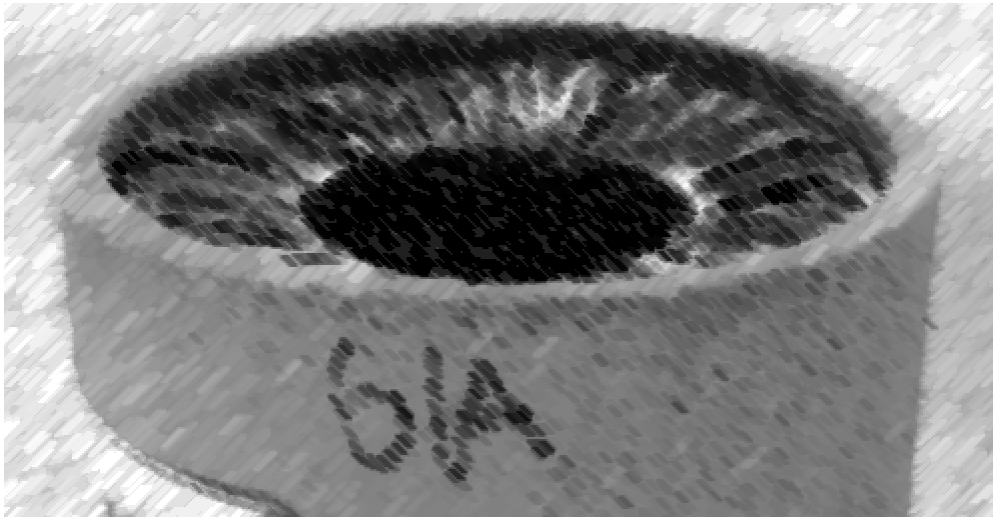

CS61A Lecture 23

Amir Kamil

UC Berkeley

March 15, 2013

Announcements

- Ants project due Monday
- HW8 due next Wednesday at 7pm
- Midterm 2 next Thursday at 7pm
  - Review session Sat. 3/16 at 2pm in 2050 VLSB
  - Office hours Sun. 3/17 12-4pm in 310 Soda
  - HKN review session Sun. 3/17 at 4pm in 145 Dwinelle
  - See course website for more information

The Independence of Data Types

Data abstraction and class definitions keep types separate

Some operations need to cross type boundaries

How do we add a complex number and a rational number together?

`add_rational  mul_rational`

Rational numbers as numerators & denominators

`add_complex  mul_complex`

Complex numbers as two-dimensional vectors

There are many different techniques for doing this!

Type Dispatching

Define a different function for each possible combination of types for which an operation (e.g., addition) is valid

```
def iscomplex(z):
    return type(z) in (ComplexRI, ComplexMA)
def isrational(z):
    return type(z) is Rational
def add_complex_and_rational(z, r):
    return ComplexRI(z.real + r.numerator / r.denominator,
                     z.imag)
def add_by_type_dispatching(z1, z2):
    """Add z1 and z2, which may be complex or rational."""
    if iscomplex(z1) and iscomplex(z2):
        return add_complex(z1, z2)
    elif iscomplex(z1) and isrational(z2):
        return add_complex_and_rational(z1, z2)
    elif isrational(z1) and iscomplex(z2):
        return add_complex_and_rational(z2, z1)
    else:
        add_rational(z1, z2)
```

Converted to a real number (float)

Tag-Based Type Dispatching

Idea: Use dictionaries to dispatch on type (like we did for message passing)

```
def type_tag(x):
    return type_tags[type(x)]

type_tags = {ComplexRI:  'com',
             ComplexMA:  'com',
             Rational:   'rat'}
```

Declares that **ComplexRI** and **ComplexMA** should be treated uniformly

```
def add(z1, z2):
    types = (type_tag(z1), type_tag(z2))
    return add_implementations[types](z1, z2)

add_implementations = {}
add_implementations[('com', 'com')] = add_complex
add_implementations[('rat', 'rat')] = add_rational
add_implementations[('com', 'rat')] = add_complex_and_rational
add_implementations[('rat', 'com')] = add_rational_and_complex
```

```
lambda r, z: add_complex_and_rational(z, r)
```

