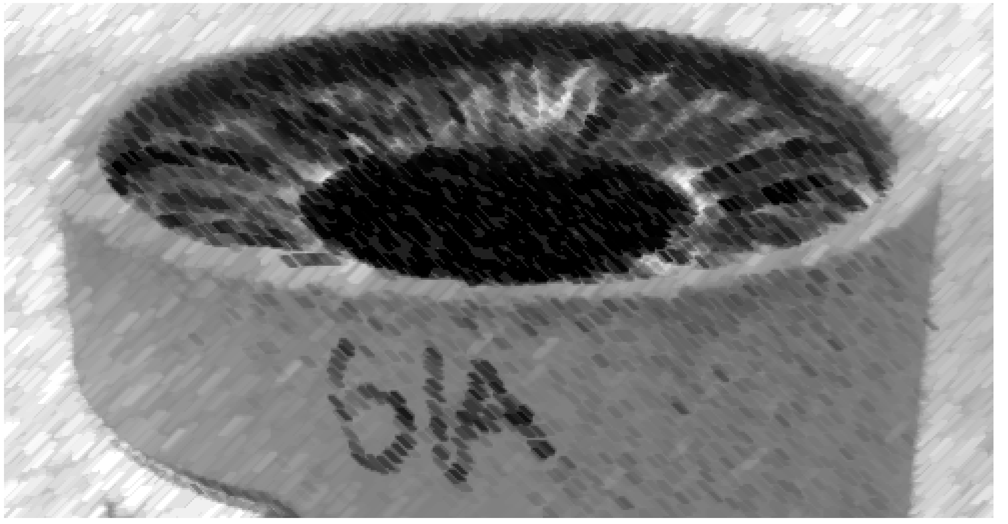

CS61A Lecture 13

Amir Kamil
UC Berkeley
February 20, 2013

Announcements

- HW4 due today at 11:59pm

- Hog contest deadline on Friday
  - Completely optional, opportunity for extra credit
  - See website for details

Converting Recursion to Iteration

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def summation(n, term):
    if n == 0:
        return 0
    return summation(n - 1, term) + term(n)
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What's summed so far?

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def summation(n, term):
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Termination condition: `n == 0`

Initial value: `0`

What's summed so far?: `summation(n - 1, term)`

How to get each incremental piece: `term(n)`

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def summation(n, term):
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Termination condition (`n == 0`)

Initial value (`0`)

What's summed so far? (`summation(n - 1, term)`)

How to get each incremental piece (`term(n)`)

```
def summation_iter(n, term):
    total = 0
    while n > 0:
        total, n = total + term(n), n – 1
    return total
```

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```
def fib_iter(n):
    if n == 0:
        return 0
    fib_n, fib_n_1, k = 1, 0, 1
    while k < n:
        fib_n, fib_n_1 = fib_n + fib_n_1, fib_n
        k = k + 1
    return fib_n
```

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        k = k + 1
    return fib_n
```

Local names become...

```
def fib_rec(n, fib_n, fib_n_1, k):
    if n == 0:
        return 0
    if k >= n:
        return fib_n
    return fib_rec(n, fib_n + fib_n_1, fib_n, k + 1)
```

Converting Iteration to Recursion

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Parameters in a recursive function

Mutual Recursion

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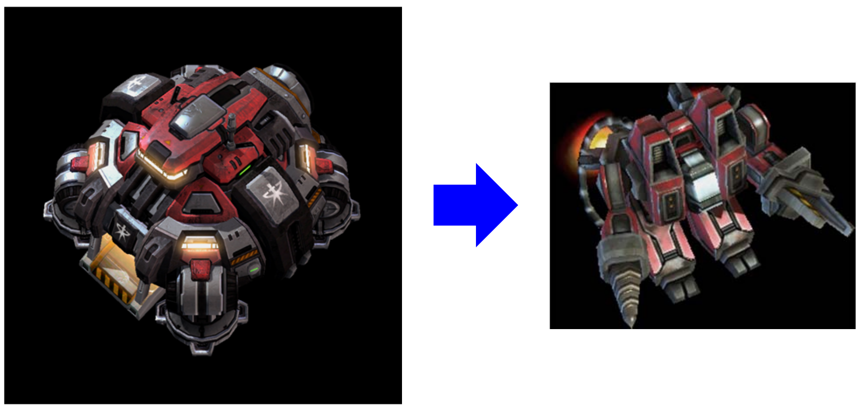

