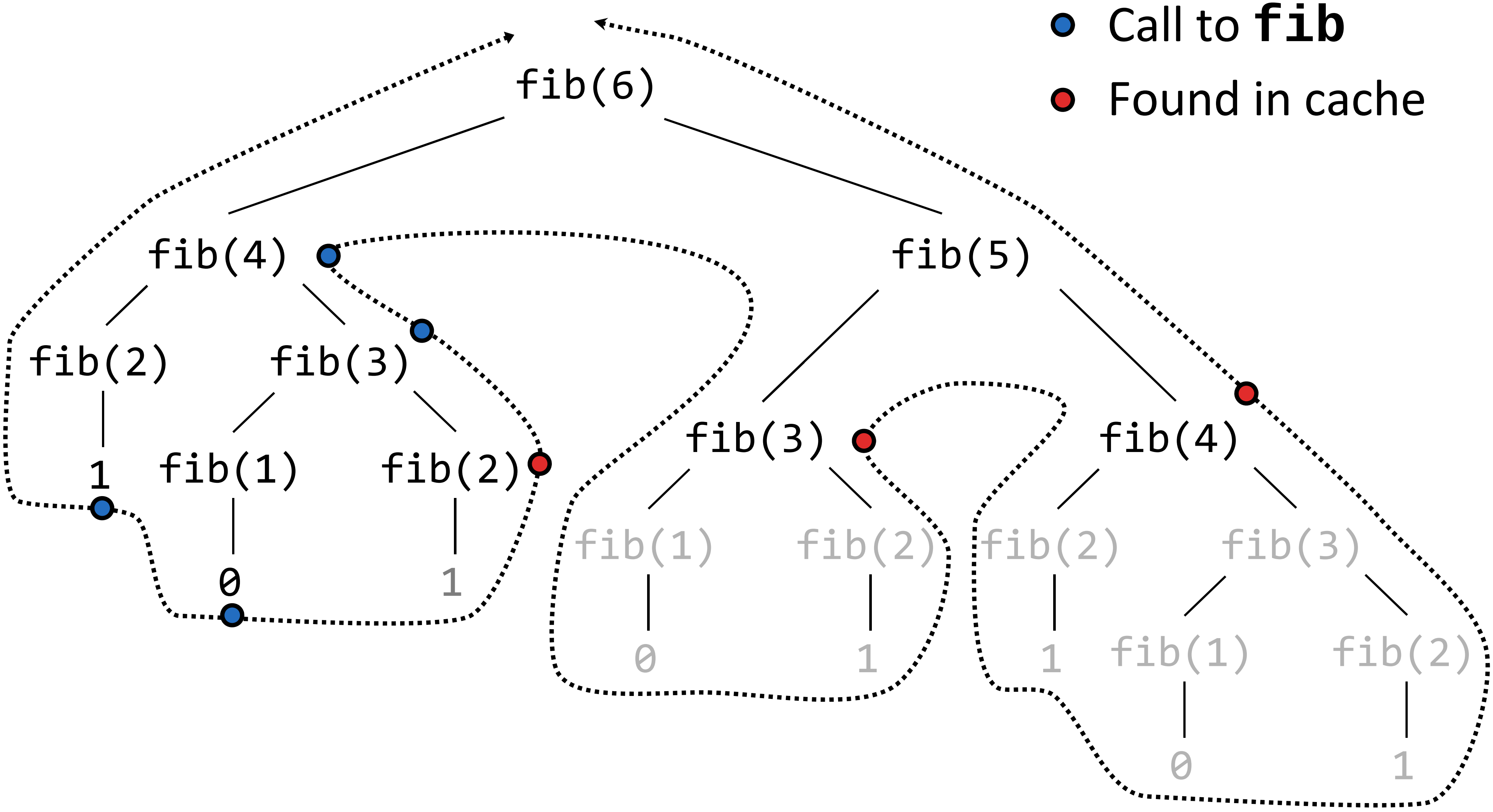