Type Dispatching Analysis

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integer, rational, real, complex

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integer, rational, real, complex

$$m \cdot (m-1) \cdot n$$

add, subtract, multiply, divide

$$4 \cdot (4-1) \cdot 4 = 48$$

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Type Dispatching

Message Passing

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Idea: One dispatch function for (operator, types) pairs

```
def apply(operator_name, x, y):
    tags = (type_tag(x), type_tag(y))
    key = (operator_name, tags)
    return apply_implementations[key](x, y)
```

Data-Directed Programming

There's nothing addition-specific about **add**

Idea: One dispatch function for (operator, types) pairs

```
def apply(operator_name, x, y):
    tags = (type_tag(x), type_tag(y))
    key = (operator_name, tags)
    return apply_implementations[key](x, y)

apply_implementations = {
    ('add', ('com', 'com')): add_complex,
    ('add', ('rat', 'rat')): add_rational,
    ('add', ('com', 'rat')): add_complex_and_rational,
    ('add', ('rat', 'com')): add_rational_and_complex,
    ('mul', ('com', 'com')): mul_complex,
    ('mul', ('rat', 'rat')): mul_rational,
    ('mul', ('com', 'rat')): mul_complex_and_rational,
    ('mul', ('rat', 'com')): mul_rational_and_complex
    }
```

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Idea: Some types can be converted into other types

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```
def rational_to_complex(x):
```

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```
def rational_to_complex(x):
    return ComplexRI(x.numerator / x.denominator, 0)
```

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Takes advantage of structure in the type system

```
def rational_to_complex(x):
    return ComplexRI(x.numerator / x.denominator, 0)

coercions = {('rat', 'com'): rational_to_complex}
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```
def rational_to_complex(x):
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Question: Can any numeric type be coerced into any other?

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```
def rational_to_complex(x):
    return ComplexRI(x.numerator / x.denominator, 0)

coercions = {('rat', 'com'): rational_to_complex}
```

Question: Can any numeric type be coerced into any other?

Question: Have we been repeating ourselves with data-directed programming?

