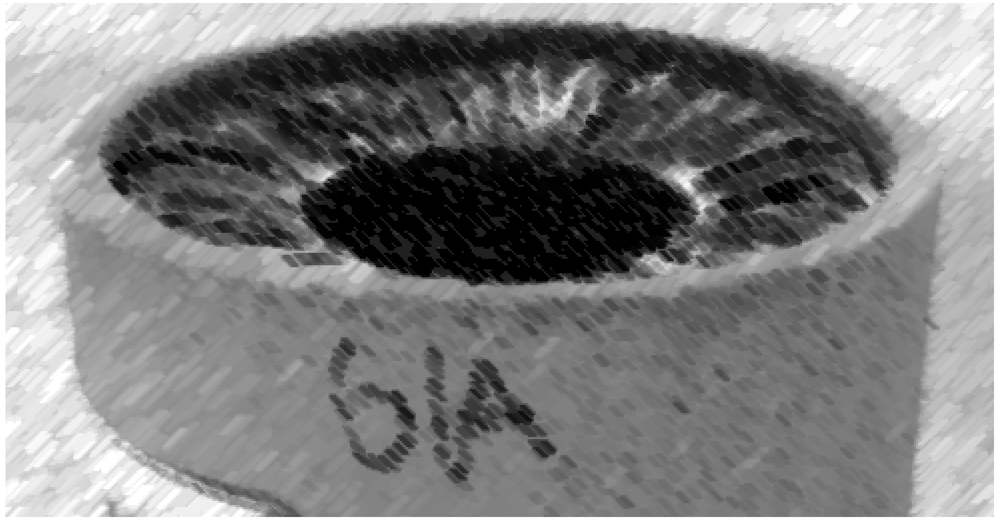
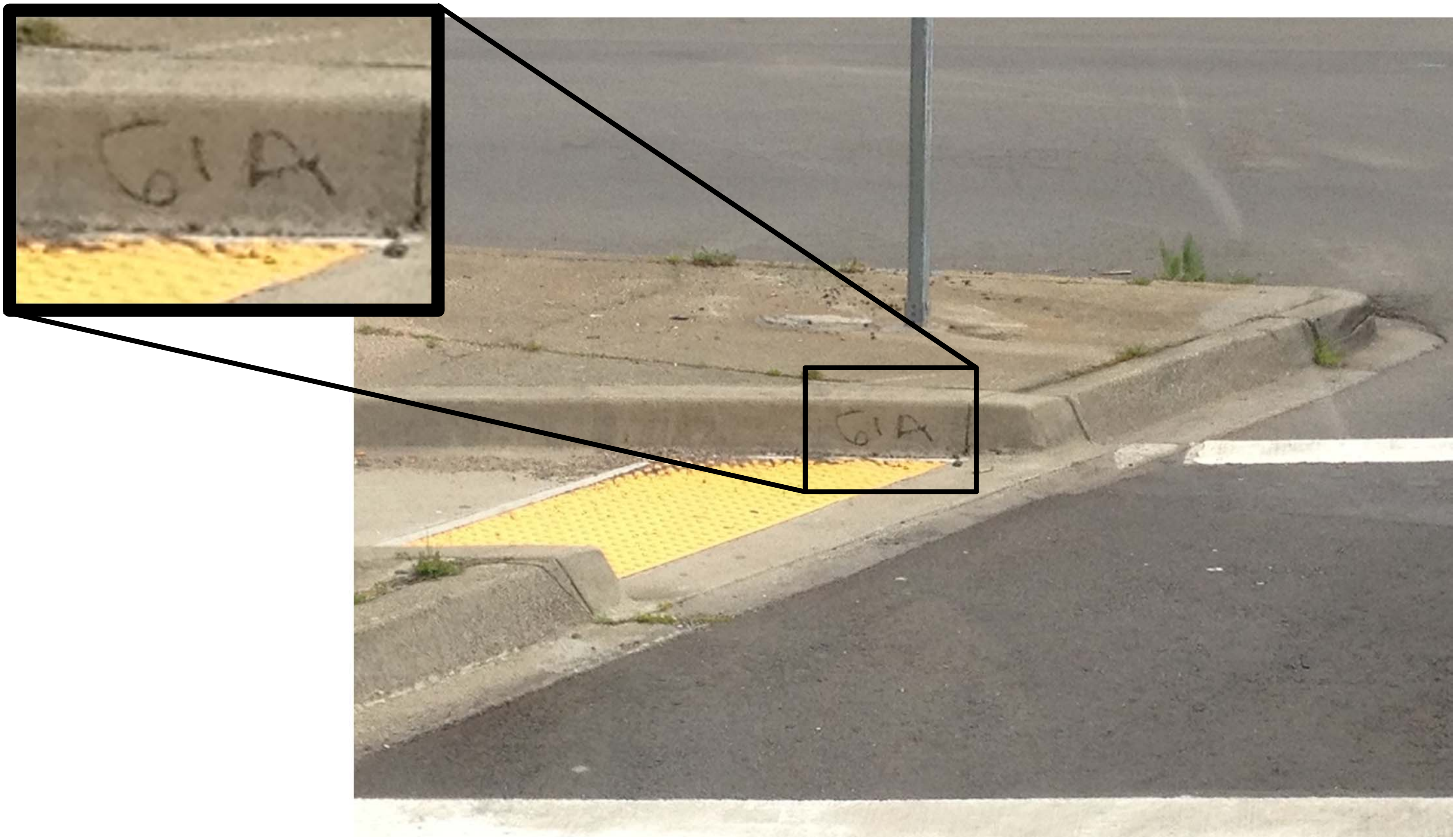


