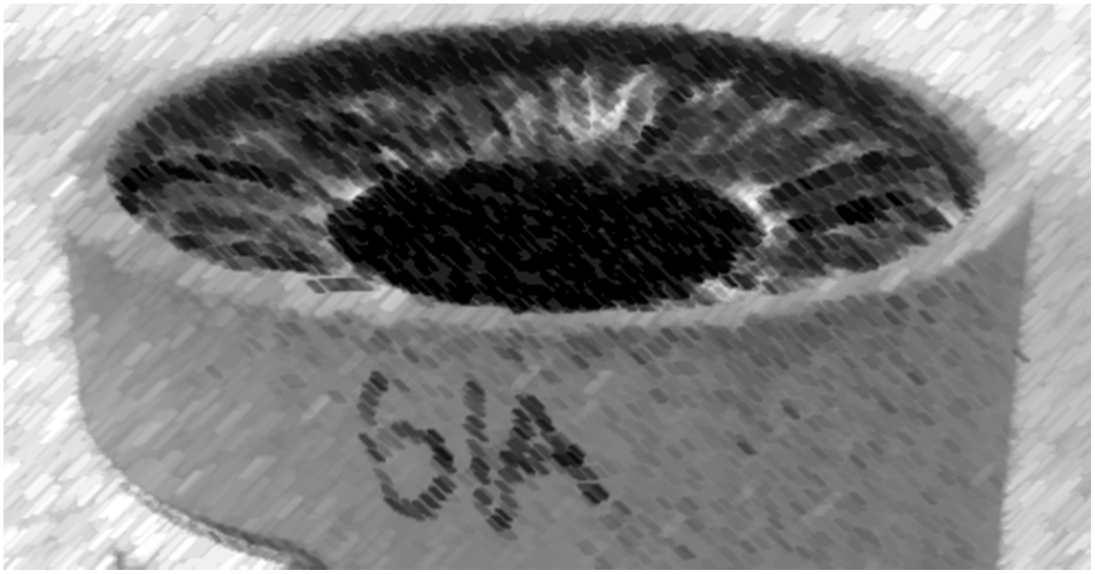

CS61A Lecture 22

Amir Kamil

UC Berkeley

March 13, 2013

Announcements

- HW7 due tonight
- Ants project due Monday
- HW8 due next Wednesday at 7pm
- Midterm 2 next Thursday at 7pm

Interfaces

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Message passing allows **different data types** to respond to the **same message**.

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In languages like Python and Ruby, interfaces are implicitly implemented by providing the right methods with the correct behavior

- *If it quacks like a duck…*

Other languages require interfaces to be explicitly implemented

Example: Rational Numbers

```
class Rational(object):
    def __init__(self, numer, denom):
        g = gcd(numer, denom)
        self.numerator = numer // g
        self.denominator = denom // g
    def __repr__(self):
        return 'Rational({0}, {1})'.format(self.numerator,
                                           self.denominator)
    def __str__(self):
        return '{0}/{1}'.format(self.numerator,
                                self.denominator)
    def __add__(self, num):
        return add_rational(self, num)
    def __mul__(self, num):
        return mul_rational(self, num)
    def __eq__(self, num):
        return eq_rational(self, num)
```

Property Methods

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```
>>> f = Rational(3, 5)
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Often, we want the value of instance attributes to be linked.

```
>>> f = Rational(3, 5)
>>> f.float_value
0.6
>>> f.numerator = 4
>>> f.float_value
0.8
```

Property Methods

Often, we want the value of instance attributes to be linked.

```
>>> f = Rational(3, 5)
>>> f.float_value
0.6
>>> f.numerator = 4
>>> f.float_value
0.8
>>> f.denominator -= 3
```

Property Methods

Often, we want the value of instance attributes to be linked.

```
>>> f = Rational(3, 5)
>>> f.float_value
0.6
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Property Methods

Often, we want the value of instance attributes to be linked.

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>>> f = Rational(3, 5)
>>> f.float_value
0.6
>>> f.numerator = 4
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0.8
>>> f.denominator -= 3
>>> f.float_value
2.0
```

Property Methods

Often, we want the value of instance attributes to be linked.

```
>>> f = Rational(3, 5)
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0.6
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The **`@property`** decorator on a method designates that it will be called whenever it is *looked up* on an instance.

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```

```
@property
def float_value(self):
    return (self.numerator //
            self.denominator)
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>>> f.denominator -= 3
>>> f.float_value
2.0
```