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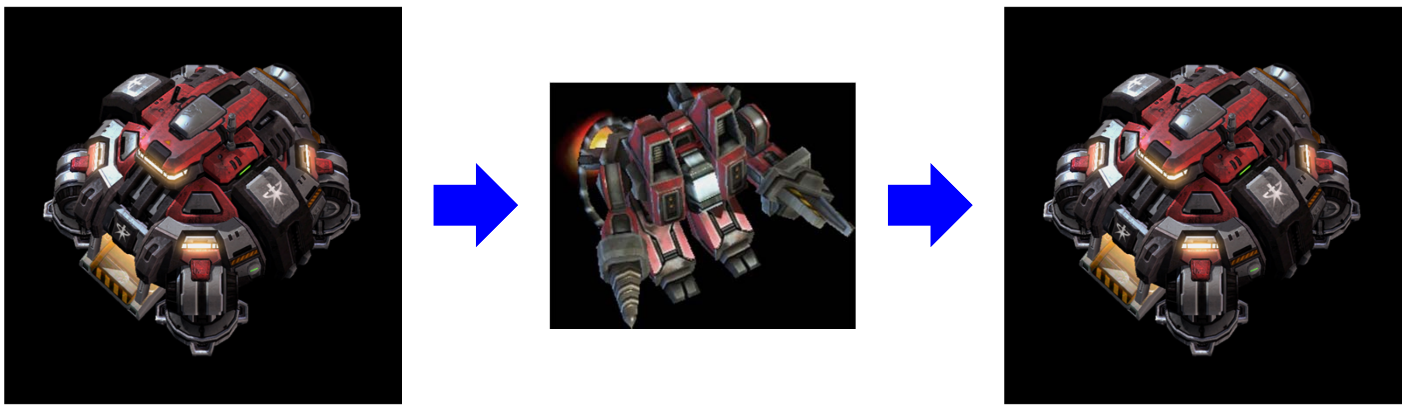

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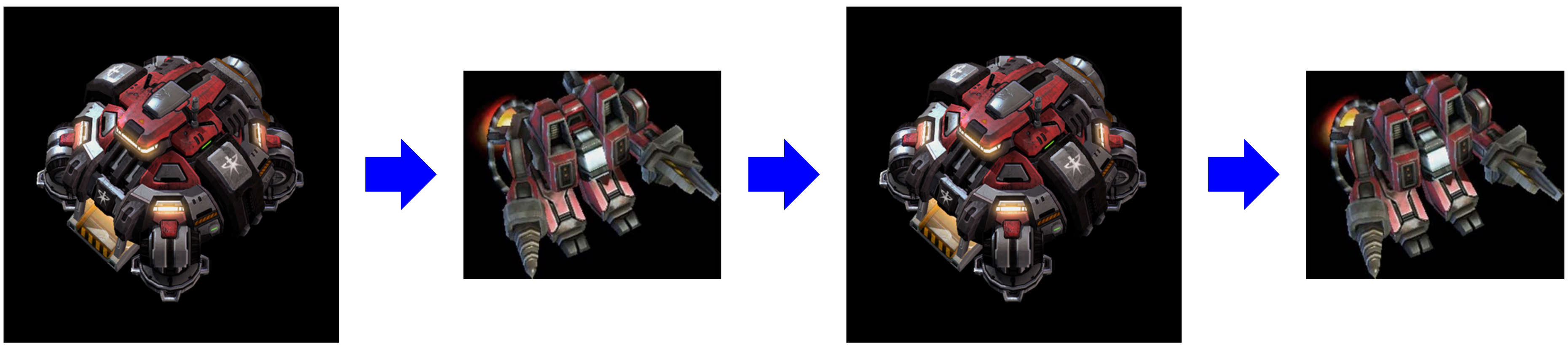