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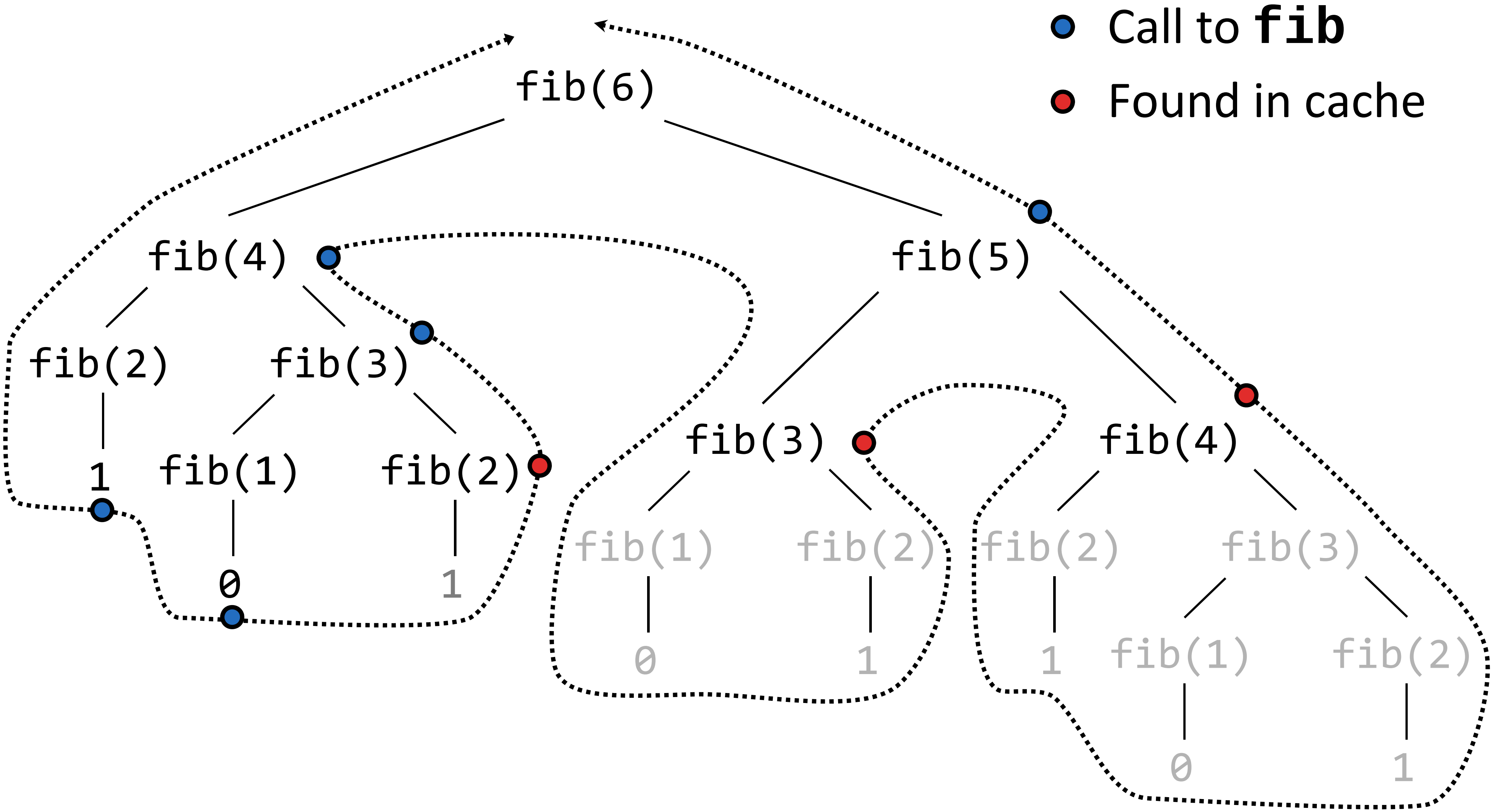

