CS61A Lecture 7

Amir Kamil
UC Berkeley
February 6, 2013

Announcements

- HW3 out, due Tuesday at 7pm
- Midterm next Wednesday at 7pm
  - Keep an eye out for your assigned location
  - Old exams posted soon
  - Review sessions
    - Saturday 2-4pm in TBA
    - Extend office hours Sunday 11-3pm in TBA
    - HKN review session Sunday 3-6pm in 145 Dwinelle
- Environment diagram handout on website
- Code review system online
  - See Piazza post for details

How to Draw an Environment Diagram

When defining a function:

Create a function value with signature
<name>(<formal parameters>)

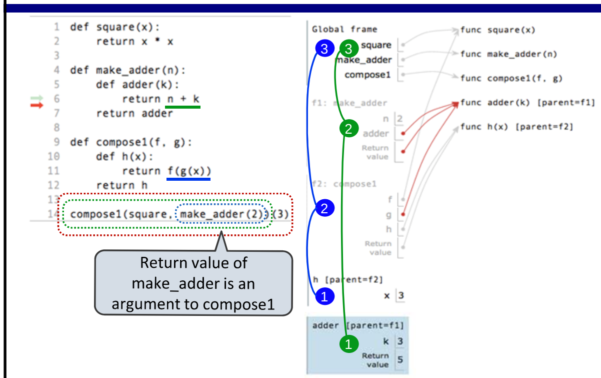
For nested definitions, label the parent as the first frame of the current environment

Bind <name> to the function value in the first frame of the current environment

When calling a function:

1. Add a local frame labeled with the <name> of the function
2. If the function has a parent label, copy it to this frame
3. Bind the <formal parameters> to the arguments in this frame
4. Execute the body of the function in the environment that starts with this frame

Environment for Function Composition



Example: http://goo.gl/5zcug

Lambda Expressions

```
>>> ten = 10
```

An expression: this one evaluates to a number

```
>>> square = x * x
```

Also an expression: evaluates to a function

```
>>> square = lambda x: x * x
```

Notice: no "return"

A function
with formal parameter x
and body "return x * x"

Must be a single expression

```
>>> square(4)
16
```

Lambda expressions are rare in Python, but important in general

Evaluation of Lambda vs. Def

```
lambda x: x * x
```

VS

```
def square(x):
    return x * x
```

Execution procedure for def *statements*:

1. Create a function value with signature <name>(<formal parameters>) and the current frame as parent
2. Bind <name> to that value in the current frame

Evaluation procedure for lambda *expressions*:

1. Create a function value with signature λ(<formal parameters>) and the current frame as parent
2. Evaluate to that value

No intrinsic name

Lambda vs. Def Statements

```
square = lambda x: x * x
```

VS

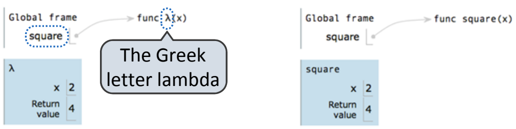
```
def square(x):
    return x * x
```

Both create a function with the same arguments & behavior

Both of those functions are associated with the environment in which they are defined

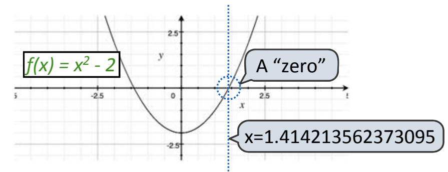
Both bind that function to the name "square"

Only the def statement gives the function an intrinsic name



Newton's Method Background

Finds approximations to zeroes of differentiable functions

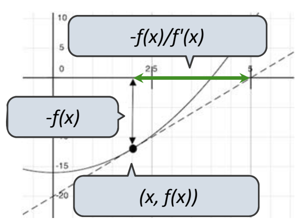


Application: a method for (approximately) computing square roots, using only basic arithmetic.

The positive zero of $f(x) = x^2 - a$ is $\sqrt{a}$

Newton's Method

Begin with a function f and an initial guess x



Compute the value of *f* at the guess: *f(x)*

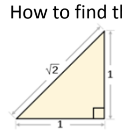
Compute the derivative of *f* at the guess: *f'(x)*

Update guess to be: $x - \frac{f(x)}{f'(x)}$

Visualization: http://en.wikipedia.org/wiki/File:NewtonIteration_Ani.gif

Using Newton's Method

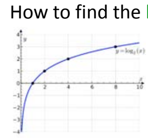
How to find the square root of 2?




```
>>> f = lambda x: x*x - 2
>>> find_zero(f)
1.4142135623730951
```

$f(x) = x^2 - 2$

How to find the log base 2 of 1024?



```
>>> g = lambda x: pow(2, x) - 1024
>>> find_zero(g)
10.0
```

$g(x) = 2^x - 1024$

Special Case: Square Roots

How to compute **`square_root(a)`**

Idea: Iteratively refine a guess *x* about the square root of *a*

Update: $x = \frac{x + \frac{a}{x}}{2}$

x - f(x)/f'(x)

Babylonian Method

Implementation questions:

What guess should start the computation?

How do we know when we are finished?

Special Case: Cube Roots

How to compute **`cube_root(a)`**

Idea: Iteratively refine a guess *x* about the cube root of *a*

Update: $x = \frac{2x + \frac{a}{x^2}}{3}$

x - f(x)/f'(x)

Implementation questions:

What guess should start the computation?

How do we know when we are finished?

Iterative Improvement

First, identify common structure.

Then define a function that generalizes the procedure.

```
def iter_improve(update, done, guess=1, max_updates=1000):
    """Iteratively improve guess with update until done
    returns a true value.

    >>> iter_improve(golden_update, golden_test)
    1.618033988749895
    """
    k = 0
    while not done(guess) and k < max_updates:
        guess = update(guess)
        k = k + 1
    return guess
```

Newton's Method for nth Roots

```
def nth_root_func_and_derivative(n, a):
    def root_func(x):
        return pow(x, n) - a
    def derivative(x):
        return n * pow(x, n-1)
    return root_func, derivative

def nth_root_newton(a, n):
    """Return the nth root of a.

    >>> nth_root_newton(8, 3)
    2.0
    """
    root_func, deriv = nth_root_func_and_derivative(n, a)
    def update(x):
        return x - root_func(x) / deriv(x)
    def done(x):
        return root_func(x) == 0
    return iter_improve(update, done)
```

Exact derivative

$x - f(x)/f'(x)$

Definition of a function zero