```
@property
def float_value(self):
    return (self.numerator //
            self.denominator)
```

The `@property` decorator on a method designates that it will be called whenever it is *looked up* on an instance.

It allows zero-argument methods to be called without an explicit call expression.

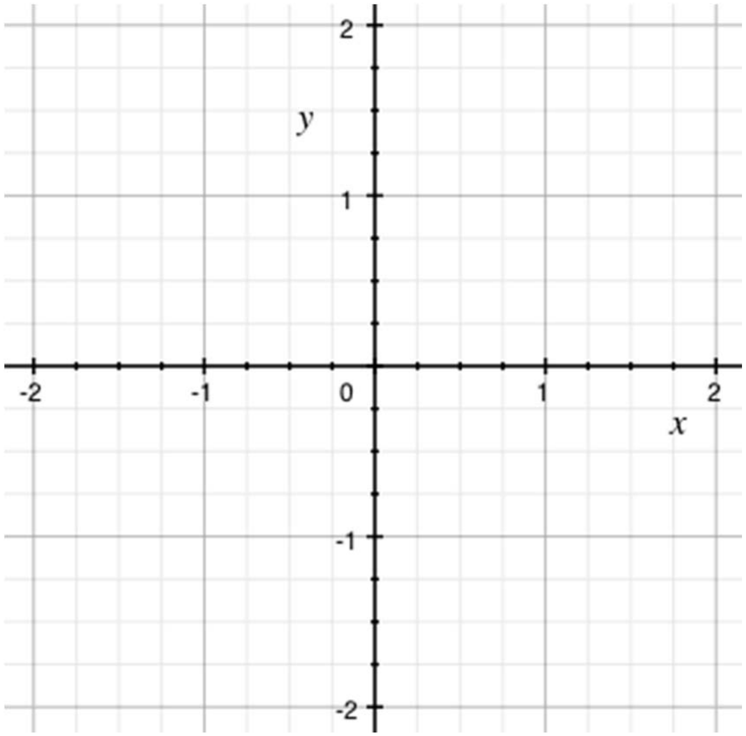
Multiple Representations of Abstract Data

Multiple Representations of Abstract Data

Rectangular and polar representations for complex numbers

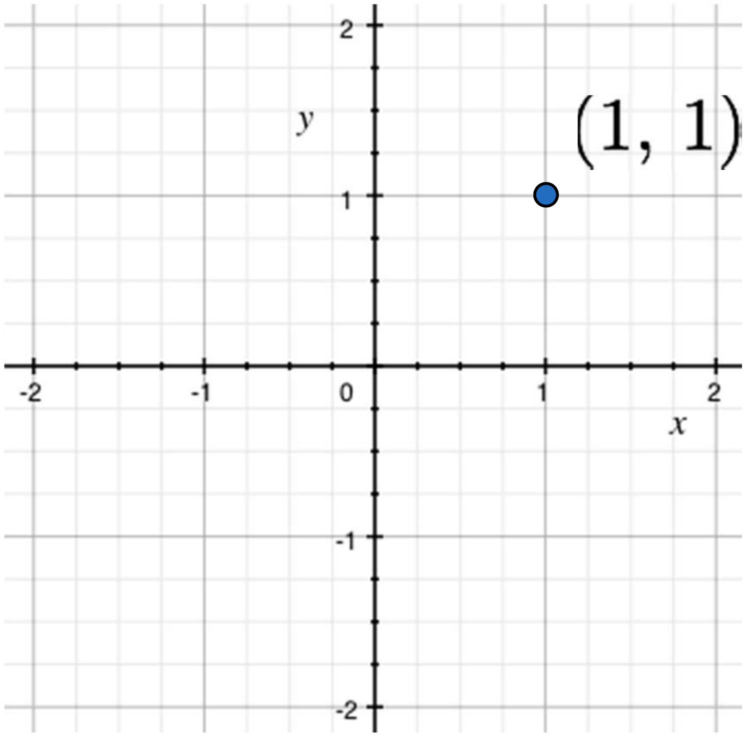
Multiple Representations of Abstract Data