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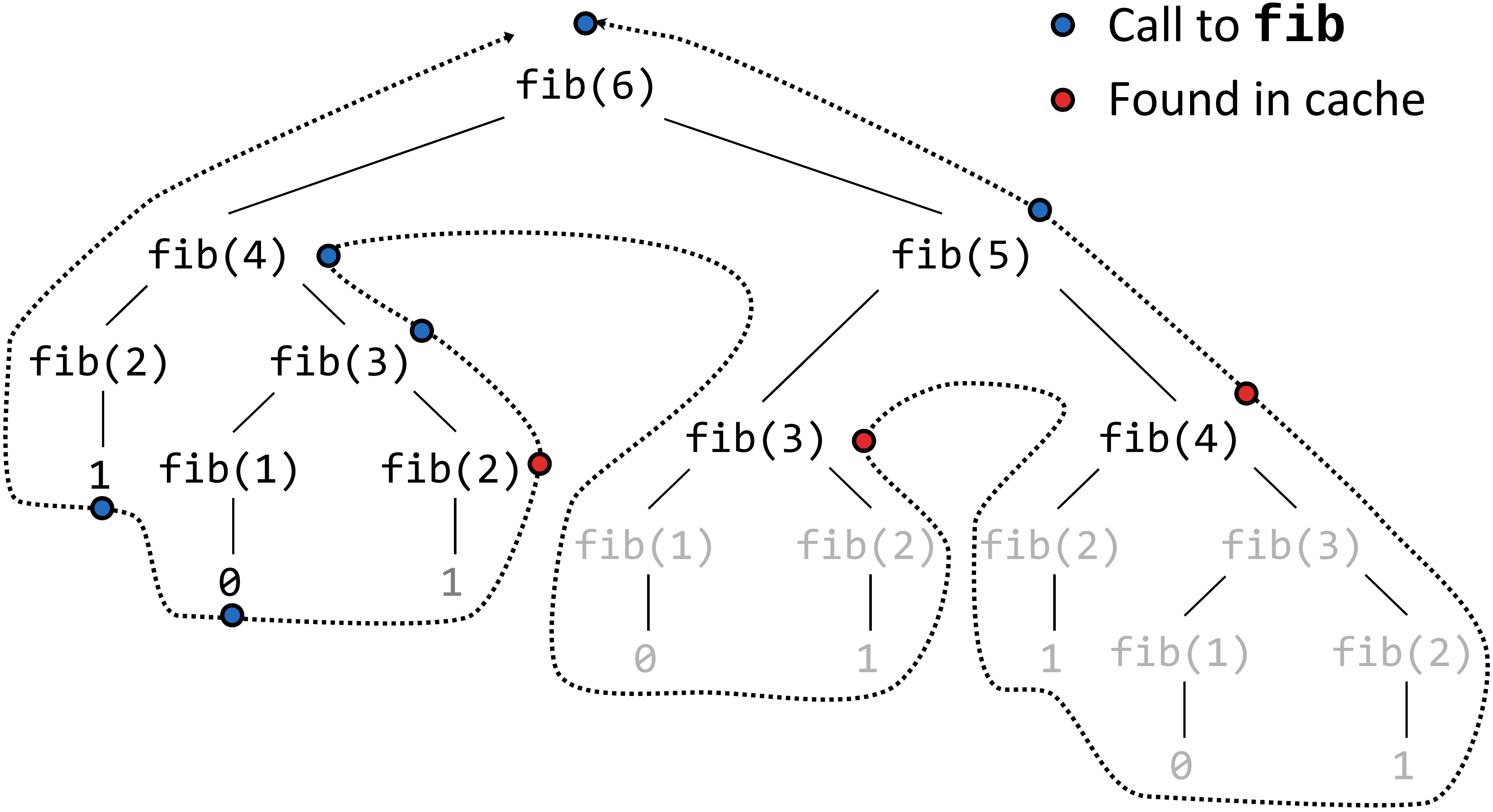

