



CS61A Lecture 15

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Announcements



- HW5 due on Wednesday

- Trends project out
 - Partners are required; find one in lab or on Piazza
 - Will not work in IDLE
 - New bug submission policy; see Piazza

The Sequence Abstraction



red, orange, yellow, green, blue, indigo, violet.

0, 1, 2, 3, 4, 5, 6.

There isn't just one sequence type (in Python or in general)

This abstraction is a collection of behaviors:

Length. A sequence has a finite length.

Element selection. A sequence has an element corresponding to any non-negative integer index less than its length, starting at 0 for the first element.

The sequence abstraction is shared among several types, including tuples.

Recursive Lists



Constructor:

```
def rlist(first, rest):  
    """Return a recursive list from its first element and  
    the rest."""
```

Selectors:

```
def first(s):  
    """Return the first element of recursive list s."""
```

```
def rest(s):  
    """Return the remaining elements of recursive list s."""
```

Behavior condition(s):

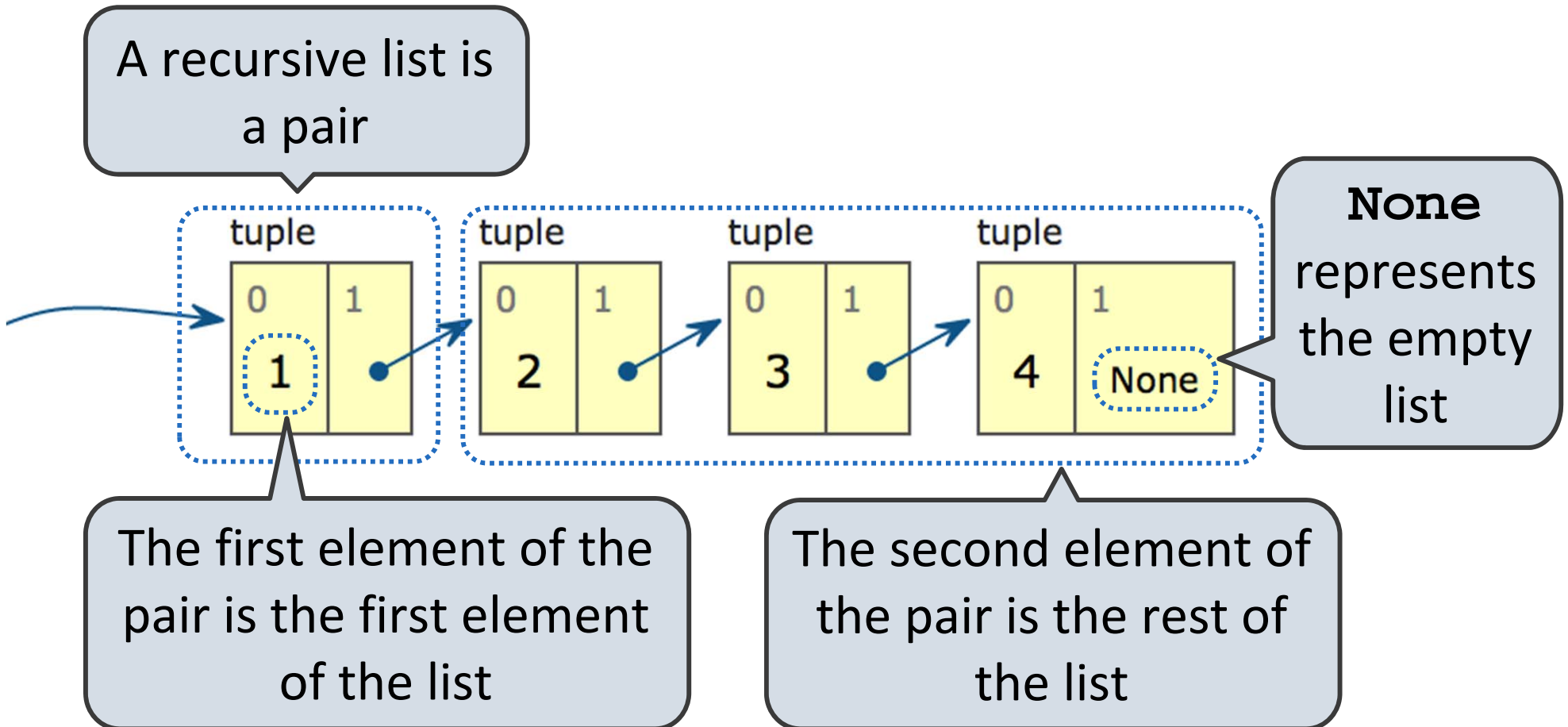
If a recursive list s is constructed from a first element f and a recursive list r , then

- $\mathbf{first}(s)$ returns f , and
- $\mathbf{rest}(s)$ returns r , which is a recursive list.