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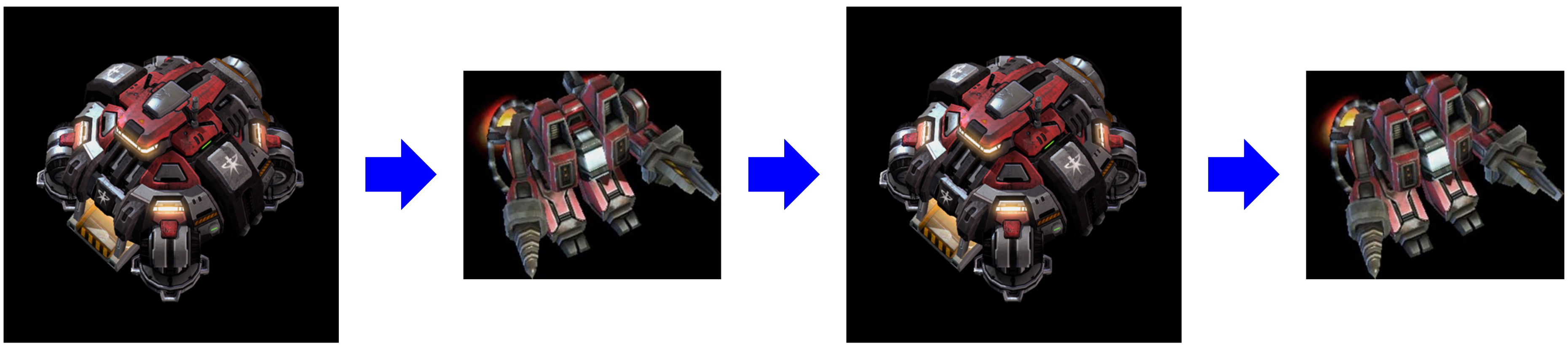

Decorating a recursive function generally results in mutual recursion

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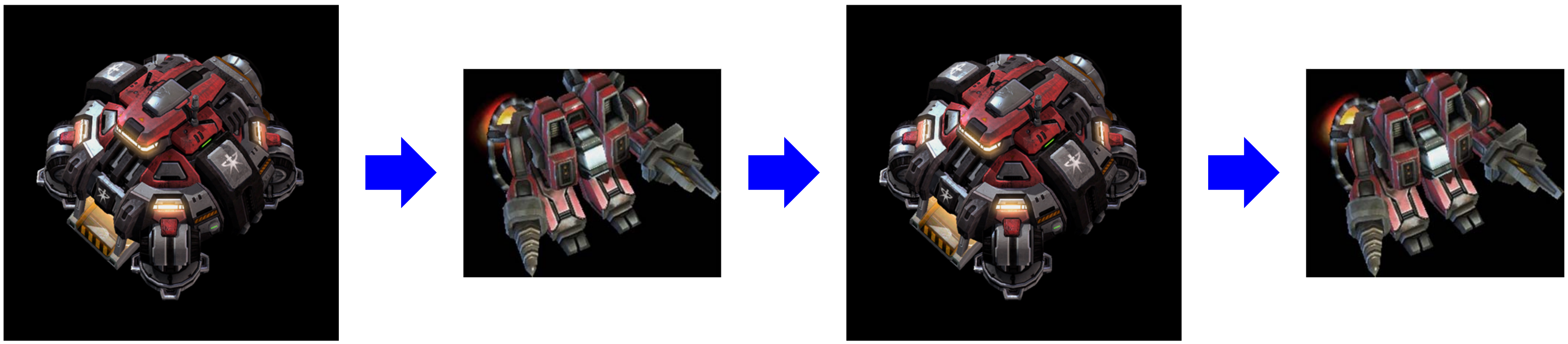