Applying Operators with Coercion

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def coerce_apply(operator_name, x, y):
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```
def coerce_apply(operator_name, x, y):
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    if tx != ty:
```

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def coerce_apply(operator_name, x, y):
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        else:
            return 'No coercion possible.'
    assert tx == ty
    key = (operator_name, tx)
    return coerce_implementations[key](x, y)
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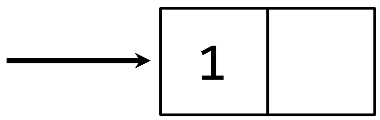

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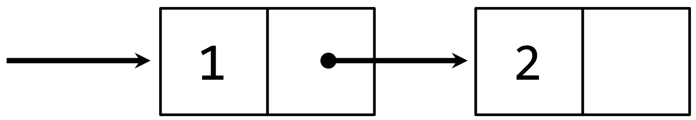

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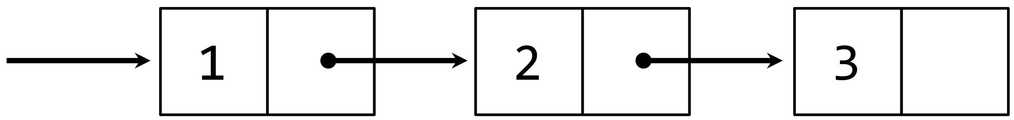

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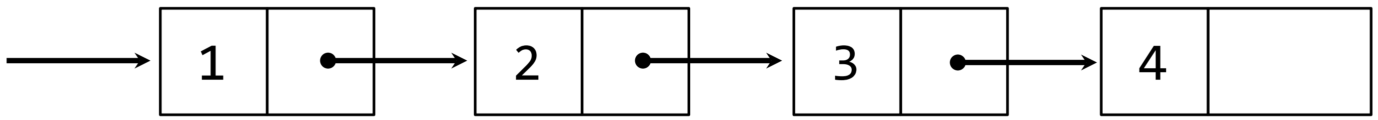

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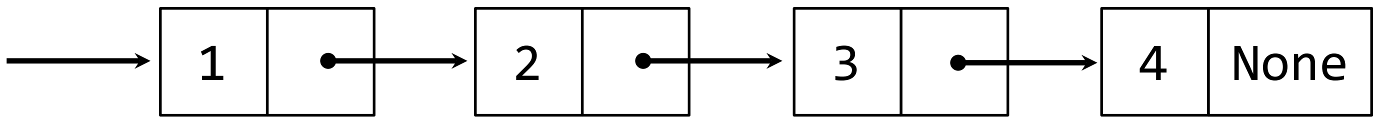

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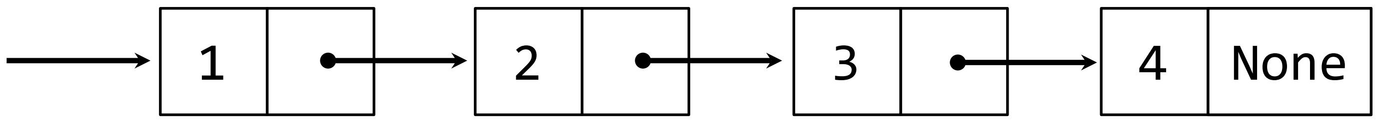

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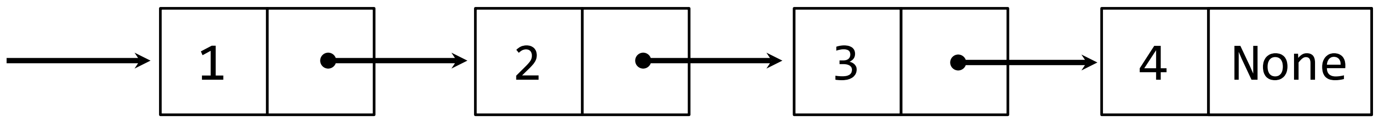

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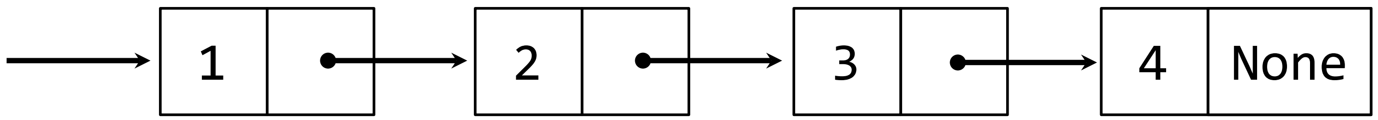

Recursive lists are recursive: the rest of the list is a list.

Nested pairs (old): `(1, (2, (3, (4, None))))`

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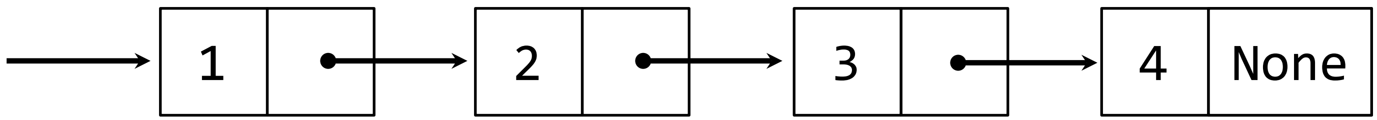

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Rlist class (new):

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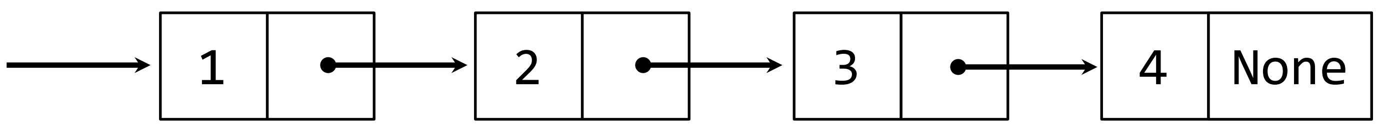

Recursive lists are recursive: the rest of the list is a list.

Nested pairs (old): `(1, (2, (3, (4, None))))`

Rlist class (new): `Rlist(1, Rlist(2, Rlist(3, Rlist(4))))`

Recursive List Class

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```
class Rlist(object):
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```
class Rlist(object):
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Recursive List Class