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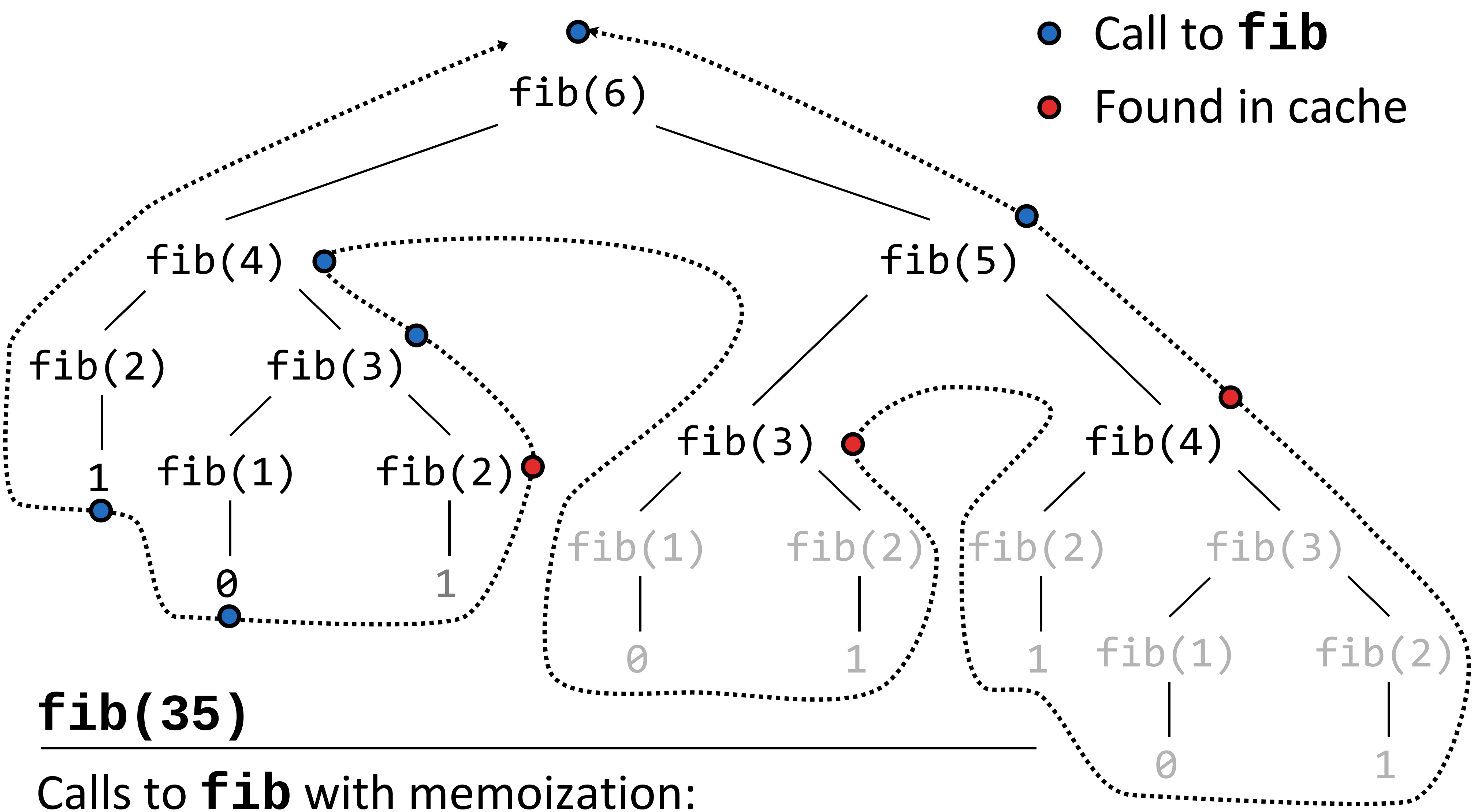
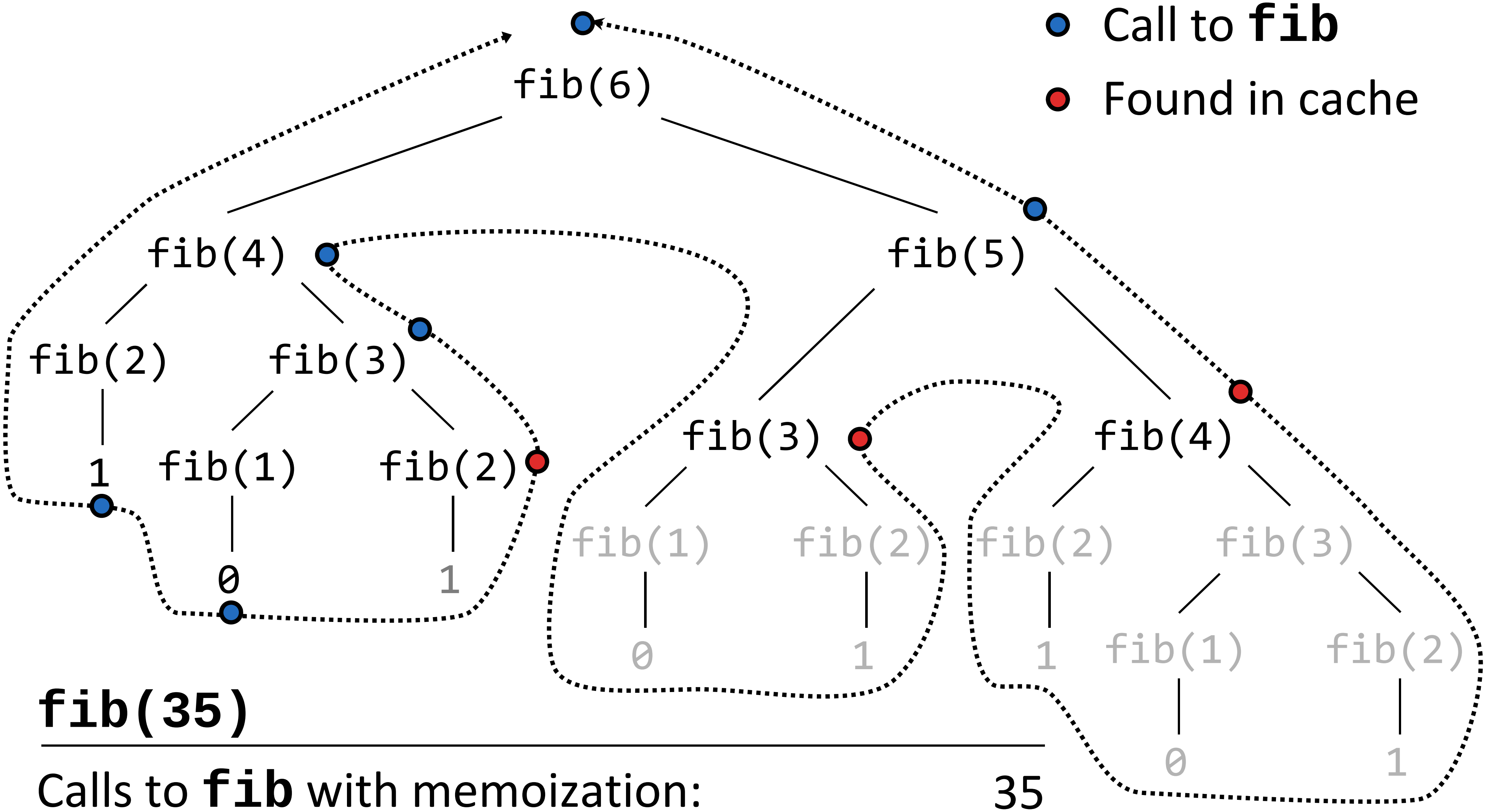