Implementing Recursive Lists Using Pairs



1, 2, 3, 4



Implementing the Sequence Abstraction



Length. A sequence has a finite length.

Element selection. A sequence has an element corresponding to any non-negative integer index less than its length, starting at 0 for the first element.

Implementing the Sequence Abstraction



```
def len_rlist(s):  
    """Return the length of recursive list s."""  
    if s == empty_rlist:  
        return 0  
    return 1 + len_rlist(rest(s))
```

Length. A sequence has a finite length.

Element selection. A sequence has an element corresponding to any non-negative integer index less than its length, starting at 0 for the first element.

Implementing the Sequence Abstraction



```
def len_rlist(s):
    """Return the length of recursive list s."""
    if s == empty_rlist:
        return 0
    return 1 + len_rlist(rest(s))

def getitem_rlist(s, i):
    """Return the element at index i of recursive list s."""
    if i == 0:
        return first(s)
    return getitem_rlist(rest(s), i - 1)
```

Length. A sequence has a finite length.

Element selection. A sequence has an element corresponding to any non-negative integer index less than its length, starting at 0 for the first element.

Python Sequence Abstraction



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Built-in sequence types provide the following behavior

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Type-specific constructor	<pre>>>> a = (1, 2, 3) >>> b = tuple([4, 5, 6, 7])</pre>
---------------------------	--

Length	<pre>>>> len(a), len(b) (3, 4)</pre>
--------	---

Element selection	<pre>>>> a[1], b[-1] (2, 7)</pre>
-------------------	--

Slicing	<pre>>>> a[1:3], b[1:1], a[:2], b[1:] ((2, 3), (), (1, 2), (5, 6, 7))</pre>
---------	--

Membership	<pre>>>> 2 in a, 4 in a, 4 not in b (True, False, False)</pre>
------------	---

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A list; more on this later

Element selection	<pre>>>> a[1], b[-1] (2, 7)</pre>
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Python Sequence Abstraction



Built-in sequence types provide the following behavior

Type-specific constructor
`>>> a = (1, 2, 3)`
`>>> b = tuple([4, 5, 6, 7])`

Length
`>>> len(a), len(b)`
`(3, 4)`

A list; more on this later

Element selection
`>>> a[1], b[-1]`
`(2, 7)`

Count from the end; -1 is last element

Slicing
`>>> a[1:3], b[1:1], a[:2], b[1:]`
`((2, 3), (), (1, 2), (5, 6, 7))`

Membership
`>>> 2 in a, 4 in a, 4 not in b`
`(True, False, False)`

Sequence Iteration



Sequence Iteration



Python has a special statement for iterating over the elements in a sequence

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Python has a special statement for iterating over the elements in a sequence

```
def count(s, value):  
    total = 0  
  
    for elem in s:  
        if elem == value:  
            total += 1  
    return total
```

Sequence Iteration



Python has a special statement for iterating over the elements in a sequence

```
def count(s, value):  
    total = 0
```

Name bound in the first
frame of the current
environment

```
    for elem in s:  
        if elem == value:  
            total += 1  
    return total
```

For Statement Execution Procedure



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```
for <name> in <expression>:  
    <suite>
```

For Statement Execution Procedure



```
for <name> in <expression>:  
    <suite>
```

1. Evaluate the header **<expression>**, which must yield an iterable value.

For Statement Execution Procedure



```
for <name> in <expression> :  
    <suite>
```

1. Evaluate the header **<expression>**, which must yield an iterable value.
2. For each element in that sequence, in order:
 - A. Bind **<name>** to that element in the first frame of the current environment.
 - B. Execute the **<suite>**.

Sequence Unpacking in For Statements



Sequence Unpacking in For Statements



```
>>> pairs = ((1, 2), (2, 2), (2, 3), (4, 4))  
  
>>> same_count = 0
```


Sequence Unpacking in For Statements



A sequence of
fixed-length sequences

```
>>> pairs = ((1, 2), (2, 2), (2, 3), (4, 4))
```

```
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```

Sequence Unpacking in For Statements



A sequence of
fixed-length sequences

```
>>> pairs = ((1, 2), (2, 2), (2, 3), (4, 4))
```

```
>>> same_count = 0
```

```
>>> for x, y in pairs:  
    if x == y:  
        same_count = same_count + 1
```

```
>>> same_count  
2
```