Rectangular and polar representations for complex numbers



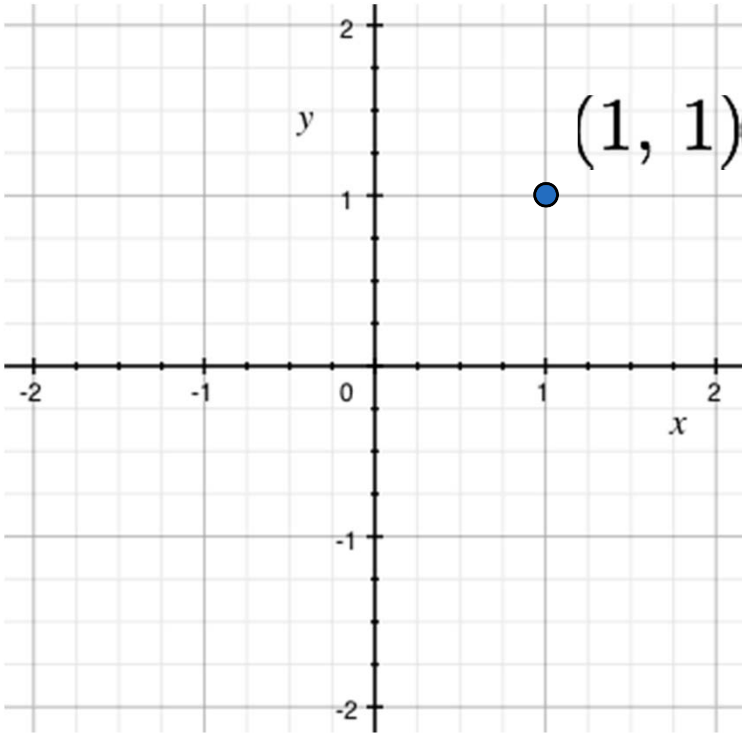
Multiple Representations of Abstract Data

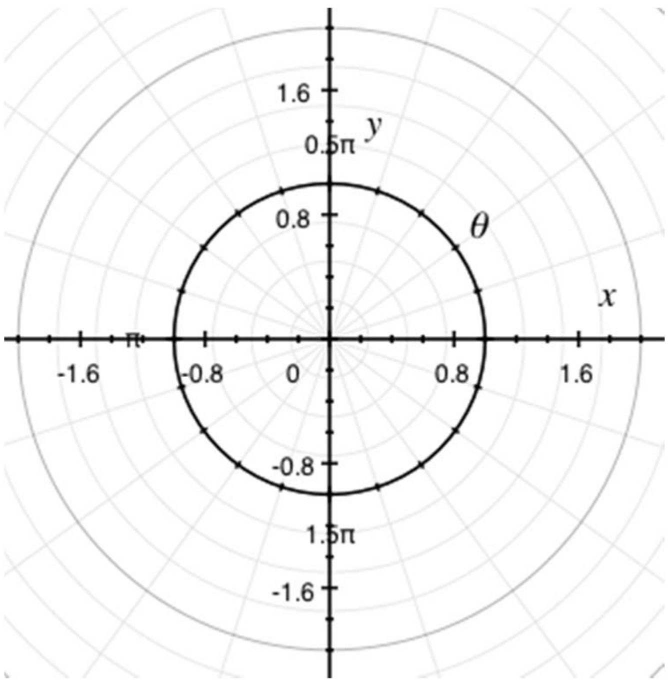
Rectangular and polar representations for complex numbers



Multiple Representations of Abstract Data

Rectangular and polar representations for complex numbers





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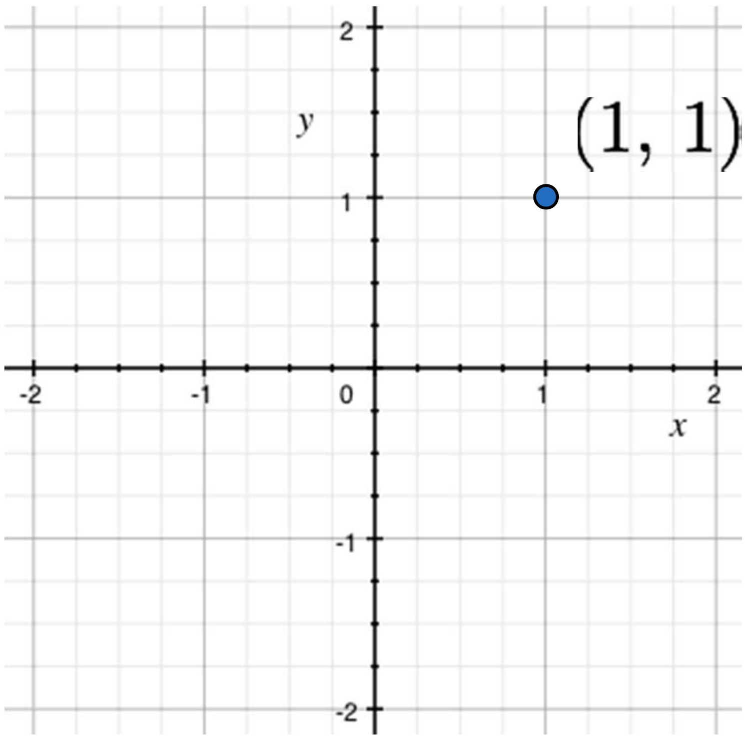

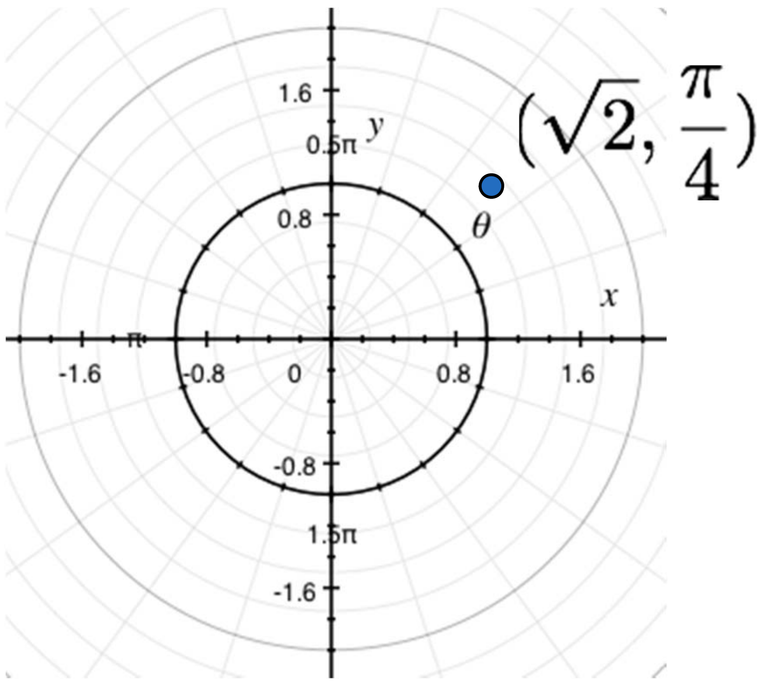

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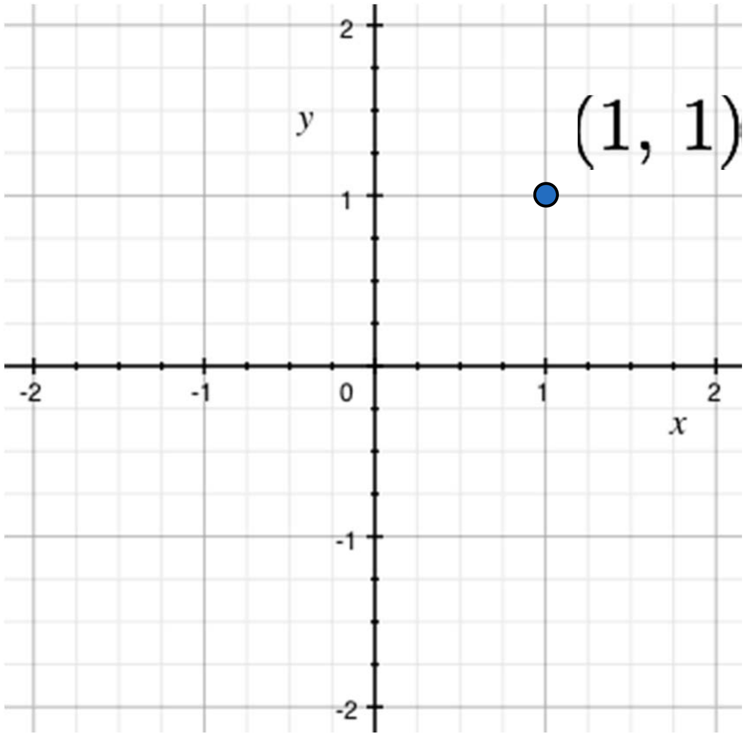