fib(35)

Calls to **fib** with memoization:

Calls to **fib** without memoization:

Memoized Tree Recursion



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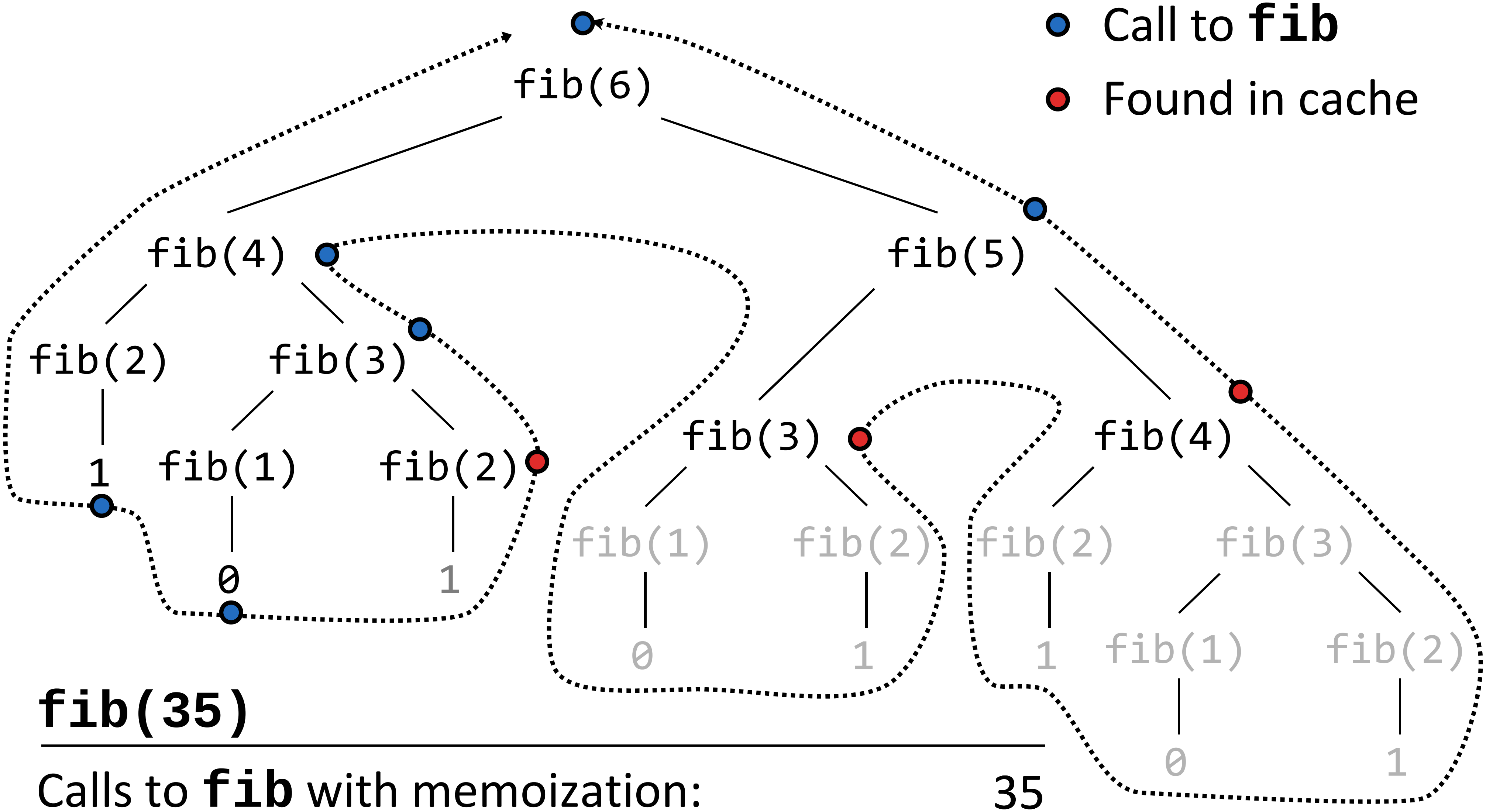
`fib(35)`