Sequence Unpacking in For Statements



A sequence of
fixed-length sequences

```
>>> pairs = ((1, 2), (2, 2), (2, 3), (4, 4))
```

```
>>> same_count = 0
```

A name for each element in
a fixed-length sequence

```
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    if x == y:  
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```

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>>> same_count  
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Sequence Unpacking in For Statements



A sequence of
fixed-length sequences

```
>>> pairs = ((1, 2), (2, 2), (2, 3), (4, 4))
```

```
>>> same_count = 0
```

A name for each element in
a fixed-length sequence

Each name is bound to a value,
as in multiple assignment

```
>>> for x, y in pairs:  
    if x == y:  
        same_count = same_count + 1
```

```
>>> same_count  
2
```

The Range Type



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A range is a sequence of consecutive integers.*

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* Ranges can actually represent more general integer sequences.

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..., -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, ...

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`range(-2, 3)`

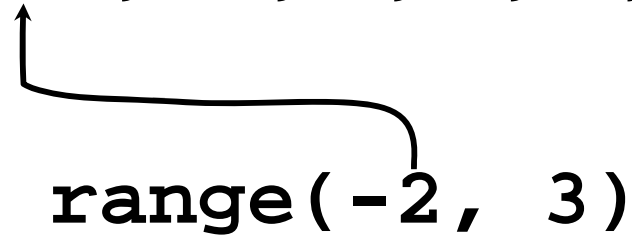
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..., -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, ...

A diagram consisting of a horizontal line with an arrowhead pointing to the left, starting from the number 3 and ending at the number -2. Below this line is the text `range(-2, 3)`.
`range(-2, 3)`

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The Range Type



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..., -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, ...

range(-2, 3)

The diagram shows a sequence of integers from -5 to 5. A bracket below the numbers -2, -1, 0, 1, 2, and 3 is labeled "range(-2, 3)". Two arrows point from the ends of this bracket up to the numbers -2 and 3, indicating that the range function generates the sequence of integers from -2 to 3.

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range(-2, 3)

Length: ending value - starting value

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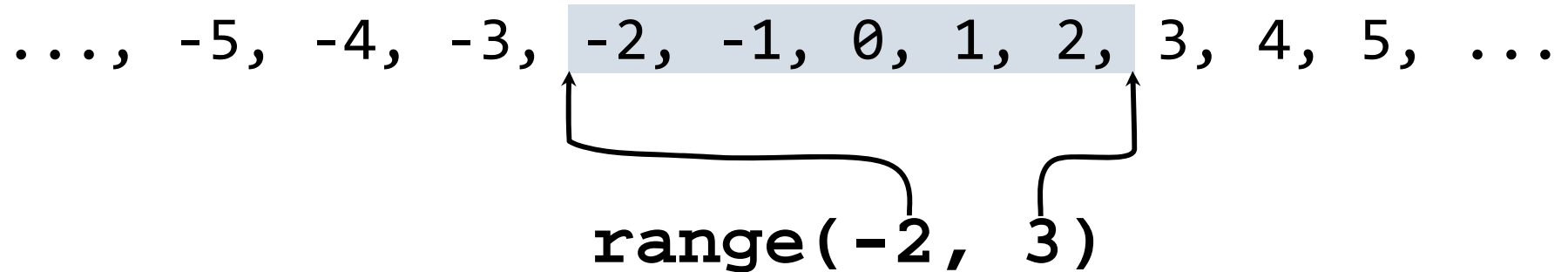
Element selection: starting value + index

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Length: ending value - starting value

Element selection: starting value + index

```
>>> tuple(range(-2, 3))  
(-2, -1, 0, 1, 2)
```

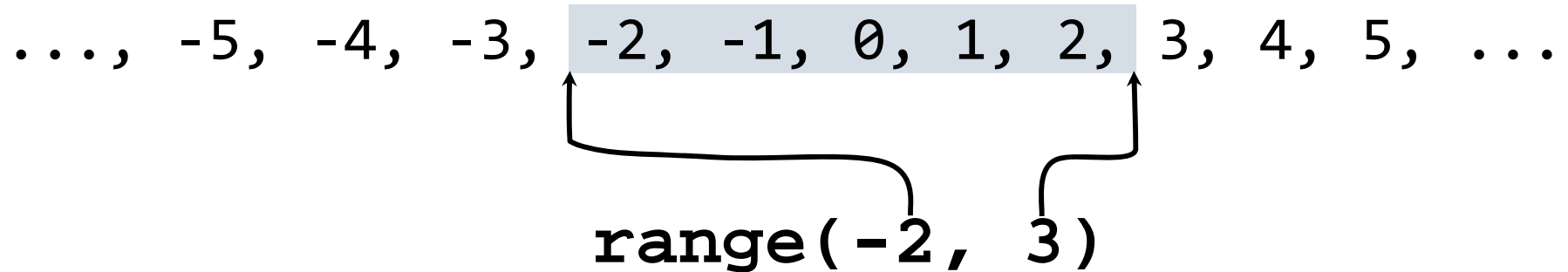
```
>>> tuple(range(4))  
(0, 1, 2, 3)
```

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Length: ending value - starting value

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>>> tuple(range(-2, 3))  
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Tuple constructor

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>>> tuple(range(4))  
(0, 1, 2, 3)
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Length: ending value - starting value

Element selection: starting value + index

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>>> tuple(range(-2, 3))  
(-2, -1, 0, 1, 2)
```