```
class Rlist(object):
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        def __len__(self):
```

Recursive List Class

```
class Rlist(object):
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        def __len__(self):
            return 0
```

Recursive List Class

```
class Rlist(object):
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        def __len__(self):
            return 0
    empty = EmptyList()
```

Recursive List Class

```
class Rlist(object):
    class EmptyList(object):
        def __len__(self):
            return 0
    empty = EmptyList()
    def __init__(self, first, rest=empty):
```

Recursive List Class

```
class Rlist(object):
    class EmptyList(object):
        def __len__(self):
            return 0
    empty = EmptyList()
    def __init__(self, first, rest=empty):
        self.first = first
```

Recursive List Class

```
class Rlist(object):
    class EmptyList(object):
        def __len__(self):
            return 0
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    def __init__(self, first, rest=empty):
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        self.rest = rest
```

Recursive List Class

```
class Rlist(object):
    class EmptyList(object):
        def __len__(self):
            return 0
    empty = EmptyList()
    def __init__(self, first, rest=empty):
        self.first = first
        self.rest = rest
    def __len__(self):
```

Recursive List Class

```
class Rlist(object):
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        def __len__(self):
            return 0
    empty = EmptyList()
    def __init__(self, first, rest=empty):
        self.first = first
        self.rest = rest
    def __len__(self):
        return 1 + len(self.rest)
```

Recursive List Class

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class Rlist(object):
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            return 0
    empty = EmptyList()
    def __init__(self, first, rest=empty):
        self.first = first
        self.rest = rest
    def __len__(self):
        return 1 + len(self.rest)
    def __getitem__(self, i):
```

Recursive List Class

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class Rlist(object):
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        def __len__(self):
            return 0
    empty = EmptyList()
    def __init__(self, first, rest=empty):
        self.first = first
        self.rest = rest
    def __len__(self):
        return 1 + len(self.rest)
    def __getitem__(self, i):
        if i == 0:
```

Recursive List Class

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class Rlist(object):
    class EmptyList(object):
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Methods can be recursive as well!

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    def __getitem__(self, i):
        if i == 0:
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        return self.rest[i - 1]
```

There's the base case!

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Recursive Operations on Rlists

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```

```
def extend_rlist(s1, s2):
    if s1 is Rlist.empty:
```

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Rlist(2, Rlist(3, Rlist(1, Rlist(2, Rlist(3)))))
```

```
def extend_rlist(s1, s2):
    if s1 is Rlist.empty:
        return s2
```

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Rlist(2, Rlist(3, Rlist(1, Rlist(2, Rlist(3)))))
```

```
def extend_rlist(s1, s2):
    if s1 is Rlist.empty:
        return s2
    return Rlist(s1.first, extend_rlist(s1.rest, s2))
```

Map and Filter on Rlists

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We want operations on a whole list, not an element at a time.

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    if s is Rlist.empty:
        return s
    return Rlist(fn(s.first), map_rlist(s.rest, fn))
```

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def map_rlist(s, fn):
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    if s is Rlist.empty:
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    rest = filter_rlist(s.rest, fn)
    if fn(s.first):
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    if fn(s.first):
        return Rlist(s.first, rest)
    return rest
```