Calls to **`fib`** with memoization: 35

Calls to **`fib`** without memoization:

Memoized Tree Recursion



`fib(35)`	
Calls to **`fib`** with memoization:	35
Calls to **`fib`** without memoization:	18,454,929

Orders of Growth

Iterative, recursive, and memoized implementations are not the same.

Time	Space

```
def fib_iter(n):
    prev, curr = 1, 0
    for _ in range(n - 1):
         prev, curr = curr, prev + curr
    return curr

def fib(n):
    if n == 1:
        return 0
    if n == 2:
        return 1
    return fib(n - 2) + fib(n - 1)

fib = memo(fib)
```

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Time	**Space**
$\Theta(n)$	

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Sets

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One more built-in Python container type

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{1, 2, 3, 4}
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>>> 3 in s
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True
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>>> len(s)
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>>> s
{1, 2, 3, 4}
>>> 3 in s
True
>>> len(s)
4
```

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```
>>> s = {3, 2, 1, 4, 4}
>>> s
{1, 2, 3, 4}
>>> 3 in s
True
>>> len(s)
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>>> s.union({1, 5})
```

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>>> s.intersection({6, 5, 4, 3})
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```

Implementing Sets

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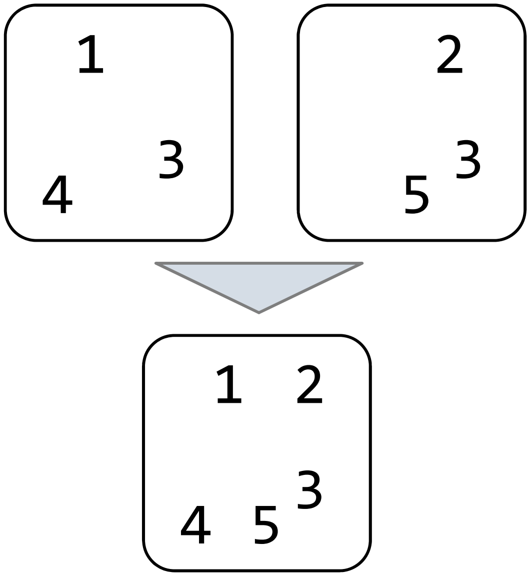

