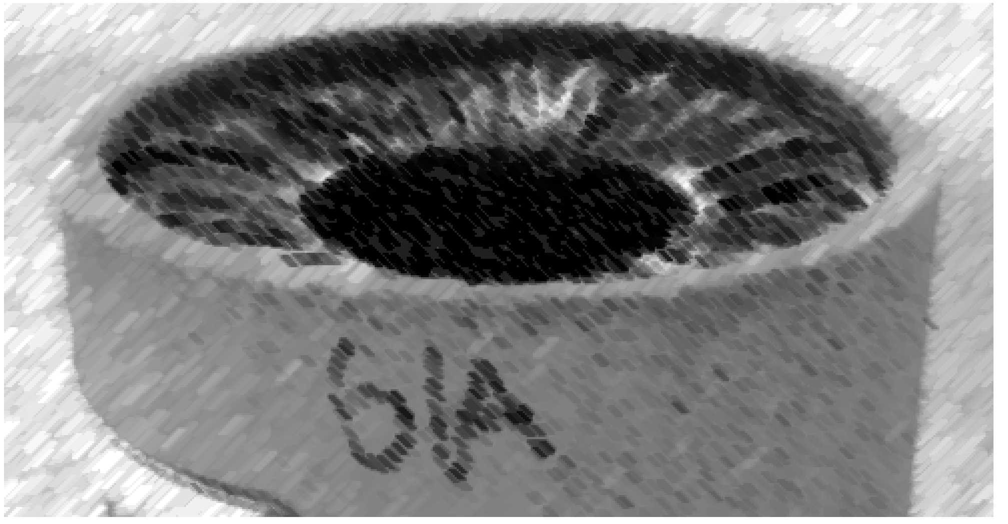

CS61A Lecture 14

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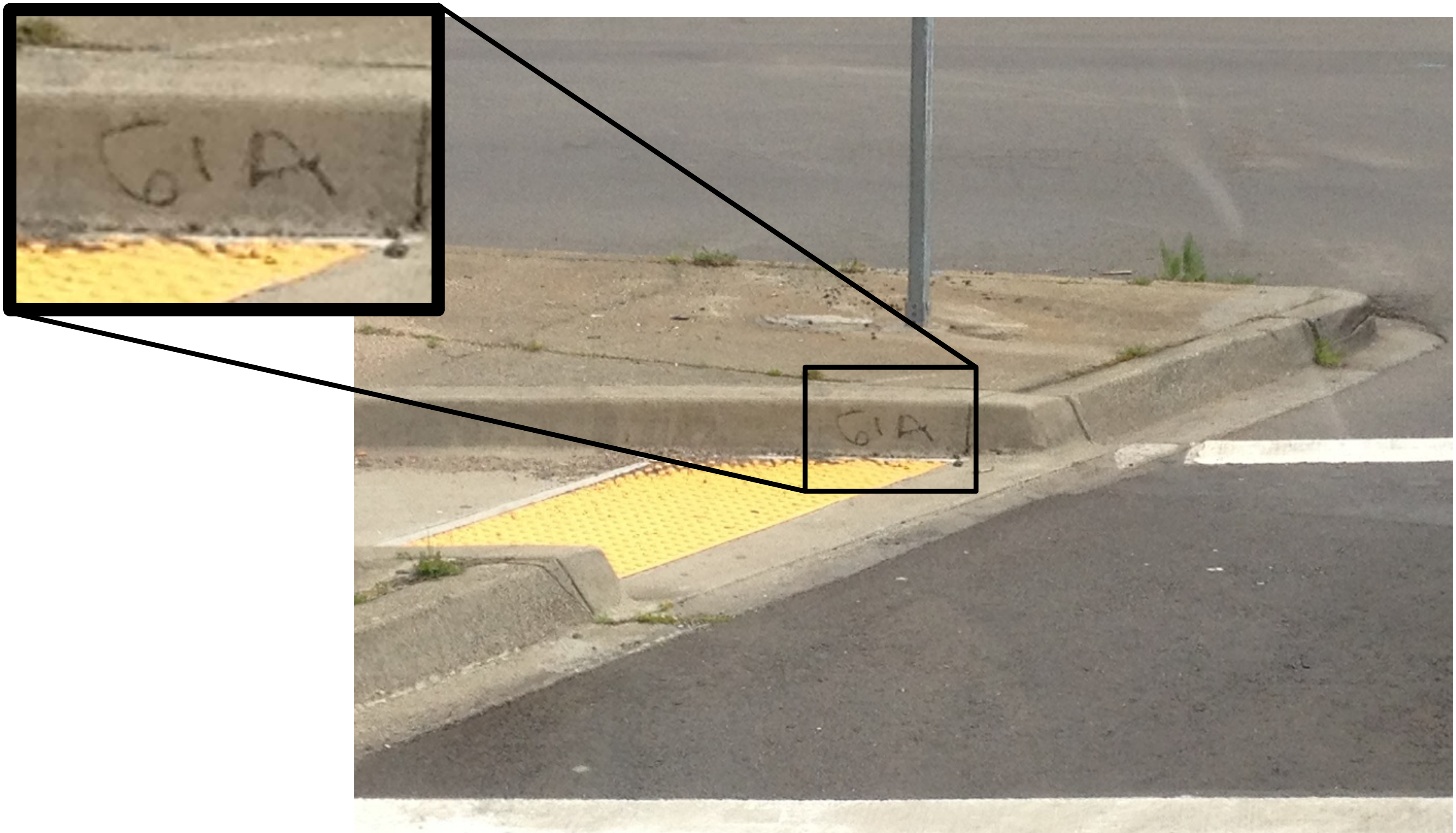
The 61A Graffiti Bandit Strikes Again!