Decorating a recursive function generally results in mutual recursion

```
@trace1
def factorial(n):
    if n == 0:
        return 1
    return n * factorial(n-1)
```

Example: http://goo.gl/4LZZv

Currying

Currying

We have used higher-order functions to produce a function to add a constant to its argument

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```
def make_adder(n):
    def adder(k):
        return add(n, k)
    return adder

>>> make_adder(2)(3)
5
>>> add(2, 3)
5
```

Currying

We have used higher-order functions to produce a function to add a constant to its argument

What if we wanted to do the same for multiplication?

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def make_multiplier(n):
    def multiplier(k):
        return mul(n, k)
    return multiplier

>>> make_multiplier(2)(3)
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>>> mul(2, 3)
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Same relationship between functions

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```

Same relationship between functions

How can we do this in general without repeating ourselves?

Currying

```
def make_adder(n):
    def adder(k):
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    return adder

>>> make_adder(2)(3)
5
>>> add(2, 3)
5
```

Currying

First, identify common structure.

```
def make_adder(n):
    def adder(k):
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Currying

First, identify common structure.

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def make_adder(n):
    def adder(k):
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>>> make_adder(2)(3)
5
>>> add(2, 3)
5
```

```
def outer(n):
    def inner(k):
        return f(n, k)
    return inner
```

Currying

First, identify common structure.

Then define a function that generalizes the procedure.

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def make_adder(n):
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```

```
def curry2(f):
    def outer(n):
        def inner(k):
            return f(n, k)
        return inner
    return outer
```

Currying

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Then define a function that generalizes the procedure.

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def make_adder(n):
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```

```
def curry2(f):
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>>> curry2(mul)(2)(3)
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```

Currying

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>>> curry2(mul)(2)(3)
6
>>> mul(2, 3)
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```

This process of converting a multi-argument function to consecutive single-argument functions is called *currying*.

Functional Abstractions

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```
def square(x):
    return mul(x, x)
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def square(x):
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```
def sum_squares(x, y):
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def square(x):              def sum_squares(x, y):
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What does **sum_squares** need to know about **square**?

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What does **sum_squares** need to know about **square**?

- **square** takes one argument.

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What does **sum_squares** need to know about **square**?

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If the name "square" were bound to a built-in function, sum_squares would still work identically

What is Data?

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Data: the things that programs fiddle with

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Primitive values are the simplest type of data

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Primitive values are the simplest type of data

- Integers: 2, 3, 2013, -837592010
- Floating point (decimal) values: -4.5, 98.6
- Booleans: True, False

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How do we represent more complex data?

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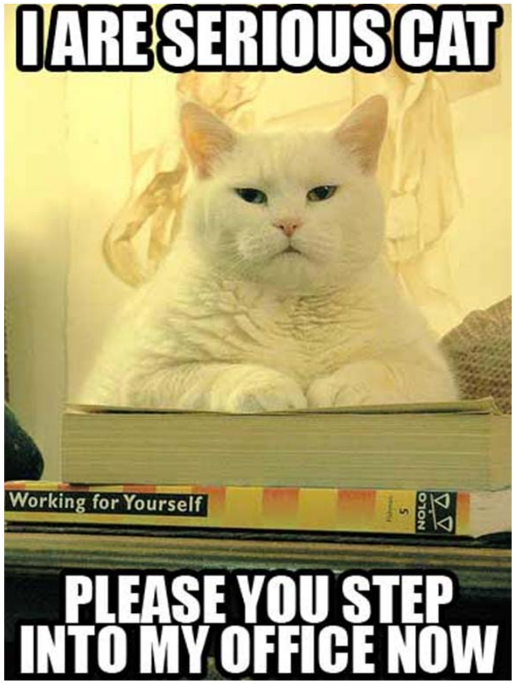
How do we represent more complex data?