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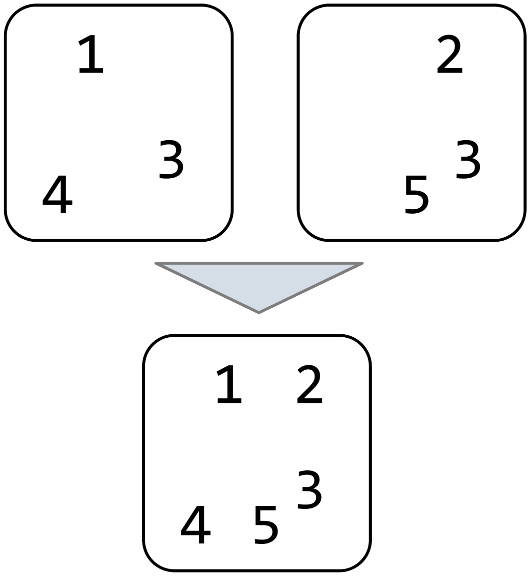
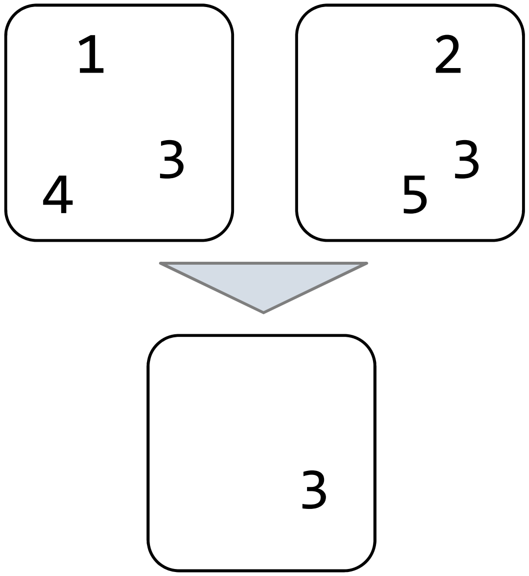


Intersection



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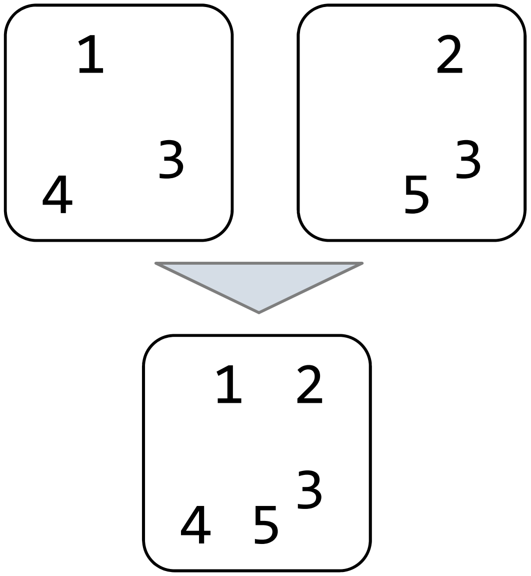
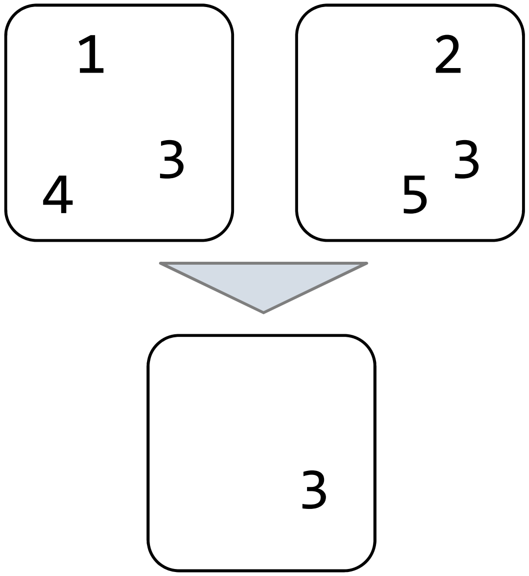
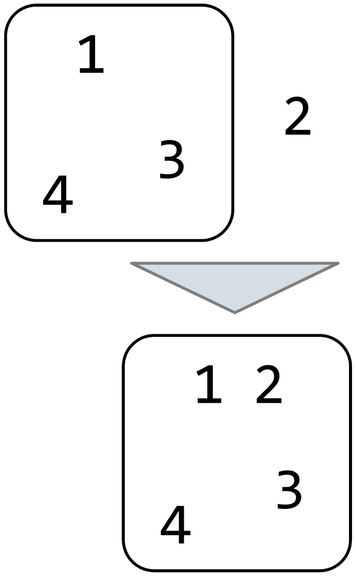


Intersection



Adjunction



Sets as Unordered Sequences

Proposal 1: A set is represented by a recursive list that contains no duplicate items

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This is how we implemented dictionaries

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```
def empty(s):
```

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```
def empty(s):
    return s is Rlist.empty
```

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```
def empty(s):
    return s is Rlist.empty

def set_contains(s, v):
```

Sets as Unordered Sequences

Proposal 1: A set is represented by a recursive list that contains no duplicate items

This is how we implemented dictionaries

