

### CS61A Lecture 17

Amir Kamil UC Berkeley March 1, 2013

#### **Announcements**



☐ HW6 due next Thursday

- □ Trends project due on Tuesday
  - ☐ Partners are required; find one in lab or on Piazza
  - □ Will not work in IDLE
  - □ New bug submission policy; see Piazza





Names typically don't matter for correctness, but they matter tremendously for legibility



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boolean d play\_helper



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boolean turn\_is\_over d dice play\_helper

📥 take\_turn



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```
boolean turn_is_over d dice play_helper take_turn
```

Use names for repeated compound expressions



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boolean turn_is_over d dice play_helper take_turn
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Use names for repeated compound expressions

```
if sqrt(square(a) + square(b)) > 1:
    x = x + sqrt(square(a) + square(b))
```



Names typically don't matter for correctness, but they matter tremendously for legibility

```
boolean turn_is_over d dice play_helper take_turn
```

Use names for repeated compound expressions

```
if sqrt(square(a) + square(b)) > 1:
    x = x + sqrt(square(a) + square(b))
    h = sqrt(square(a) + square(b))
    if h > 1:
        x = x + h
```



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```
boolean turn_is_over d dice play_helper take_turn
```

Use names for repeated compound expressions

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if sqrt(square(a) + square(b)) > 1:
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Use names for meaningful parts of compound expressions



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if sqrt(square(a) + square(b)) > 1:
    x = x + sqrt(square(a) + square(b))
    h = sqrt(square(a) + square(b))
    if h > 1:
        x = x + h
```

Use names for meaningful parts of compound expressions

```
x = (-b + sqrt(square(b) - 4 * a * c)) / (2 * a)
```



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```
boolean turn_is_over d dice play_helper take_turn
```

Use names for repeated compound expressions

```
if sqrt(square(a) + square(b)) > 1:
    x = x + sqrt(square(a) + square(b))
    h = sqrt(square(a) + square(b))
    if h > 1:
        x = x + h
```

Use names for meaningful parts of compound expressions

```
x = (-b + sqrt(square(b) - 4 * a * c)) / (2 * a)

disc_term = sqrt(square(b) - 4 * a * c)
x = (-b + disc_term) / (2 * a)
```







```
def find_quadratic_root(a, b, c, plus=True):
    """Applies the quadratic formula to the polynomial
    ax^2 + bx + c."""
    if plus:
        return (-b + sqrt(square(b) - 4 * a * c)) / (2 * a)
    else:
        return (-b - sqrt(square(b) - 4 * a * c)) / (2 * a)
```



```
def find_quadratic_root(a, b, c, plus=True):
    """Applies the quadratic formula to the polynomial
    ax^2 + bx + c."""
    if plus:
        return (-b + sqrt(square(b) - 4 * a * c)) / (2 * a)
    else:
        return (-b - sqrt(square(b) - 4 * a * c)) / (2 * a)
```





```
def find quadratic root(a, b, c, plus=True):
    """Applies the quadratic formula to the polynomial
    ax^2 + bx + c."""
    if plus:
        return (-b + sqrt(square(b) - 4 * a * c)) / (2 * a)
    else:
        return (-b - sqrt(square(b) - 4 * a * c)) / (2 * a)
def find_quadratic_root(a, b, c, plus=True):
    """Applies the quadratic formula to the polynomial
    ax^2 + bx + c."""
    disc term = sqrt(square(b) - 4 * a * c)
    if not plus:
        disc term *=-1
    return (-b + disc_term) / (2 * a)
```





Write the test of a function before you write a function



Write the test of a function before you write a function

A test will clarify the (one) job of the function



# Write the test of a function before you write a function

A test will clarify the (one) job of the function

Your tests can help identify tricky edge cases



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Develop incrementally and test each piece before moving on



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# Develop incrementally and test each piece before moving on

You can't depend upon code that hasn't been tested



# Write the test of a function before you write a function

A test will clarify the (one) job of the function Your tests can help identify tricky edge cases

# Develop incrementally and test each piece before moving on

You can't depend upon code that hasn't been tested Run your old tests again after you make new changes

## **Hog Contest**



#### **Hog Contest**



#### Contest rules:

- □ All entries run against every other entry
- $\square$  An entry wins a match if its true win rate is > 0.5
- All strategies must be deterministic, pure functions and must not use pre-computed data
- Extra credit for entries with the most wins or the highest cumulative win rate
- □ Total of 54 valid submissions

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- □ Total of 54 valid submissions

We used itertools.combinations to determine the set of matches





Congratulations to the team of Colin Lockard and Sherry Xu, who achieved a perfect 53-0 record <u>and</u> the highest win rate (28.77)!