Tuple constructor

```
>>> tuple(range(4))  
(0, 1, 2, 3)
```

With a 0 starting value

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String Literals



String Literals



```
>>> 'I am string!'
'I am string!'
>>> "I've got an apostrophe"
"I've got an apostrophe"
>>> '您好'
'您好'
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Single- and double-quoted strings are equivalent

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Single- and double-quoted strings are equivalent

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claims, Readability counts.
Read more: import this."""
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A backslash "escapes" the following character

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"Line feed" character represents a new line

Strings Are Sequences



Strings Are Sequences



```
>>> city = 'Berkeley'  
>>> len(city)  
8  
>>> city[3]  
'k'
```

Strings Are Sequences

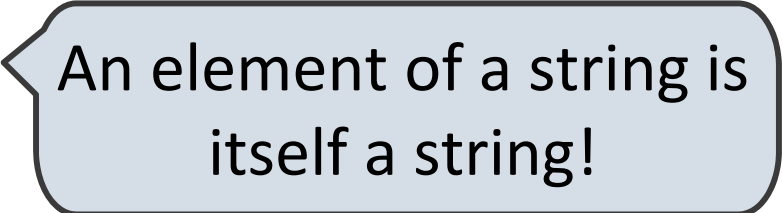


```
>>> city = 'Berkeley'
```

```
>>> len(city)
```

```
8
```

```
>>> city[3]  
'k'
```

A light blue callout box with a black border and a pointer pointing to the left, containing the text "An element of a string is itself a string!".

An element of a string is
itself a string!

Strings Are Sequences



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>>> city = 'Berkeley'  
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The **in** and **not in** operators match substrings

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The **in** and **not in** operators match substrings

```
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True
```

Why? Working with strings, we care about words, not characters

Sequence Arithmetic



Sequence Arithmetic



Some Python sequences support arithmetic operations

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```
>>> city = 'Berkeley'  
>>> city + ', CA'  
'Berkeley, CA'
```


Sequence Arithmetic



Some Python sequences support arithmetic operations

```
>>> city = 'Berkeley'  
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'Berkeley, CA'
```

A light blue rounded rectangular callout box with a pointer on the left side pointing towards the plus sign in the code above.

Concatenate

Sequence Arithmetic



Some Python sequences support arithmetic operations

```
>>> city = 'Berkeley'
```

```
>>> city + ', CA'
```

```
'Berkeley, CA'
```

Concatenate

```
>>> "Don't repeat yourself! " * 2
```

```
"Don't repeat yourself! Don't repeat yourself! "
```

Sequence Arithmetic



Some Python sequences support arithmetic operations

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>>> city = 'Berkeley'
```

```
>>> city + ', CA'
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```
'Berkeley, CA'
```

Concatenate

```
>>> "Don't repeat yourself! " * 2
```

```
"Don't repeat yourself! Don't repeat yourself! "
```

Repeat twice

Sequence Arithmetic



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>>> city = 'Berkeley'  
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'Berkeley, CA'
```

Concatenate

```
>>> "Don't repeat yourself! " * 2  
"Don't repeat yourself! Don't repeat yourself! "
```

Repeat twice

```
>>> (1, 2, 3) * 3  
(1, 2, 3, 1, 2, 3, 1, 2, 3)
```

```
>>> (1, 2, 3) + (4, 5, 6, 7)  
(1, 2, 3, 4, 5, 6, 7)
```

Sequences as Conventional Interfaces



Sequences as Conventional Interfaces



We can apply a function to every element in a sequence

Sequences as Conventional Interfaces



We can apply a function to every element in a sequence

This is called *mapping* the function over the sequence

Sequences as Conventional Interfaces



We can apply a function to every element in a sequence

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```
>>> fibs = tuple(map(fib, range(8)))
```