```
def empty(s):
    return s is Rlist.empty

def set_contains(s, v):
    if empty(s):
```

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Proposal 1: A set is represented by a recursive list that contains no duplicate items

This is how we implemented dictionaries

```
def empty(s):
    return s is Rlist.empty

def set_contains(s, v):
    if empty(s):
        return False
```

Sets as Unordered Sequences

Proposal 1: A set is represented by a recursive list that contains no duplicate items

This is how we implemented dictionaries

```
def empty(s):
    return s is Rlist.empty

def set_contains(s, v):
    if empty(s):
        return False
    elif s.first == v:
```

Sets as Unordered Sequences

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This is how we implemented dictionaries

```
def empty(s):
    return s is Rlist.empty

def set_contains(s, v):
    if empty(s):
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        return True
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Proposal 1: A set is represented by a recursive list that contains no duplicate items

This is how we implemented dictionaries

```
def empty(s):
    return s is Rlist.empty

def set_contains(s, v):
    if empty(s):
        return False
    elif s.first == v:
        return True
    return set_contains(s.rest, v)
```

Sets as Unordered Sequences

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```
def adjoin_set(s, v):
```

Sets as Unordered Sequences

```
def adjoin_set(s, v):
    if set_contains(s, v):
```

Sets as Unordered Sequences

```
def adjoin_set(s, v):
    if set_contains(s, v):
        return s
```

Sets as Unordered Sequences

```
def adjoin_set(s, v):
    if set_contains(s, v):
        return s
    return Rlist(v, s)
```

Sets as Unordered Sequences

Time order of growth

```
def adjoin_set(s, v):
    if set_contains(s, v):
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def adjoin_set(s, v):
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$$\Theta(n)$$

Sets as Unordered Sequences

Time order of growth

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def adjoin_set(s, v):
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$\Theta(n)$

The size of the set

Sets as Unordered Sequences

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def adjoin_set(s, v):
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The size of the set

```
def intersect_set(set1, set2):
```

Sets as Unordered Sequences

Time order of growth

```
def adjoin_set(s, v):
    if set_contains(s, v):
        return s
    return Rlist(v, s)
```

$\Theta(n)$

The size of the set

```
def intersect_set(set1, set2):
    f = lambda v: set_contains(set2, v)
```

Sets as Unordered Sequences

Time order of growth

$\Theta(n)$

The size of the set

```
def adjoin_set(s, v):
    if set_contains(s, v):
        return s
    return Rlist(v, s)


def intersect_set(set1, set2):
    f = lambda v: set_contains(set2, v)
    return filter_rlist(set1, f)
```

Sets as Unordered Sequences

Time order of growth

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def adjoin_set(s, v):
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Assume sets are the same size

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def intersect_set(set1, set2):
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Assume sets are the same size

```
def union_set(set1, set2):
```

Sets as Unordered Sequences

Time order of growth

```
def adjoin_set(s, v):
    if set_contains(s, v):
        return s
    return Rlist(v, s)
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$\Theta(n)$

The size of the set

```
def intersect_set(set1, set2):
    f = lambda v: set_contains(set2, v)
    return filter_rlist(set1, f)
```

$\Theta(n^2)$

Assume sets are the same size

```
def union_set(set1, set2):
    f = lambda v: not set_contains(set2, v)
```

Sets as Unordered Sequences

Time order of growth

```
def adjoin_set(s, v):
    if set_contains(s, v):
        return s
    return Rlist(v, s)
```

$\Theta(n)$

The size of the set

```
def intersect_set(set1, set2):
    f = lambda v: set_contains(set2, v)
    return filter_rlist(set1, f)
```

$\Theta(n^2)$

Assume sets are the same size

```
def union_set(set1, set2):
    f = lambda v: not set_contains(set2, v)
    set1_not_set2 = filter_rlist(set1, f)
```

Sets as Unordered Sequences

Time order of growth

```
def adjoin_set(s, v):
    if set_contains(s, v):
        return s
    return Rlist(v, s)
```

$\Theta(n)$

The size of the set

```
def intersect_set(set1, set2):
    f = lambda v: set_contains(set2, v)
    return filter_rlist(set1, f)
```

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Assume sets are the same size

```
def union_set(set1, set2):
    f = lambda v: not set_contains(set2, v)
    set1_not_set2 = filter_rlist(set1, f)
    return extend_rlist(set1_not_set2, set2)
```

Sets as Unordered Sequences

Time order of growth

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def adjoin_set(s, v):
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