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Second-most wins (51-2): Eric Holt and Anna Carey



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Third-highest in both (50-3, 28.67): Sean Scofield and Frank Lu



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Second-highest win rate (28.70): Don Mai and Jeechee Chen

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Complete rankings will be posted on the website

### Computing Win Rates Exactly



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Requires access to both strategies, which must be deterministic

#### Computing Win Rates Exactly



A state in the game:

(who rolls next?, player score, opponent score)

Requires access to both strategies, which must be deterministic



A state in the game: (who rolls next?, player score, opponent score)

A strategy is a table



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A strategy is a table

(me,0,0): 5



A state in the game: (who rolls next?, player score, opponent score)

A strategy is a table

(me,0,0): 5 (me,0,70): 9



A state in the game: (who rolls next?, player score, opponent score)

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• • •



A state in the game: (who rolls next?, player score, opponent score)

#### A strategy is a table

```
(me,0,0): 5
(me,0,70): 9
...
(me,96,99): 0
```



```
A state in the game: (who rolls next?, player score, opponent score)
```

#### A strategy is a table

```
(me,0,0): 5
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(me,96,99): 0
...
(me,99,99): 10
```



A state in the game: (who rolls next?, player score, opponent score)

#### A strategy is a table

Each state has a chance to win

```
(me,0,0): 5
(me,0,70): 9
...
(me,96,99): 0
...
(me,99,99): 10
```



A state in the game:

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```
(me,0,0): 5
(me,0,70): 9
...
(me,96,99): 0
...
(me,99,99): 10
```

(me,99,100+)



A state in the game:

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```
(me,0,0): 5
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```

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(me,99,100+) 0



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```



A state in the game:

(who rolls next?, player score, opponent score)

#### A strategy is a table

Each state has a chance to win

```
(me,0,0): 5
(me,0,70): 9
...
(me,96,99): 0
...
(me,99,99): 10
```

```
(you,98,99)
(you,100+,99)
(me,99,100+)
```



A state in the game:

(who rolls next?, player score, opponent score)

#### A strategy is a table

Each state has a chance to win

```
(me,0,0): 5
(me,0,70): 9
...
(me,96,99): 0
...
(me,99,99): 10
```



### A state in the game:

(who rolls next?, player score, opponent score)

#### A strategy is a table

Each state has a chance to win

```
(me,0,0): 5
(me,0,70): 9
...
(me,96,99): 0
...
(me,99,99): 10
```

... (you,98,99) (you,100+,99) (me,99,100+)



### A state in the game:

(who rolls next?, player score, opponent score)

#### A strategy is a table

Each state has a chance to win

```
(me,0,0): 5
(me,0,70): 9
...
(me,96,99): 0
...
(me,99,99): 10
```

```
... (you,98,99) (you,100+,99) 1 (me,99,100+)
```



### A state in the game:

(who rolls next?, player score, opponent score)

#### A strategy is a table

Each state has a chance to win

```
(me,0,0): 5

(me,0,70): 9

...

(me,96,99): 0

...

(me,99,99): 10

(you,90,99)

...

(you,98,99)

(you,100+,99)

(me,99,100+)
```



### A state in the game:

(who rolls next?, player score, opponent score)

#### A strategy is a table

Each state has a chance to win

```
(me,0,0): 5

(me,0,70): 9

...

(me,96,99): 0

...

(me,99,99): 10

(you,90,99)

...

(you,98,99)

(you,100+,99)

(me,99,100+)
```



### A state in the game:

(who rolls next?, player score, opponent score)

#### A strategy is a table

Each state has a chance to win

```
(me,0,0): 5

(me,0,70): 9

...