```
>>> fibs
```

```
(0, 1, 1, 2, 3, 5, 8, 13)
```


Sequences as Conventional Interfaces



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We can extract elements that satisfy a given condition

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```
>>> even_fibs = tuple(filter(is_even, fibs))
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(0, 2, 8)
```

Sequences as Conventional Interfaces



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We can compute the sum of all elements

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We can compute the sum of all elements

```
>>> sum(even_fibs)
10
```

Sequences as Conventional Interfaces



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>>> even_fibs
(0, 2, 8)
```

We can compute the sum of all elements

```
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10
```

Both **map** and **filter** produce an iterable, not a sequence

Iterables



Iterables



Iterables provide access to some elements in order but do not provide length or element selection

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Python-specific construct; more general than a sequence

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Many built-in functions take iterables as argument

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Python-specific construct; more general than a sequence

Many built-in functions take iterables as argument

tuple	Construct a tuple containing the elements
map	Construct a map that results from applying the given function to each element
filter	Construct a filter with elements that satisfy the given condition
sum	Return the sum of the elements
min	Return the minimum of the elements
max	Return the maximum of the elements

Iterables



Iterables provide access to some elements in order but do not provide length or element selection

Python-specific construct; more general than a sequence

Many built-in functions take iterables as argument

tuple	Construct a tuple containing the elements
map	Construct a map that results from applying the given function to each element
filter	Construct a filter with elements that satisfy the given condition
sum	Return the sum of the elements
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For statements also operate on iterable values.

Generator Expressions



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One large expression that combines mapping and filtering to produce an iterable

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```
( <map exp> for <name> in <iter exp> if <filter exp> )
```

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- Evaluates to an iterable.

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( <map exp> for <name> in <iter exp> if <filter exp> )
```

- Evaluates to an iterable.
- `<iter exp>` is evaluated when the generator expression is evaluated.

Generator Expressions



One large expression that combines mapping and filtering to produce an iterable

```
( <map exp> for <name> in <iter exp> if <filter exp> )
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- Evaluates to an iterable.
- `<iter exp>` is evaluated when the generator expression is evaluated.
- Remaining expressions are evaluated when elements are accessed.

Generator Expressions



One large expression that combines mapping and filtering to produce an iterable

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No-filter version: (`<map exp>` for `<name>` in `<iter exp>`)

Generator Expressions



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- `<iter exp>` is evaluated when the generator expression is evaluated.
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No-filter version: (`<map exp>` for `<name>` in `<iter exp>`)

Precise evaluation rule introduced in Chapter 4.

Reducing a Sequence



Reducing a Sequence



Reduce is a higher-order generalization of max, min, and sum.

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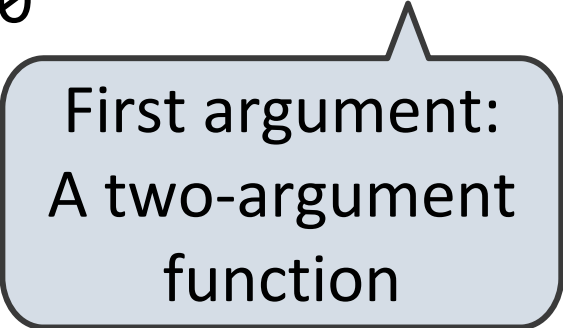
```
>>> from operator import mul
>>> from functools import reduce
>>> reduce(mul, (1, 2, 3, 4, 5), 1)
120
```

Reducing a Sequence



Reduce is a higher-order generalization of max, min, and sum.

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>>> from operator import mul
>>> from functools import reduce
>>> reduce(mul, (1, 2, 3, 4, 5), 1)
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```

A light blue callout box with a black border and a pointer pointing to the first argument of the reduce function in the code above.

First argument:
A two-argument
function

Reducing a Sequence



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Optional initial
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Like accumulate from Homework 2, but with iterables

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Optional initial value as third argument

First argument:
A two-argument function

Second argument:
an iterable object

Like accumulate from Homework 2, but with iterables

```
def accumulate(combiner, start, n, term):
    return reduce(combiner,
                  map(term, range(1, n + 1)),
                  start)
```

More Functions on Iterables (Bonus)



Create an iterable of fixed-length sequences

```
>>> a, b = (1, 2, 3), (4, 5, 6, 7)
>>> for x, y in zip(a, b):
...     print(x + y)
...
5
7
9
```

Produces tuples with one element from each argument, up to length of smallest argument

The **itertools** module contains many useful functions for working with iterables

```
>>> from itertools import product, combinations
>>> tuple(product(a, b[:2]))
((1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5))
>>> tuple(combinations(a, 2))
((1, 2), (1, 3), (2, 3))
```