CS61A Lecture 14

Amir Kamil
UC Berkeley
February 22, 2013

The 61A Graffiti Bandit Strikes Again!

Thanks to Colin Lockard for the picture (and the title)!



Announcements

- HW5 out
- Hog contest due today
  - Completely optional, opportunity for extra credit
  - See website for details
- Trends project out today

Rational Number Arithmetic Code

```
def mul_rational(x, y):
    return rational(numer(x) * numer(y),
                    denom(x) * denom(y))
```

Constructor

Selectors

```
def add_rational(x, y):
    nx, dx = numer(x), denom(x)
    ny, dy = numer(y), denom(y)
    return rational(nx * dy + ny * dx, dx * dy)

def eq_rational(x, y):
    return numer(x) * denom(y) == numer(y) * denom(x)
```

Wishful thinking

- **`rational(n, d)`** returns a rational number *x*
- **`numer(x)`** returns the numerator of *x*
- **`denom(x)`** returns the denominator of *x*

Tuples

```
>>> pair = (1, 2)
>>> pair
(1, 2)
```

A tuple literal:
Comma-separated expression

```
>>> x, y = pair
>>> x
1
>>> y
2
```

"Unpacking" a tuple

```
>>> pair[0]
1
>>> pair[1]
2
>>> from operator import getitem
>>> getitem(pair, 0)
1
>>> getitem(pair, 1)
2
```

Element selection

More on tuples today

Representing Rational Numbers

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```
def rational(n, d):
    """Construct a rational number x that represents
    n/d."""
    return (n, d)
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```

Select from a tuple

Reducing to Lowest Terms

Example:

Reducing to Lowest Terms

Example:

$$\frac{3}{2} * \frac{5}{3}$$

Reducing to Lowest Terms

Example:

$$\frac{3}{2} * \frac{5}{3} = \frac{5}{2}$$

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$$\frac{3}{2} * \frac{5}{3} = \frac{5}{2}$$

$$\frac{15}{6} * \frac{1/3}{1/3} = \frac{5}{2}$$

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$$\frac{2}{5} + \frac{1}{10}$$

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Example:

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Example:

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```
from fractions import gcd
```

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```
from fractions import gcd

def rational(n, d):
    """Construct a rational number x that represents
    n/d."""
    g = gcd(n, d)
    return (n//g, d//g)
```

Reducing to Lowest Terms

Example:

$$\frac{3}{2} * \frac{5}{3} = \frac{5}{2} \qquad \frac{2}{5} + \frac{1}{10} = \frac{1}{2}$$

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```
from fractions import gcd
```

Greatest common divisor

```
def rational(n, d):
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    n/d."""
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```

Abstraction Barriers

Rational numbers as whole data values

`add_rational   mul_rational   eq_rational`

Rational numbers as numerators & denominators

`rational   numer   denom`

Rational numbers as tuples

`tuple   getitem`

However tuples are implemented in Python

Violating Abstraction Barriers

```
add_rational( (1, 2), (1, 4) )

def divide_rational(x, y):
    return (x[0] * y[1], x[1] * y[0])
```

Violating Abstraction Barriers

Does not use constructors

```
add_rational( (1, 2), (1, 4) )

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Twice!

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No selectors!

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add_rational( (1, 2), (1, 4) )
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No selectors!

And no constructor!

What is an Abstract Data Type?

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You can recognize data types by behavior, not by bits

Behavior Conditions of a Pair

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Generally true of container types.

Not true for rational numbers because of GCD

Functional Pair Implementation

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```
def pair(x, y):
    """Return a functional pair."""
    def dispatch(m):
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def getitem_pair(p, i):
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Selector defers to the functional pair

Using a Functionally Implemented Pair

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This pair representation is valid!

The Sequence Abstraction

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red, orange, yellow, green, blue, indigo, violet.

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Length. A sequence has a finite length.

Element selection. A sequence has an element corresponding to any non-negative integer index less than its length, starting at 0 for the first element.

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0 , 1 , 2 , 3 , 4 , 5 , 6 .

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This abstraction is a collection of behaviors:

Length. A sequence has a finite length.

Element selection. A sequence has an element corresponding to any non-negative integer index less than its length, starting at 0 for the first element.

The sequence abstraction is shared among several types, including tuples.

Tuples in Environment Diagrams

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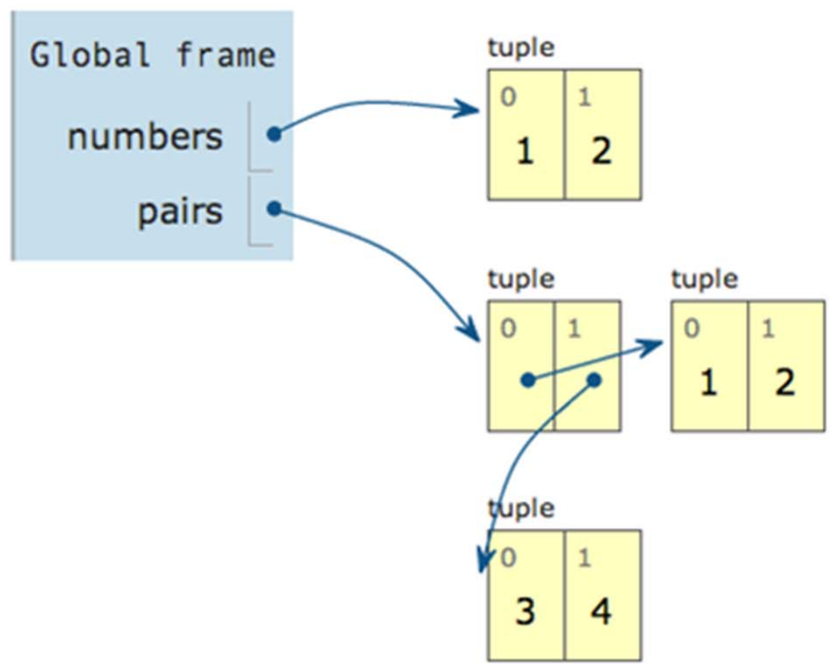
```
numbers = (1, 2)
pairs = ((1, 2), (3, 4))
```



Example: http://goo.gl/iFHx0

The Closure Property of Data Types

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Hierarchical structures are made up of parts, which themselves are made up of parts, and so on.

Tuples can contain tuples as elements

Recursive Lists

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If a recursive list **s** is constructed from a first element **f** and a recursive list **r**, then

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If a recursive list **`s`** is constructed from a first element **`f`** and a recursive list **`r`**, then

- **`first(s)`** returns **`f`**, and

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Selectors:

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def rest(s):
    """Return the remaining elements of recursive list s."""
```