(me,96,99): 0

(you,88,99)

...

(you,90,99) 0

(you,98,99) 0

(you,98,99) 1
```



### A state in the game:

(who rolls next?, player score, opponent score)

#### A strategy is a table

Each state has a chance to win

(me,0,0): 5 (me,0,70): 9		
 (me,96,99): 0		
(1110,30,33).	(you,88,99)	0
 (me,99,99): 10	(you,90,99)	0
	•••	• • •
	(you,98,99)	0
	(you,100+,99)	1
	(me.99.100+)	0



### A state in the game:

(who rolls next?, player score, opponent score)

#### A strategy is a table

### (me,0,0): 5 (me,0,70): 9 ... (me,96,99): 0 ... (me,99,99): 10

#### Each state has a chance to win

```
...
(me,87,99)
...
(you,88,99)
(you,90,99)
...
(you,98,99)
(you,100+,99)
```



### A state in the game:

(who rolls next?, player score, opponent score)

#### A strategy is a table

```
(me,0,0): 5
(me,0,70): 9
...
(me,96,99): 0
...
(me,99,99): 10
```

#### Each state has a chance to win

```
(me,87,99)
...
(you,88,99)
(you,90,99)
...
(you,98,99)
(you,100+,99)
(me,99,100+)
```



### A state in the game:

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#### A strategy is a table

```
(me,0,0): 5
(me,0,70): 9
...
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...
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```

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```
(me,87,99)
...
(you,88,99)
(you,90,99)
...
(you,98,99)
(you,100+,99)
(me,99,100+)
```



### A state in the game:

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```
(me,0,0): 5
(me,0,70): 9
...
(me,96,99): 0
...
(me,99,99): 10
```

#### Each state has a chance to win

```
...
(me,87,99)
...
(you,88,99) 0
(you,90,99) 0
... ...
(you,98,99) 0
(you,100+,99) 1
(me,99,100+) 0
```



#### A state in the game:

(who rolls next?, player score, opponent score)

#### A strategy is a table

```
(me,0,0): 5
(me,0,70): 9
...
(me,96,99): 0
...
(me,99,99): 10
```

#### Each state has a chance to win

```
...
(me,87,99)
...
(you,88,99)
(you,90,99)
...
(you,98,99)
(you,100+,99)
(me,99,100+)
```



#### A state in the game:

(who rolls next?, player score, opponent score)

#### A strategy is a table

(me,0,0): 5 (me,0,70): 9 ... (me,96,99): 0 ... (me,99,99): 10

#### Each state has a chance to win





Optimal strategy given an opponent:



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□ At each state, compute probability of winning for each allowed number of dice



Optimal strategy given an opponent:

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- ☐ Choose the number of dice that maximizes the probability



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The perfect strategy: use iterative improvement!



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- □ Update to: optimal opponent of current strategy



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- □ Update to: optimal opponent of current strategy
- □ Done when: 0.5 win rate against optimal opponent



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( )ntimai	l strategy	$\sigma N \Delta n$	<b>2</b> n	annan	Δnt·
Obullia	וטנומנכצע	RIVEII	all	UDUUII	CIIL.
- p		$\mathbf{O}$		- 1- 1	

- □ At each state, compute probability of winning for each allowed number of dice
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The perfect strategy: use iterative improvement!

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- □ Done when: 0.5 win rate against optimal opponent

Takes only 16 steps to converge!

#### Achieving the Perfect Strategy



$\sim$ 1.		•	
( Intima	LCTratagy	GIVAN 2N	opponent:
Obullia	ısılatek	RIVEII all	ODDONELL
		0	

- □ At each state, compute probability of winning for each allowed number of dice
- ☐ Choose the number of dice that maximizes the probability

The perfect strategy: use iterative improvement!

- ☐ Initial guess: always roll 5
- □ Update to: optimal opponent of current strategy
- □ Done when: 0.5 win rate against optimal opponent

Takes only 16 steps to converge!