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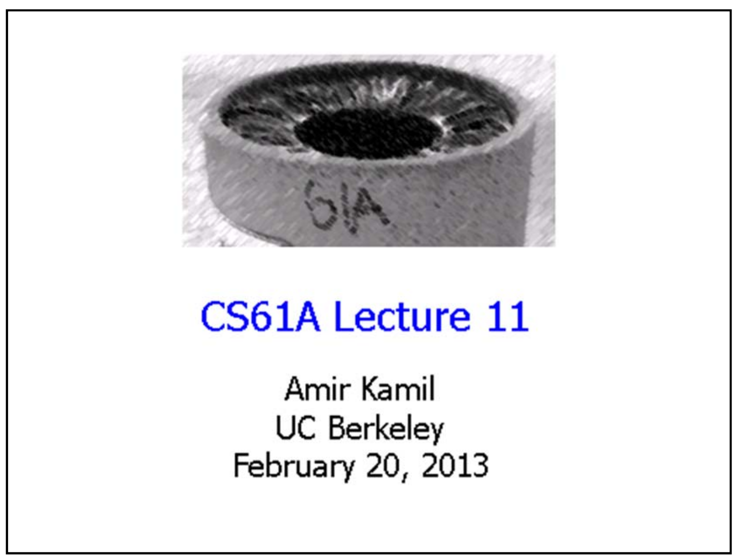