Behavior condition(s):

If a recursive list **s** is constructed from a first element **f** and a recursive list **r**, then

- **first(s)** returns **f**, and
- **rest(s)** returns **r**, which is a recursive list.

Implementing Recursive Lists Using Pairs

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1 , 2 , 3 , 4

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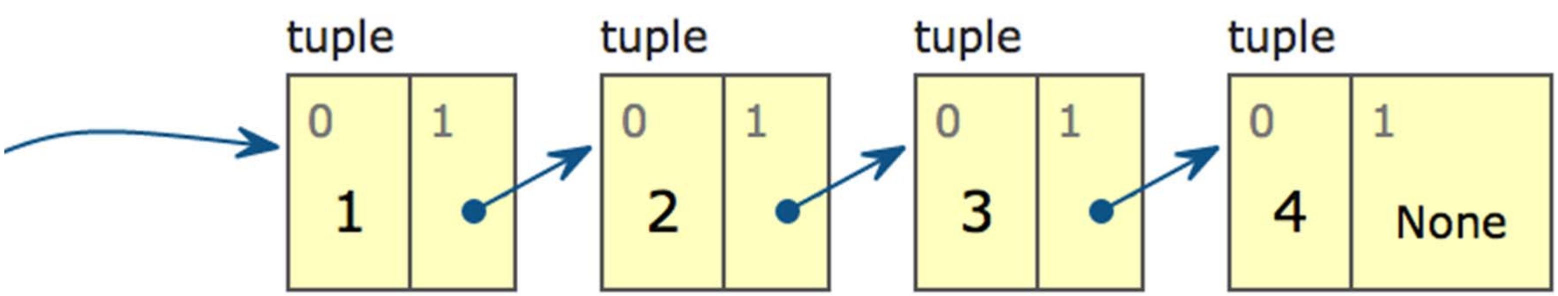