Can also compute perfect strategy directly using table





Let's model a bank account that has a balance of \$100



Let's model a bank account that has a balance of \$100

>>> withdraw(25)



Let's model a bank account that has a balance of \$100



Let's model a bank account that has a balance of \$100

```
>>> withdraw(25)
```

75

>>> withdraw(25)



Let's model a bank account that has a balance of \$100

```
>>> withdraw(25)
75
```

75

>>> withdraw(25)

50



Let's model a bank account that has a balance of \$100

```
>>> withdraw(25)
```

75

>>> withdraw(25)

50

>>> withdraw(60)



Let's model a bank account that has a balance of \$100

```
>>> withdraw(25)
75

>>> withdraw(25)
50

>>> withdraw(60)
'Insufficient funds'
```



Let's model a bank account that has a balance of \$100

```
>>> withdraw(25)
75

>>> withdraw(25)
50

>>> withdraw(60)
'Insufficient funds'
>>> withdraw(15)
```



#### Let's model a bank account that has a balance of \$100

Return value: remaining balance

>>> withdraw(25)
75

Argument: amount to withdraw

Different return value!

>>> withdraw(25) 50

Second withdrawal of the same amount

>>> withdraw(60)
'Insufficient funds'

>>> withdraw(15)
35



#### Let's model a bank account that has a balance of \$100

Return value: remaining balance

>>> withdraw(25)
75

Argument: amount to withdraw

Different return value!

>>> withdraw(25) 50

Second withdrawal of the same amount

>>> withdraw(60)
'Insufficient funds'

>>> withdraw(15)
35

Where's this balance stored?



#### Let's model a bank account that has a balance of \$100

Return value: remaining balance

>>> withdraw(25)
75

Argument: amount to withdraw

Different return value!

>>> withdraw(25) 50

Second withdrawal of the same amount

>>> withdraw(60)
'Insufficient funds'

>>> withdraw(15)
35

Where's this balance stored?

>>> withdraw = make\_withdraw(100)



#### Let's model a bank account that has a balance of \$100

Return value: remaining balance

```
>>> withdraw(25)
75
```

Argument: amount to withdraw

Different return value!

```
>>> withdraw(25)
50
```

Second withdrawal of the same amount

>>> withdraw(60)
'Insufficient funds'

```
>>> withdraw(15)
35
```

Where's this balance stored?

>>> withdraw = make\_withdraw(100)

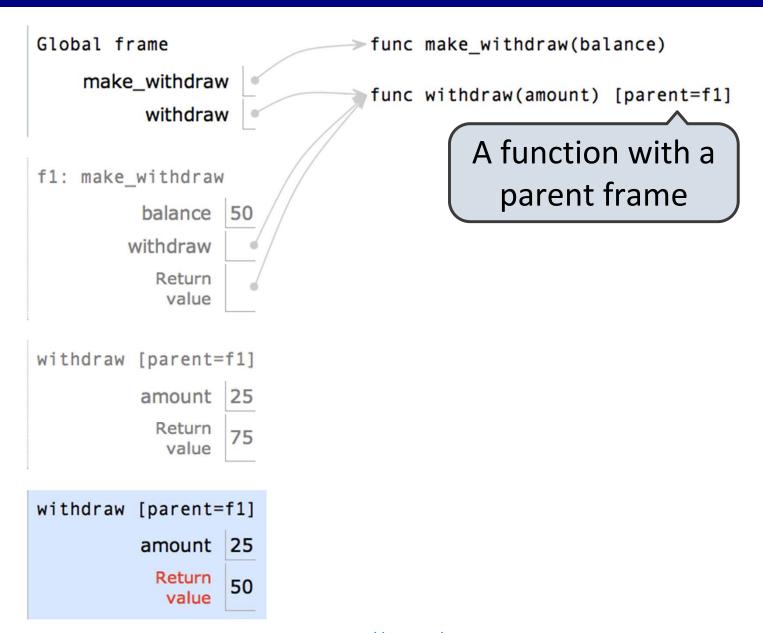
Within the function!



```
Global frame
                              > func make_withdraw(balance)
    make_withdraw
                                func withdraw(amount) [parent=f1]
          withdraw
f1: make_withdraw
          balance
                  50
        withdraw
           Return
            value
withdraw [parent=f1]
          amount
           Return
            value
withdraw [parent=f1]
          amount 25
           Return
                  50
            value
```