Thanks to Colin Lockard for the picture (and the title)!



Announcements

- HW5 out
- Hog contest due today
  - Completely optional, opportunity for extra credit
  - See website for details
- Trends project out today

Rational Number Arithmetic Code

```
def mul_rational(x, y):
    return rational(numer(x) * numer(y),
                    denom(x) * denom(y))
```

Constructor

Selectors

```
def add_rational(x, y):
    nx, dx = numer(x), denom(x)
    ny, dy = numer(y), denom(y)
    return rational(nx * dy + ny * dx, dx * dy)

def eq_rational(x, y):
    return numer(x) * denom(y) == numer(y) * denom(x)
```

Wishful thinking

- `rational(n, d)` returns a rational number *x*
- `numer(x)` returns the numerator of *x*
- `denom(x)` returns the denominator of *x*

Tuples

```
>>> pair = (1, 2)
>>> pair
(1, 2)
```

A tuple literal:
Comma-separated expression

```
>>> x, y = pair
>>> x
1
>>> y
2
```

"Unpacking" a tuple

```
>>> pair[0]
1
>>> pair[1]
2
>>> from operator import getitem
>>> getitem(pair, 0)
1
>>> getitem(pair, 1)
2
```

Element selection

More on tuples today

Representing Rational Numbers

```
def rational(n, d):
    """Construct a rational number x that represents
    n/d."""
    return (n, d)
```

Construct a tuple

```
from operator import getitem

def numer(x):
    """Return the numerator of rational number x."""
    return getitem(x, 0)

def denom(x):
    """Return the denominator of rational number
    x."""
    return getitem(x, 1)
```

Select from a tuple

Reducing to Lowest Terms

Example:

$$\frac{3}{2} * \frac{5}{3} = \boxed{\frac{5}{2}}$$

$$\frac{15}{6} * \frac{1/3}{1/3} = \frac{5}{2}$$

$$\frac{2}{5} + \frac{1}{10} = \boxed{\frac{1}{2}}$$

$$\frac{25}{50} * \frac{1/25}{1/25} = \frac{1}{2}$$

```
from fractions import gcd
```

Greatest common divisor

```
def rational(n, d):
    """Construct a rational number x that represents
    n/d."""
    g = gcd(n, d)
    return (n//g, d//g)
```

Abstraction Barriers

Rational numbers as whole data values

`add_rational   mul_rational   eq_rational`

Rational numbers as numerators & denominators

`rational   numer   denom`

Rational numbers as tuples

`tuple   getitem`

However tuples are implemented in Python

Violating Abstraction Barriers

Does not use constructors

Twice!

```
add_rational( (1, 2), (1, 4) )
```

```
def divide_rational(x, y):
    return (x[0] * y[1], x[1] * y[0])
```

No selectors!

And no constructor!

What is an Abstract Data Type?

- We need to guarantee that constructor and selector functions together specify the right behavior.
- Behavior condition: If we construct rational number *x* from numerator *n* and denominator *d*, then **`numer(x)/denom(x)`** must equal *n/d*.
- An abstract data type is some collection of selectors and constructors, together with some behavior condition(s).
- If behavior conditions are met, the representation is valid.