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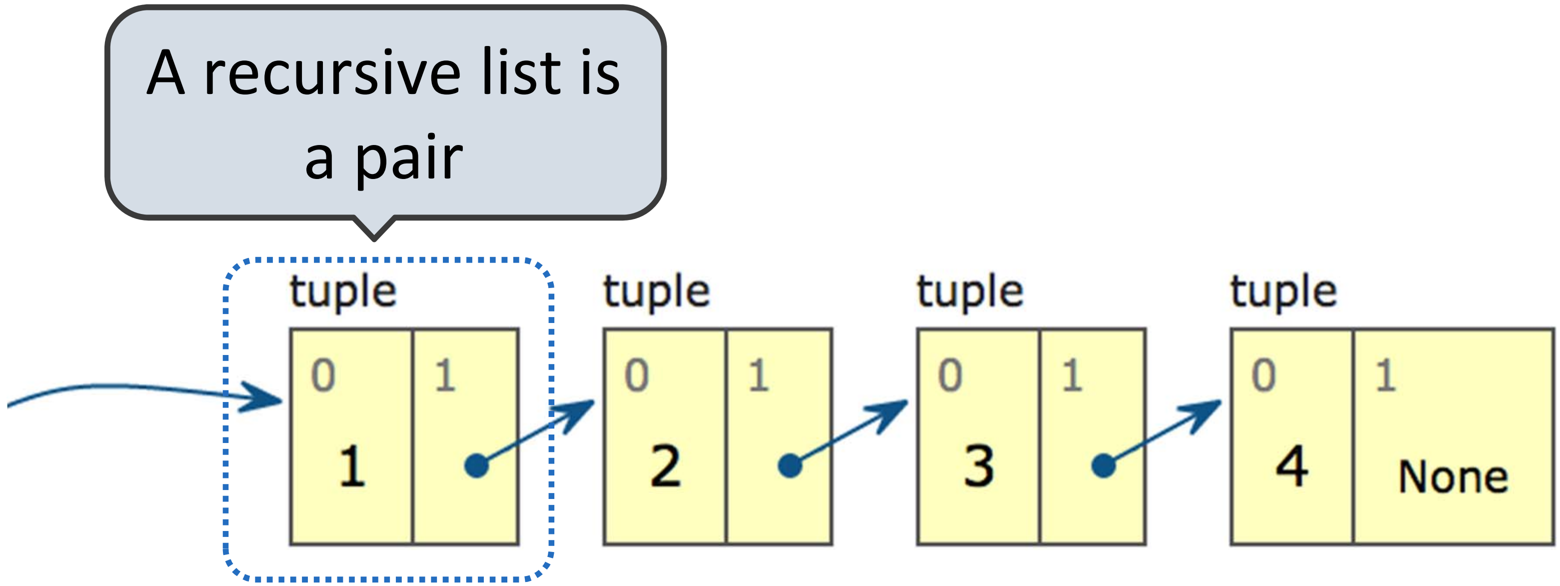