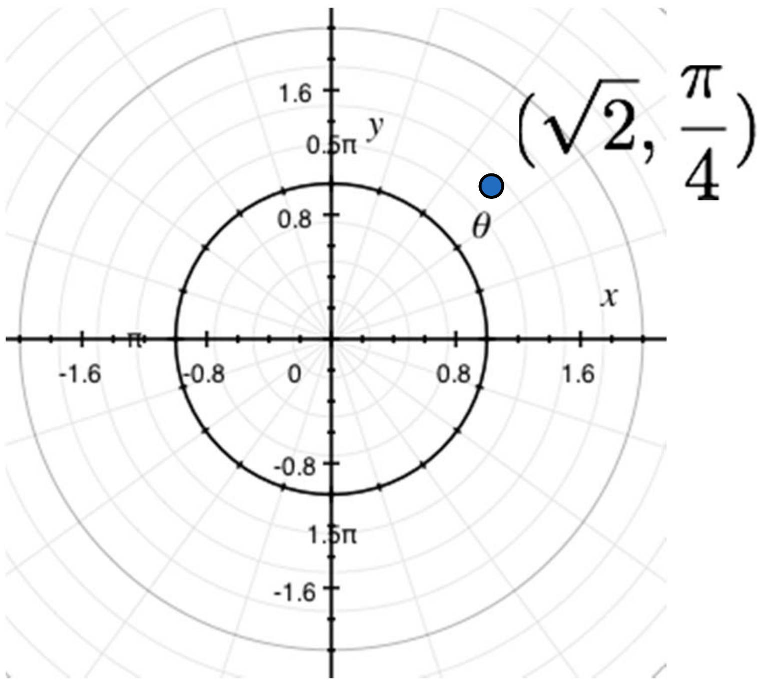

Most operations don't care about the representation.

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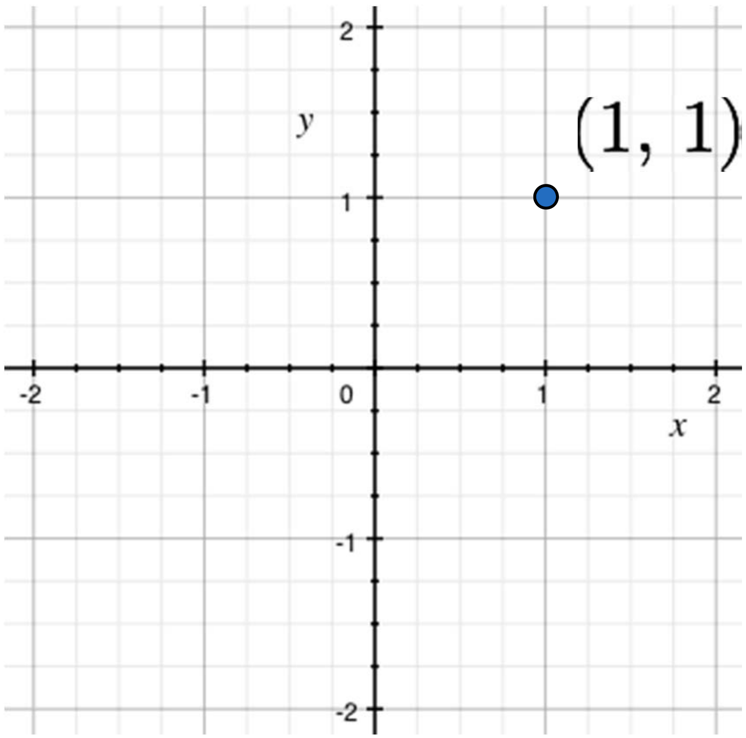
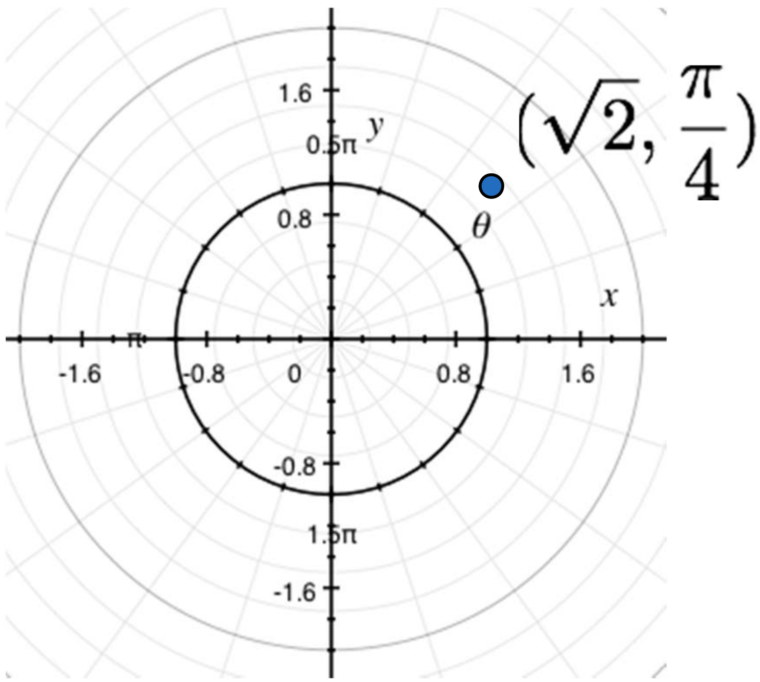
Rectangular and polar representations for complex numbers





Most operations don't care about the representation.

Some mathematical operations are easier on one than the other.

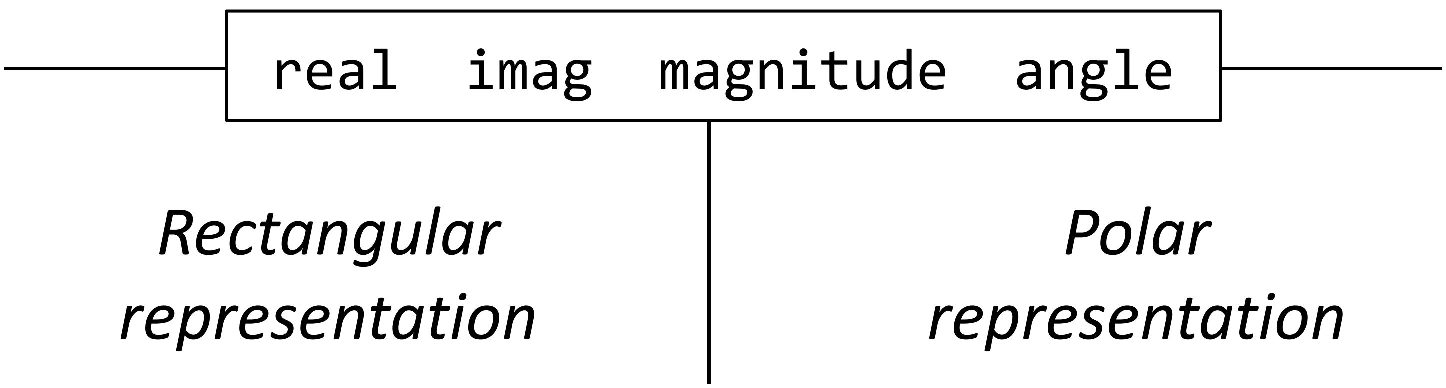
Arithmetic Abstraction Barriers

Arithmetic Abstraction Barriers

Rectangular representation

Polar representation

Arithmetic Abstraction Barriers



Arithmetic Abstraction Barriers

Complex numbers as two-dimensional vectors

`real` `imag` `magnitude` `angle`

Rectangular representation

Polar representation

Arithmetic Abstraction Barriers

`add_complex` `mul_complex`

Complex numbers as two-dimensional vectors

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Arithmetic Abstraction Barriers

Complex numbers as whole data values

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An Interface for Complex Numbers

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All complex numbers should have real and imag components.

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def add_complex(z1, z2):
    return ComplexRI(z1.real + z2.real,
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def add_complex(z1, z2):
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def mul_complex(z1, z2):
    return ComplexMA(z1.magnitude * z2.magnitude,
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The Rectangular Representation

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```
class ComplexRI(object):
```