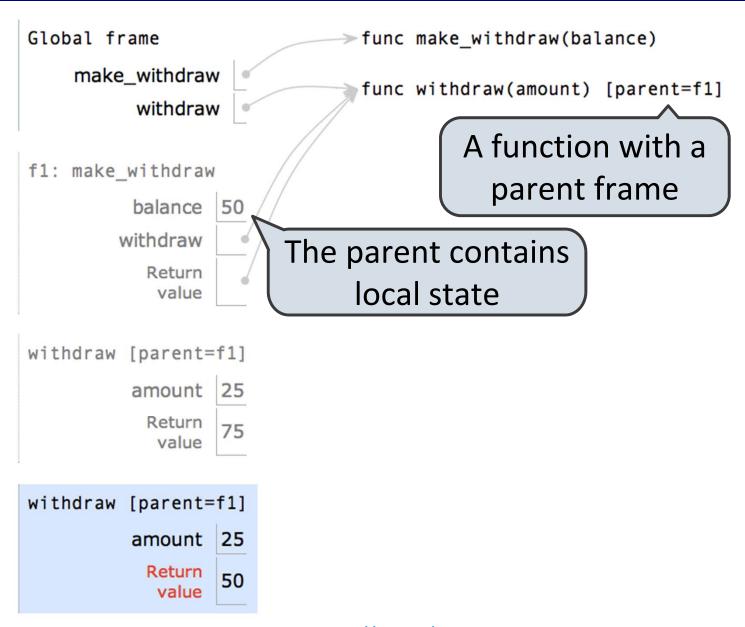
Example: http://goo.gl/5LZ6F





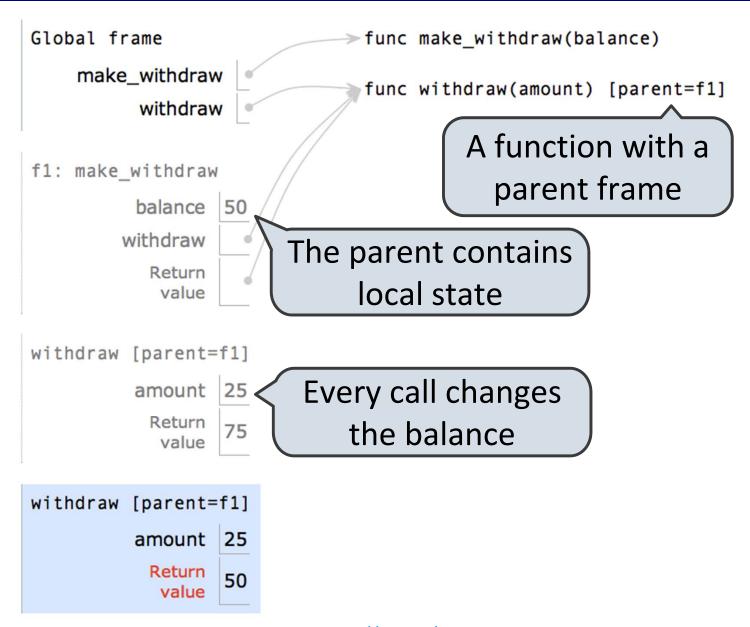
Example: <a href="http://goo.gl/5LZ6F">http://goo.gl/5LZ6F</a>





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Example: <a href="http://goo.gl/5LZ6F">http://goo.gl/5LZ6F</a>





Assignment binds name(s) to value(s) in the first frame of the current environment



Example: <a href="http://goo.gl/xkYgN">http://goo.gl/xkYgN</a>



#### Execution rule for assignment statements:

- 1. Evaluate all expressions right of =, from left to right.
- 2. Bind the names on the left the resulting values in the first frame of the current environment.