You can recognize data types by behavior, not by bits

Behavior Conditions of a Pair

To implement our rational number abstract data type, we used a two-element tuple (also known as a pair).

What is a pair?

Constructors, selectors, and behavior conditions:

If a pair *p* was constructed from elements *x* and *y*, then

- **`getitem_pair(p, 0)`** returns *x*, and
- **`getitem_pair(p, 1)`** returns *y*.

Together, selectors are the inverse of the constructor

Generally true of container types.

Not true for rational numbers because of GCD

Functional Pair Implementation

```
def pair(x, y):
    """Return a functional pair."""
    def dispatch(m):
        if m == 0:
            return x
        elif m == 1:
            return y
    return dispatch
```

This function represents a pair

Constructor is a higher-order function

```
def getitem_pair(p, i):
    """Return the element at index i of pair p."""
    return p(i)
```

Selector defers to the functional pair

Using a Functionally Implemented Pair

```
>>> p = pair(1, 2)

>>> getitem_pair(p, 0)
1

>>> getitem_pair(p, 1)
2
```

As long as we do not violate the abstraction barrier, we don't need to know that pairs are just functions

If a pair *p* was constructed from elements *x* and *y*, then

- **`getitem_pair(p, 0)`** returns *x*, and
- **`getitem_pair(p, 1)`** returns *y*.

This pair representation is valid!

The Sequence Abstraction

red, orange, yellow, green, blue, indigo, violet.

0 , 1 , 2 , 3 , 4 , 5 , 6 .

There isn't just one sequence type (in Python or in general)

This abstraction is a collection of behaviors:

Length. A sequence has a finite length.

Element selection. A sequence has an element corresponding to any non-negative integer index less than its length, starting at 0 for the first element.

The sequence abstraction is shared among several types, including tuples.

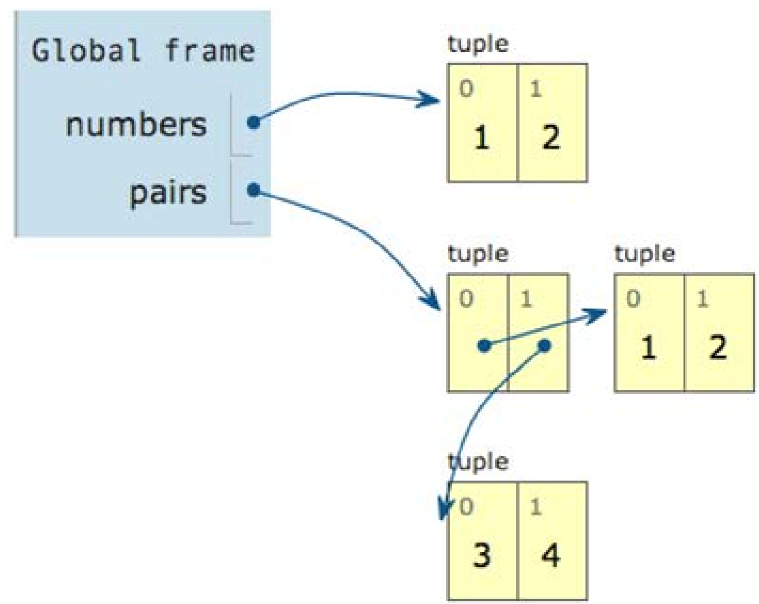
Tuples in Environment Diagrams

Tuples introduce new memory locations outside of a frame

We use *box-and-pointer* notation to represent a tuple

- Tuple itself represented by a set of boxes that hold values
- Tuple value represented by a pointer to that set of boxes

```
numbers = (1, 2)
pairs = ((1, 2), (3, 4))
```



Example: http://goo.gl/iFHx0

The Closure Property of Data Types

A method for combining data values satisfies the closure property if:

> The result of combination can itself be combined using the same method.

Closure is the key to power in any means of combination because it permits us to create hierarchical structures.

Hierarchical structures are made up of parts, which themselves are made up of parts, and so on.