The Rectangular Representation

```
class ComplexRI(object):

    def __init__(self, real, imag):
```

The Rectangular Representation

```
class ComplexRI(object):

    def __init__(self, real, imag):
        self.real = real
```

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class ComplexRI(object):

    def __init__(self, real, imag):
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    @property
```

The Rectangular Representation

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class ComplexRI(object):

    def __init__(self, real, imag):
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    @property
    def magnitude(self):
```

The Rectangular Representation

```
class ComplexRI(object):

    def __init__(self, real, imag):
        self.real = real
        self.imag = imag

    @property
    def magnitude(self):
        return (self.real ** 2 + self.imag ** 2) ** 0.5
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Property decorator: "Call this function on attribute look-up"

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    @property
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`math.atan2(y,x)`: Angle between x-axis and the point (x,y)

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    def __repr__(self):
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    @property
    def magnitude(self):
        return (self.real ** 2 + self.imag ** 2) ** 0.5

    @property
    def angle(self):
        return atan2(self.imag, self.real)

    def __repr__(self):
        return 'ComplexRI({0}, {1})'.format(self.real,
```

Property decorator: "Call this function on attribute look-up"

math.atan2(y,x): Angle between x-axis and the point (x,y)

The Rectangular Representation

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class ComplexRI(object):

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```

Property decorator: "Call this function on attribute look-up"

math.atan2(y,x): Angle between x-axis and the point (x,y)

The Polar Representation

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```
class ComplexMA(object):
```

The Polar Representation

```
class ComplexMA(object):

    def __init__(self, magnitude, angle):
```

The Polar Representation

```
class ComplexMA(object):

    def __init__(self, magnitude, angle):
        self.magnitude = magnitude
```

The Polar Representation

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class ComplexMA(object):

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The Polar Representation

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class ComplexMA(object):

    def __init__(self, magnitude, angle):
        self.magnitude = magnitude
        self.angle = angle

    @property
```

The Polar Representation

```
class ComplexMA(object):

    def __init__(self, magnitude, angle):
        self.magnitude = magnitude
        self.angle = angle

    @property
    def real(self):
```

The Polar Representation

```
class ComplexMA(object):

    def __init__(self, magnitude, angle):
        self.magnitude = magnitude
        self.angle = angle

    @property
    def real(self):
        return self.magnitude * cos(self.angle)
```

The Polar Representation

```
class ComplexMA(object):

    def __init__(self, magnitude, angle):
        self.magnitude = magnitude
        self.angle = angle

    @property
    def real(self):
        return self.magnitude * cos(self.angle)

    @property
```

The Polar Representation

```
class ComplexMA(object):

    def __init__(self, magnitude, angle):
        self.magnitude = magnitude
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    @property
    def real(self):
        return self.magnitude * cos(self.angle)

    @property
    def imag(self):
```

The Polar Representation

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class ComplexMA(object):

    def __init__(self, magnitude, angle):
        self.magnitude = magnitude
        self.angle = angle

    @property
    def real(self):
        return self.magnitude * cos(self.angle)

    @property
    def imag(self):
        return self.magnitude * sin(self.angle)
```

The Polar Representation

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class ComplexMA(object):

    def __init__(self, magnitude, angle):
        self.magnitude = magnitude
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    @property
    def real(self):
        return self.magnitude * cos(self.angle)

    @property
    def imag(self):
        return self.magnitude * sin(self.angle)

    def __repr__(self):
```

The Polar Representation

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class ComplexMA(object):

    def __init__(self, magnitude, angle):
        self.magnitude = magnitude
        self.angle = angle

    @property
    def real(self):
        return self.magnitude * cos(self.angle)

    @property
    def imag(self):
        return self.magnitude * sin(self.angle)

    def __repr__(self):
        return 'ComplexMA({0}, {1})'.format(self.magnitude,
```

The Polar Representation

```
class ComplexMA(object):

    def __init__(self, magnitude, angle):
        self.magnitude = magnitude
        self.angle = angle

    @property
    def real(self):
        return self.magnitude * cos(self.angle)

    @property
    def imag(self):
        return self.magnitude * sin(self.angle)

    def __repr__(self):
        return 'ComplexMA({0}, {1})'.format(self.magnitude,
                                            self.angle)
```

Using Complex Numbers

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Either type of complex number can be passed as either argument to **`add_complex`** or **`mul_complex`**:

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def add_complex(z1, z2):
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Using Complex Numbers

Either type of complex number can be passed as either argument to **add_complex** or **mul_complex**:

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```

```
>>> from math import pi
```

Using Complex Numbers

Either type of complex number can be passed as either argument to **add_complex** or **mul_complex**:

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def add_complex(z1, z2):
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def mul_complex(z1, z2):
    return ComplexMA(z1.magnitude * z2.magnitude,
                     z1.angle + z2.angle)
```

```
>>> from math import pi
>>> add_complex(ComplexRI(1, 2), ComplexMA(2, pi/2))
```

Using Complex Numbers

Either type of complex number can be passed as either argument to **add_complex** or **mul_complex**:

```
def add_complex(z1, z2):
    return ComplexRI(z1.real + z2.real,
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def mul_complex(z1, z2):
    return ComplexMA(z1.magnitude * z2.magnitude,
                     z1.angle + z2.angle)
```

```
>>> from math import pi
>>> add_complex(ComplexRI(1, 2), ComplexMA(2, pi/2))
ComplexRI(1.0000000000000002, 4.0)
```