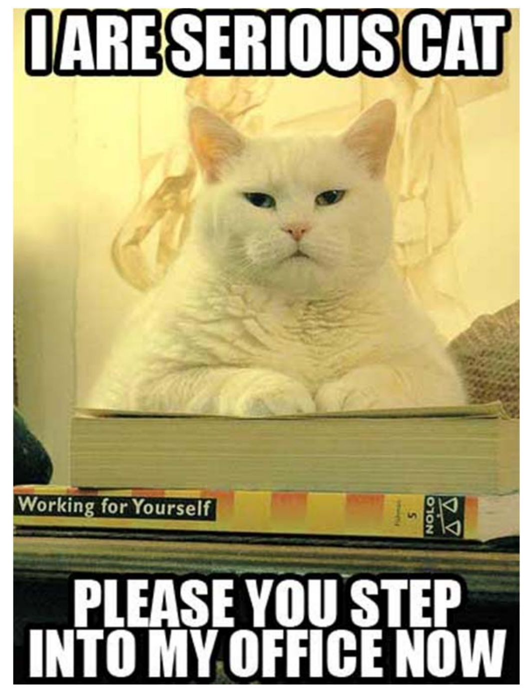

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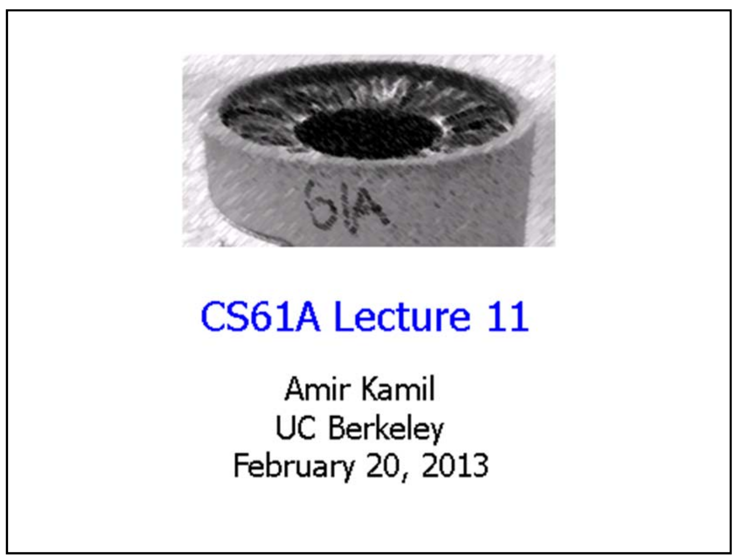
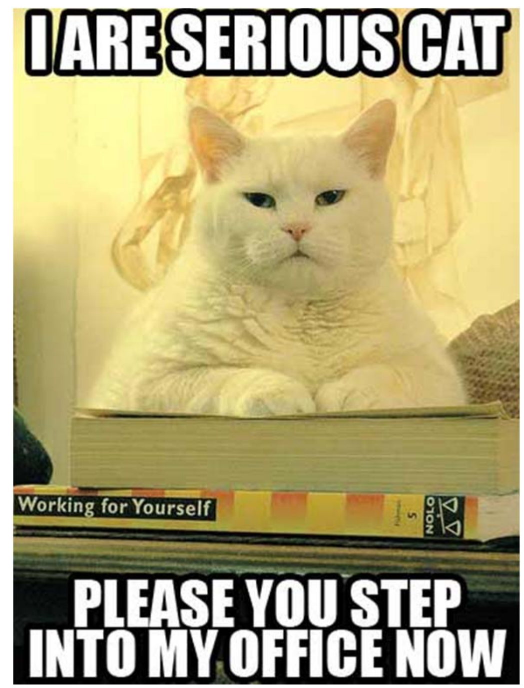
How do we represent more complex data?

We need data abstractions!





Data Abstraction

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Compound data combine smaller pieces of data together

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- A data: a year, month, and day

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- A geographic position: latitude and logitude

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An abstract data type lets us manipulate compound data as a unit

Data Abstraction

Compound data combine smaller pieces of data together

- A data: a year, month, and day
- A geographic position: latitude and logitude

An abstract data type lets us manipulate compound data as a unit

Isolate two parts of any program that uses data

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Data abstraction: A methodology by which functions enforce an abstraction barrier between representation and use

Data Abstraction

All Programmers

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All Programmers

Great Programmers

Rational Numbers

Rational Numbers

$$\frac{\text{numerator}}{\text{denominator}}$$

Rational Numbers

$$\frac{\text{numerator}}{\text{denominator}}$$

Exact representation of fractions

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A pair of integers

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Assume we can compose and decompose rational numbers:

- **`rational(n, d)`** returns a rational number *x*

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Exact representation of fractions

A pair of integers

As soon as division occurs, the exact representation is lost!

Assume we can compose and decompose rational numbers:

- **`rational(n, d)`** returns a rational number *x*
- **`numer(x)`** returns the numerator of *x*

Rational Numbers

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Exact representation of fractions

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- **`rational(n, d)`** returns a rational number *x*
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Rational Numbers

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Exact representation of fractions

A pair of integers

As soon as division occurs, the exact representation is lost!

Assume we can compose and decompose rational numbers:

Constructor **`rational(n, d)`** returns a rational number *x*

- **`numer(x)`** returns the numerator of *x*
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Rational Numbers

$$\frac{\text{numerator}}{\text{denominator}}$$

Exact representation of fractions

A pair of integers

As soon as division occurs, the exact representation is lost!

Assume we can compose and decompose rational numbers:

Constructor: **`rational(n, d)`** returns a rational number *x*

Selectors:

- **`numer(x)`** returns the numerator of *x*
- **`denom(x)`** returns the denominator of *x*

Rational Number Arithmetic

Example:

General Form:

Rational Number Arithmetic

Example:

$$\frac{3}{2} * \frac{3}{5}$$

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Example:

$$\frac{3}{2} * \frac{3}{5} = \frac{9}{10}$$

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$$\frac{3}{2} * \frac{3}{5} = \frac{9}{10}$$