Example: <a href="http://goo.gl/xkYgN">http://goo.gl/xkYgN</a>





```
def make_withdraw(balance):
```



```
def make_withdraw(balance):
    """Return a withdraw function with a starting balance."""
```



```
def make_withdraw(balance):
    """Return a withdraw function with a starting balance."""
    def withdraw(amount):
```



```
def make_withdraw(balance):
    """Return a withdraw function with a starting balance."""
    def withdraw(amount):
        if amount > balance:
```



```
def make_withdraw(balance):
    """Return a withdraw function with a starting balance."""
    def withdraw(amount):
        if amount > balance:
            return 'Insufficient funds'
```



```
def make_withdraw(balance):
    """Return a withdraw function with a starting balance."""
    def withdraw(amount):
        if amount > balance:
            return 'Insufficient funds'
        balance = balance - amount
```



```
def make withdraw(balance):
    """Return a withdraw function with a starting balance."""
    def withdraw(amount):
        if amount > balance:
            return 'Insufficient funds'
        balance = balance - amount
        return balance
```



```
def make withdraw(balance):
    """Return a withdraw function with a starting balance."""
    def withdraw(amount):
        if amount > balance:
            return 'Insufficient funds'
        balance = balance - amount
        return balance
    return withdraw
```



```
def make withdraw(balance):
    """Return a withdraw function with a starting balance."""
    def withdraw(amount):
        nonlocal balance
        if amount > balance:
            return 'Insufficient funds'
        balance = balance - amount
        return balance
    return withdraw
```



```
def make withdraw(balance):
    """Return a withdraw function with a starting balance."""
    def withdraw(amount):
                                   Declare the name
                                   "balance" nonlocal
        nonlocal balance
        if amount > balance:
            return 'Insufficient funds'
        balance = balance - amount
        return balance
    return withdraw
```



```
def make withdraw(balance):
    """Return a withdraw function with a starting balance."""
    def withdraw(amount):
                                   Declare the name
                                   "balance" nonlocal
        nonlocal balance
        if amount > balance:
            return 'Insufficient funds'
                                            Re-bind balance
        balance = balance - amount
                                              where it was
        return balance
                                            bound previously
```

return withdraw

#### The Effect of Nonlocal Statements



```
nonlocal <name>, <name 2>, ...
```

#### The Effect of Nonlocal Statements



```
nonlocal <name>, <name 2>, ...
```

Effect: Future assignments to that name change its pre-existing binding in the **first non-local frame** of the current environment in which that name is bound.



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nonlocal <name>, <name 2>, ...
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Python Docs: an "enclosing scope"



```
nonlocal <name>, <name 2>, ...
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Effect: Future assignments to that name change its pre-existing binding in the **first non-local frame** of the current environment in which that name is bound.

Python Docs: an "enclosing scope"

From the Python 3 language reference:



```
nonlocal <name>, <name 2>, ...
```

Effect: Future assignments to that name change its pre-existing binding in the **first non-local frame** of the current environment in which that name is bound.

Python Docs: an "enclosing scope"

From the Python 3 language reference:

Names listed in a <u>nonlocal</u> statement must refer to pre-existing bindings in an enclosing scope. Names listed in a nonlocal <u>statement</u> must not collide with pre-existing bindings in the local scope.



```
nonlocal <name>, <name 2>, ...
```

Effect: Future assignments to that name change its pre-existing binding in the **first non-local frame** of the current environment in which that name is bound.

Python Docs: an "enclosing scope"

From the Python 3 language reference:

Names listed in a <u>nonloca</u>l statement must refer to pre-existing bindings in an enclosing scope. Names listed in a nonlocal <u>statement</u> must not collide with pre-existing bindings in the local scope.

http://docs.python.org/release/3.1.3/reference/simple\_stmts.html#the-nonlocal-statement





Status	Effect

**Effect** 



#### Status

- No nonlocal statement
- "x" is not bound locally

 $|\mathbf{x}| = 2$ 



#### **Status**

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#### **Effect**

Create a new binding from name "x" to object 2 in the first frame of the current environment.

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	IS
bject 2 nent.	nonlocal statement is not bound locally
frame	nonlocal statement is bound locally
	nlocal x is bound in a non-local me
	is bound in a non-local



Effect
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F



<ul><li>Status</li><li>No nonlocal statement</li><li>"x" is not bound locally</li></ul>	Effect Create a new binding from name "x" to object 2 in the first frame of the current environment.
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<ul> <li>nonlocal x</li> <li>"x" is bound in a non-local frame</li> <li>"x" also bound locally</li> </ul>	<b>x</b> = 2



<ul><li>Status</li><li>No nonlocal statement</li><li>"x" is not bound locally</li></ul>	Effect Create a new binding from name "x" to object 2 in the first frame of the current environment.
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Python pre-computes which frame contains each name before executing the body of a function.



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```
def make_withdraw(balance):
    def withdraw(amount):
        if amount > balance:
            return 'Insufficient funds'
        balance = balance - amount
            return balance
        return withdraw

wd = make_withdraw(20)
wd(5)
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Local assignment
```