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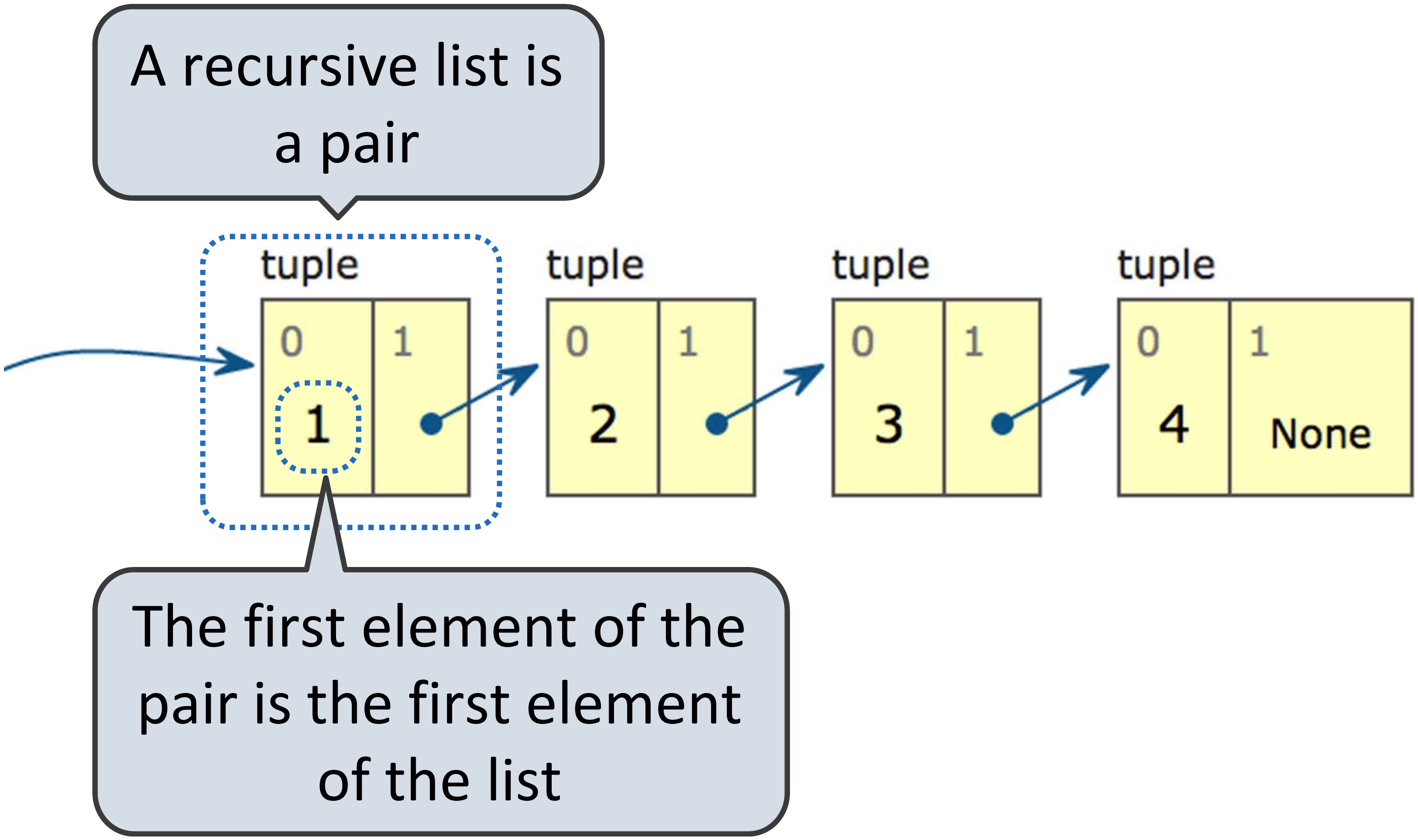
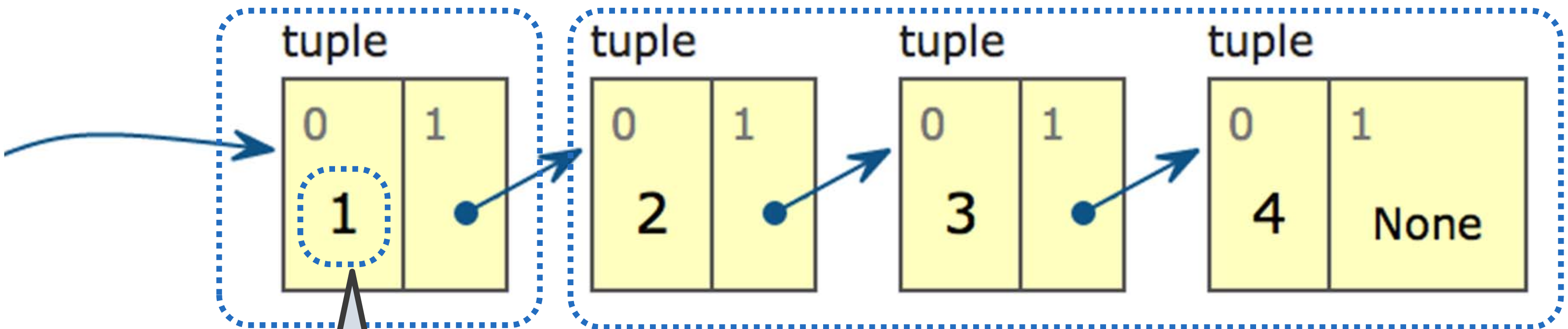