General Form:

$$\frac{nx}{dx} * \frac{ny}{dy}$$

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Example:

$$\frac{3}{2} * \frac{3}{5} = \frac{9}{10}$$

General Form:

$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

Rational Number Arithmetic

Example:

$$\frac{3}{2} * \frac{3}{5} = \frac{9}{10}$$

$$\frac{3}{2} + \frac{3}{5}$$

General Form:

$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

Rational Number Arithmetic

Example:

$$\frac{3}{2} * \frac{3}{5} = \frac{9}{10}$$

$$\frac{3}{2} + \frac{3}{5} = \frac{21}{10}$$

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General Form:

$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

$$\frac{nx}{dx} + \frac{ny}{dy} = \frac{nx*dy + ny*dx}{dx*dy}$$

Rational Number Arithmetic Code

Rational Number Arithmetic Code

- **`rational(n, d)`** returns a rational number *x*
- **`numer(x)`** returns the numerator of *x*
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Rational Number Arithmetic Code

Wishful thinking

- **`rational(n, d)`** returns a rational number *x*
- **`numer(x)`** returns the numerator of *x*
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Rational Number Arithmetic Code

```
def mul_rational(x, y):
    return rational(numer(x) * numer(y),
                    denom(x) * denom(y))
```

Wishful thinking

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Rational Number Arithmetic Code

```
def mul_rational(x, y):
    return rational(numer(x) * numer(y),
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```

Constructor

Selectors

```
def add_rational(x, y):
    nx, dx = numer(x), denom(x)
    ny, dy = numer(y), denom(y)
    return rational(nx * dy + ny * dx, dx * dy)

def eq_rational(x, y):
    return numer(x) * denom(y) == numer(y) * denom(x)
```

Wishful thinking

- `rational(n, d)` returns a rational number *x*
- `numer(x)` returns the numerator of *x*
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Tuples

Tuples

```
>>> pair = (1, 2)
```

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>>> pair = (1, 2)
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```

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```
>>> pair = (1, 2)
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>>> x
```

Tuples

```
>>> pair = (1, 2)
>>> pair
(1, 2)

>>> x, y = pair
>>> x
1
```

Tuples

```
>>> pair = (1, 2)
>>> pair
(1, 2)

>>> x, y = pair
>>> x
1
>>> y
```

Tuples

```
>>> pair = (1, 2)
>>> pair
(1, 2)

>>> x, y = pair
>>> x
1
>>> y
2
```

Tuples

```
>>> pair = (1, 2)
>>> pair
(1, 2)

>>> x, y = pair
>>> x
1
>>> y
2

>>> pair[0]
```

Tuples

```
>>> pair = (1, 2)
>>> pair
(1, 2)

>>> x, y = pair
>>> x
1
>>> y
2

>>> pair[0]
1
```

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>>> pair = (1, 2)
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(1, 2)

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(1, 2)

>>> x, y = pair
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>>> pair[0]
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>>> from operator import getitem
```

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```
>>> pair = (1, 2)
>>> pair
(1, 2)

>>> x, y = pair
>>> x
1
>>> y
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>>> pair[0]
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>>> pair[1]
2
>>> from operator import getitem
>>> getitem(pair, 0)
```

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```
>>> pair = (1, 2)
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>>> y
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```

A tuple literal:

Comma-separated expression

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A tuple literal:

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"Unpacking" a tuple

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>>> pair[0]
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Element selection

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Comma-separated expression

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>>> from operator import getitem
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>>> getitem(pair, 1)
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```

Element selection

More tuples next lecture

Representing Rational Numbers

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def rational(n, d):
    """Construct a rational number x that represents
    n/d."""
    return (n, d)
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Representing Rational Numbers

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    """Construct a rational number x that represents
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```

Construct a tuple

```
from operator import getitem

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    """Return the numerator of rational number x."""
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```

Representing Rational Numbers

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Construct a tuple

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def denom(x):
    """Return the denominator of rational number
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Select from a tuple