Using Complex Numbers

Either type of complex number can be passed as either argument to **add_complex** or **mul_complex**:

```
def add_complex(z1, z2):
    return ComplexRI(z1.real + z2.real,
                     z1.imag + z2.imag)

def mul_complex(z1, z2):
    return ComplexMA(z1.magnitude * z2.magnitude,
                     z1.angle + z2.angle)
```

```
>>> from math import pi
>>> add_complex(ComplexRI(1, 2), ComplexMA(2, pi/2))
ComplexRI(1.0000000000000002, 4.0)
>>> mul_complex(ComplexRI(0, 1), ComplexRI(0, 1))
```

Using Complex Numbers

Either type of complex number can be passed as either argument to **add_complex** or **mul_complex**:

```
def add_complex(z1, z2):
    return ComplexRI(z1.real + z2.real,
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>>> from math import pi
>>> add_complex(ComplexRI(1, 2), ComplexMA(2, pi/2))
ComplexRI(1.0000000000000002, 4.0)
>>> mul_complex(ComplexRI(0, 1), ComplexRI(0, 1))
ComplexMA(1.0, 3.141592653589793)
```

Using Complex Numbers

Either type of complex number can be passed as either argument to **add_complex** or **mul_complex**:

```
def add_complex(z1, z2):
    return ComplexRI(z1.real + z2.real,
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def mul_complex(z1, z2):
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```
>>> from math import pi
>>> add_complex(ComplexRI(1, 2), ComplexMA(2, pi/2))
ComplexRI(1.0000000000000002, 4.0)
>>> mul_complex(ComplexRI(0, 1), ComplexRI(0, 1))
ComplexMA(1.0, 3.141592653589793)
```

We can also define **__add__** and **__mul__** in both classes.

The Independence of Data Types

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Data abstraction and class definitions keep types separate

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Some operations need to cross type boundaries

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Some operations need to cross type boundaries

`add_rational` `mul_rational`

Rational numbers as numerators & denominators

The Independence of Data Types

Data abstraction and class definitions keep types separate

Some operations need to cross type boundaries

`add_rational  mul_rational`

Rational numbers as numerators & denominators

`add_complex  mul_complex`

Complex numbers as two-dimensional vectors

The Independence of Data Types

Data abstraction and class definitions keep types separate

Some operations need to cross type boundaries

How do we add a complex number and a rational number together?

`add_rational` `mul_rational`

Rational numbers as numerators & denominators

`add_complex` `mul_complex`

Complex numbers as two-dimensional vectors

The Independence of Data Types

Data abstraction and class definitions keep types separate

Some operations need to cross type boundaries

How do we add a complex number and a rational number together?

`add_rational  mul_rational`

Rational numbers as numerators & denominators

`add_complex  mul_complex`

Complex numbers as two-dimensional vectors

There are many different techniques for doing this!

Type Dispatching

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Define a different function for each possible combination of types for which an operation (e.g., addition) is valid

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def add_complex_and_rational(z, r):
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Converted to a real number (float)

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    elif isrational(z1) and iscomplex(z2):
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        return add_complex_and_rational(z1, z2)
    elif isrational(z1) and iscomplex(z2):
        return add_complex_and_rational(z2, z1)
    else:
```

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        return add_complex(z1, z2)
    elif iscomplex(z1) and isrational(z2):
        return add_complex_and_rational(z1, z2)
    elif isrational(z1) and iscomplex(z2):
        return add_complex_and_rational(z2, z1)
    else:
        add_rational(z1, z2)
```

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type_tags = {ComplexRI: 'com',
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def type_tag(x):
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type_tags = {ComplexRI: 'com',
             ComplexMA: 'com',
             Rational:  'rat'}
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def type_tag(x):
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Declares that **ComplexRI** and **ComplexMA** should be treated uniformly

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```
def type_tag(x):
    return type_tags[type(x)]

type_tags = {ComplexRI:  'com',
             ComplexMA:  'com',
             Rational:   'rat'}

def add(z1, z2):
    types = (type_tag(z1), type_tag(z2))
    return add_implementations[types](z1, z2)
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def type_tag(x):
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def add(z1, z2):
    types = (type_tag(z1), type_tag(z2))
    return add_implementations[types](z1, z2)

add_implementations = {}
add_implementations[('com', 'com')] = add_complex
add_implementations[('rat', 'rat')] = add_rational
add_implementations[('com', 'rat')] = add_complex_and_rational
add_implementations[('rat', 'com')] = add_rational_and_complex
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`lambda r, z: add_complex_and_rational(z, r)`

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add, subtract, multiply, divide

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Arg 1	Arg 2	Add	Multiply
Complex	Complex		
Rational	Rational		
Complex	Rational		
Rational	Complex		

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Message Passing