Tuples can contain tuples as elements

Recursive Lists

Constructor:

```
def rlist(first, rest):
    """Return a recursive list from its first element and
    the rest."""
```

Selectors:

```
def first(s):
    """Return the first element of recursive list s."""

def rest(s):
    """Return the remaining elements of recursive list s."""
```

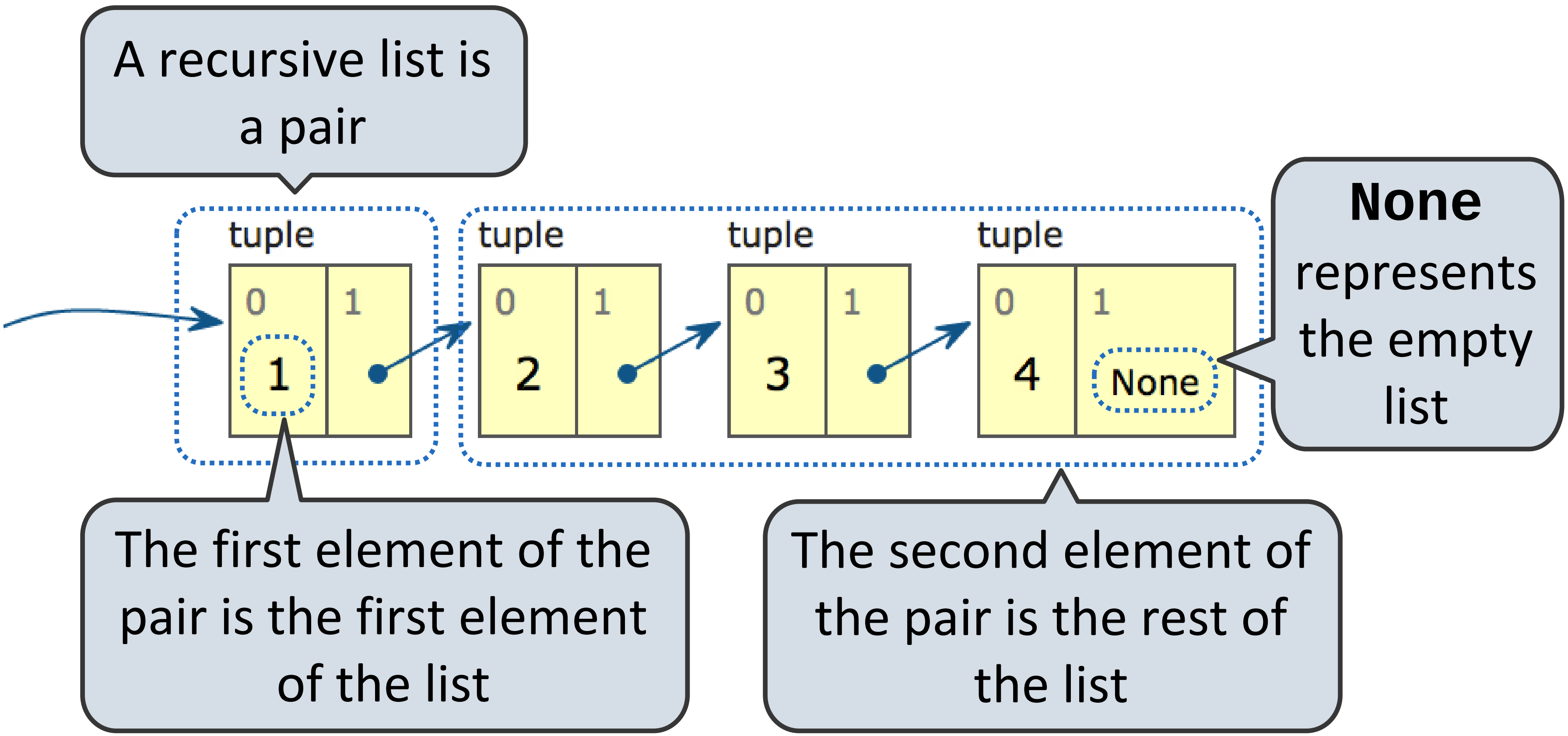
Behavior condition(s):

If a recursive list **`s`** is constructed from a first element **`f`** and a recursive list **`r`**, then

- **`first(s)`** returns **`f`**, and
- **`rest(s)`** returns **`r`**, which is a recursive list.

Implementing Recursive Lists Using Pairs

1 , 2 , 3 , 4



Example: http://goo.gl/fVhbF

Implementing the Sequence Abstraction

```
def len_rlist(s):
    """Return the length of recursive list s."""
    if s == empty_rlist:
        return 0
    return 1 + len_rlist(rest(s))

def getitem_rlist(s, i):
    """Return the element at index i of recursive list s."""
    if i == 0:
        return first(s)
    return getitem_rlist(rest(s), i - 1)
```

Length. A sequence has a finite length.

Element selection. A sequence has an element corresponding to any non-negative integer index less than its length, starting at 0 for the first element.