UnboundLocalError: local variable 'balance' referenced before assignment

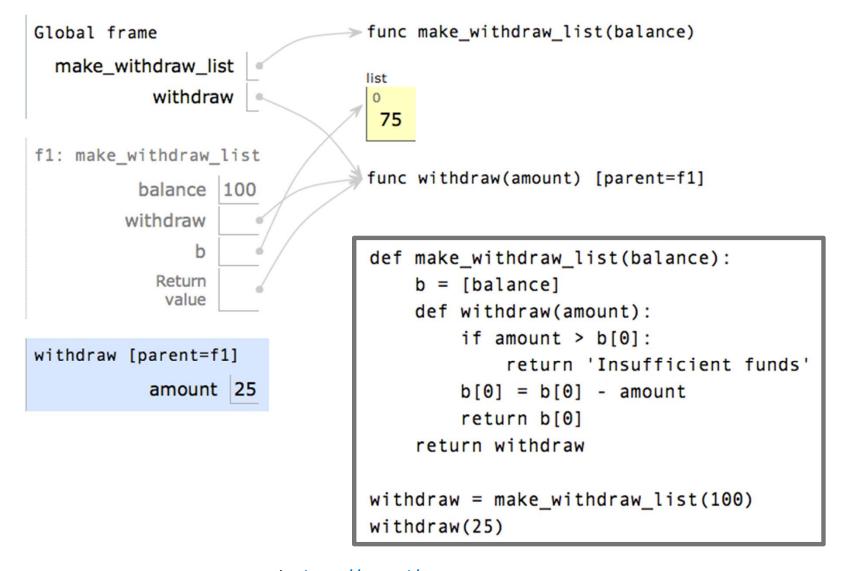




Mutable values can be changed without a nonlocal statement.



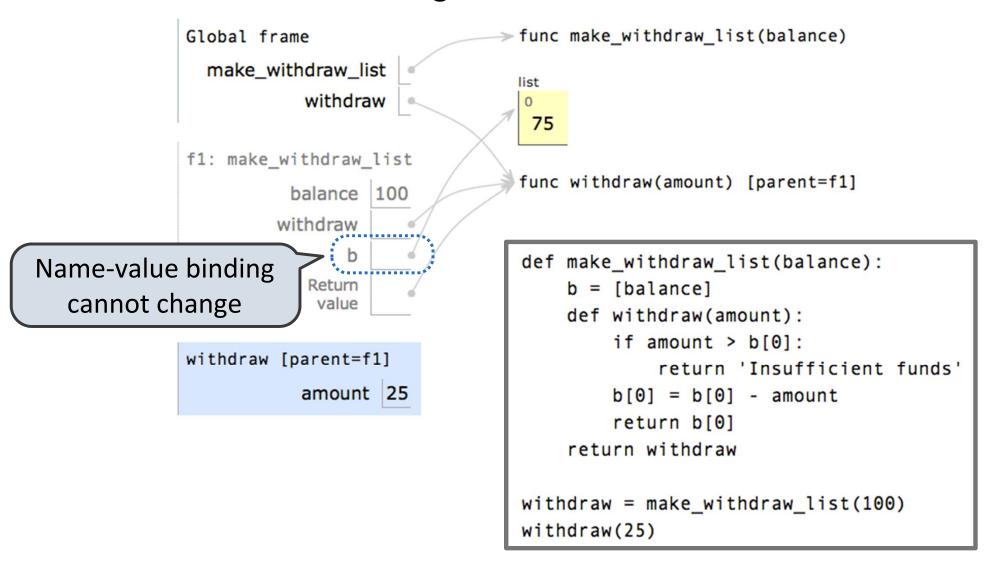
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Example: <a href="http://goo.gl/cEpmz">http://goo.gl/cEpmz</a>



