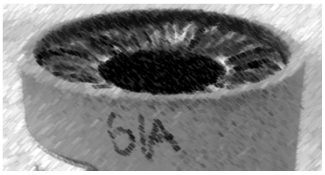

CS61A Lecture 11

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UC Berkeley
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Announcements

- HW4 due Wednesday at 11:59pm
- Hog contest deadline next week
  - Completely optional, opportunity for extra credit
  - See website for details

Fibonacci Sequence

The Fibonacci sequence is defined as

$$\text{fib}(n) = \begin{cases} 0, & n = 0 \\ 1, & n = 1 \\ \text{fib}(n-1) + \text{fib}(n-2), & n > 1 \end{cases}$$

```
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    return fib(n - 1) + fib(n - 2)
```

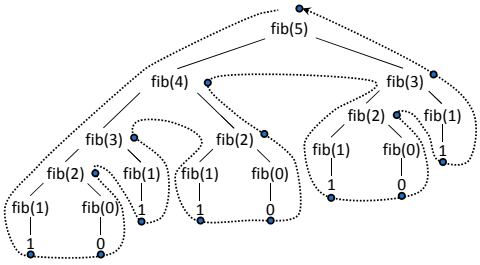
Two recursive calls!

Example: http://goo.gl/DZbRG

Tree recursion

Executing the body of a function may entail more than one recursive call to that function

This is called *tree recursion*



Tracing the Order of Calls

We can use a higher-order function to see the order in which calls are made and complete

```
def trace1(fn):
    """Return a function equivalent to fn that
    also prints trace output."""
    def traced(x):
        print('Calling', fn, '(', x, ')')
        res = fn(x)
        print('Got', res, 'from', fn, '(', x, ')')
        return res
    return traced

# Rebind the name fib to a traced version of fib
fib = trace1(fib)
```

Function Decorators

Function decorator

```
@trace1
def triple(x):
    return 3 * x
```

Decorated function

is identical to

Why not just use this?

```
def triple(x):
    return 3 * x
triple = trace1(triple)
```

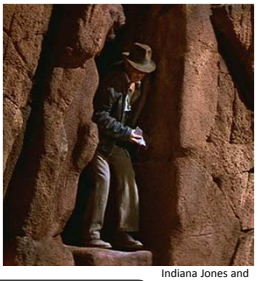
The Recursive Leap of Faith

```
def factorial(n):
    if n == 0:
        return 1
    return factorial(n-1)
```

Oops!

Is factorial implemented correctly?

1. Verify the base case.
2. Treat `factorial(n-1)` as a functional abstraction.
3. Assume that `factorial(n-1)` is correct.
4. Verify that `factorial(n)` is correct, assuming that `factorial(n-1)` is correct



Indiana Jones and The Last Crusade
© Lucasfilm, Ltd.

Simpler problem

Simplifying a Problem

Pig Latinization:

1. Move all beginning consonants to the end of the word
2. Add "ay" to the end of the word

smart → artsmay

```
def pig_latin(w):
    if starts_with_a_vowel(w):
        return w + 'ay'
    return pig_latin(rest(w) + first(w))
```

smart → marts → artsm → artsmay

2 consonants to be moved | 1 consonant to be moved | Base case

Counting Change

$1 = $0.50 + $0.25 + $0.10 + $0.10 + $0.05

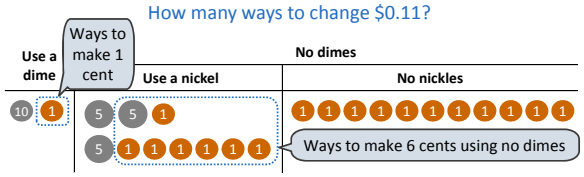
$1 = 1 half dollar, 1 quarter, 2 dimes, 1 nickel

$1 = 2 quarters, 2 dimes, 30 pennies

$1 = 100 pennies

How many ways are there to change a dollar?

How many ways to change $0.11?

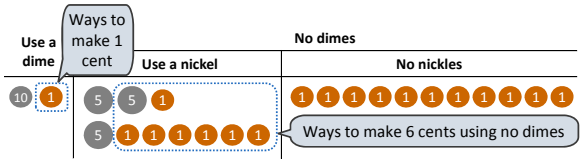


Counting Change Recursively

How many ways are there to change a dollar?

The number of ways to change an amount *a* using *n* kinds of coins is:

1. The number of ways to change *a-d* using all kinds, where *d* is the amount of the first kind of coin
2. The number of ways to change *a* using all but the first kind



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```
def count_change(a, d):
    if a == 0:
        return 1
    if a < 0 or d == 0:
        return 0
    return (count_change(a-d, d) +
            count_change(a, next_coin(d)))
```

One way to make no amount

Can't make negative amount, or any amount with no coins

Functional abstraction to get next coin