Implementing Recursive Lists Using Pairs

1 , 2 , 3 , 4

A recursive list is a pair

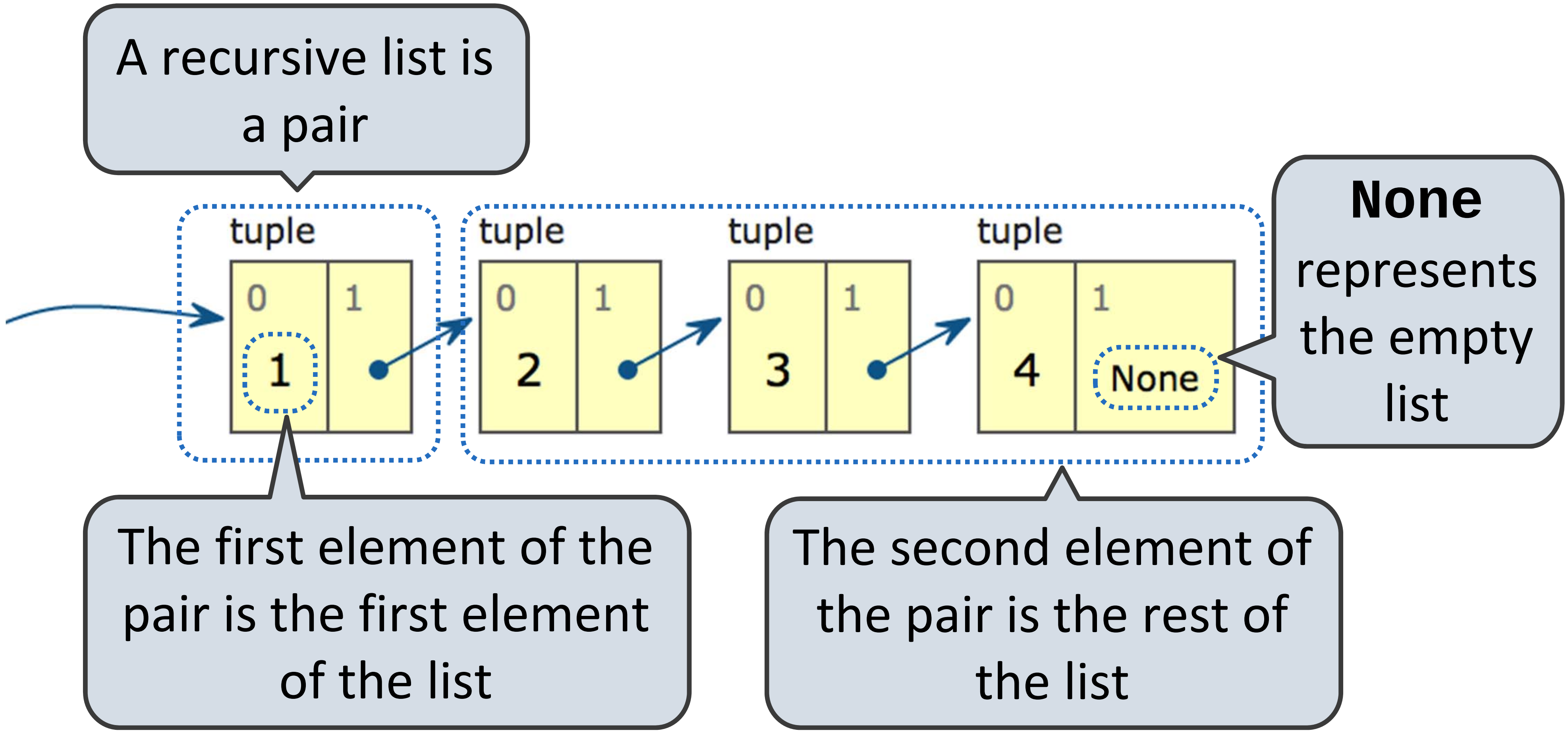


The first element of the pair is the first element of the list

The second element of the pair is the rest of the list

Implementing Recursive Lists Using Pairs

1 , 2 , 3 , 4



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Example: http://goo.gl/fVhbF

Implementing the Sequence Abstraction

```
def len_rlist(s):
    """Return the length of recursive list s."""
    if s == empty_rlist:
        return 0
    return 1 + len_rlist(rest(s))
```

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def len_rlist(s):
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        return 0
    return 1 + len_rlist(rest(s))

def getitem_rlist(s, i):
    """Return the element at index i of recursive list s."""
    if i == 0:
        return first(s)
    return getitem_rlist(rest(s), i - 1)
```

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