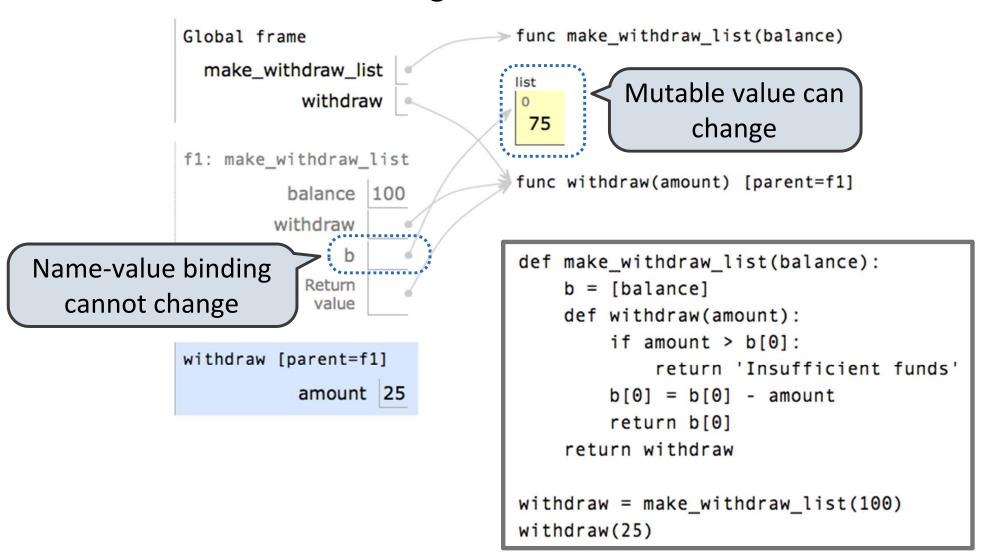
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## **Creating Two Withdraw Functions**



```
Global frame
                          >func make withdraw(balance)
    make_withdraw
                          func withdraw(amount) [parent=f1]
              wd
                          func withdraw(amount) [parent=f2]
             wd2
f1: make withdraw
         balance
                75
        withdraw
                                             def make_withdraw(balance):
          Return
           value
                                                  def withdraw(amount):
                                                       nonlocal balance
f2: make withdraw
                                                       if amount > balance:
         balance 85
                                                            return 'Insufficient funds'
        withdraw
                                                       balance = balance - amount
          Return
           value
                                                       return balance
                                                  return withdraw
withdraw [parent=f1]
         amount 25
                                             wd = make_withdraw(100)
          Return
                                             wd2 = make_withdraw(100)
            value
                                             wd(25)
withdraw [parent=f2]
                                             wd2(15)
         amount 15
           value
```

Example: <a href="http://goo.gl/glTyB">http://goo.gl/glTyB</a>

#### Multiple References to a Withdraw Function



```
Global frame
                            func make_withdraw(balance)
    make withdraw
                             func withdraw(amount) [parent=f1]
               wd
              wd2
f1: make_withdraw
          balance
                                         def make_withdraw(balance):
                 60
                                              def withdraw(amount):
        withdraw
                                                  nonlocal balance
           Return
            value
                                                  if amount > balance:
                                                       return 'Insufficient funds'
withdraw [parent=f1]
                                                  balance = balance - amount
                                                  return balance
          amount 25
                                              return withdraw
           Return
            value
                                         wd = make withdraw(100)
withdraw [parent=f1]
                                         wd2 = wd
                                         wd(25)
          amount 15
                                         wd2(15)
           Return
            value
```

Example: <a href="http://goo.gl/X2qG9">http://goo.gl/X2qG9</a>





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- The binding for balance in the first non-local frame of the environment associated with an instance of withdraw is inaccessible to the rest of the program.



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Weasley Account

\$10



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Weasley Account

\$10

Potter Account

\$1,000,000











```
mul(add(2, mul(4, 6)), 3)
mul(add(2, 24 ), 3)
mul( 26 , 3)
```



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