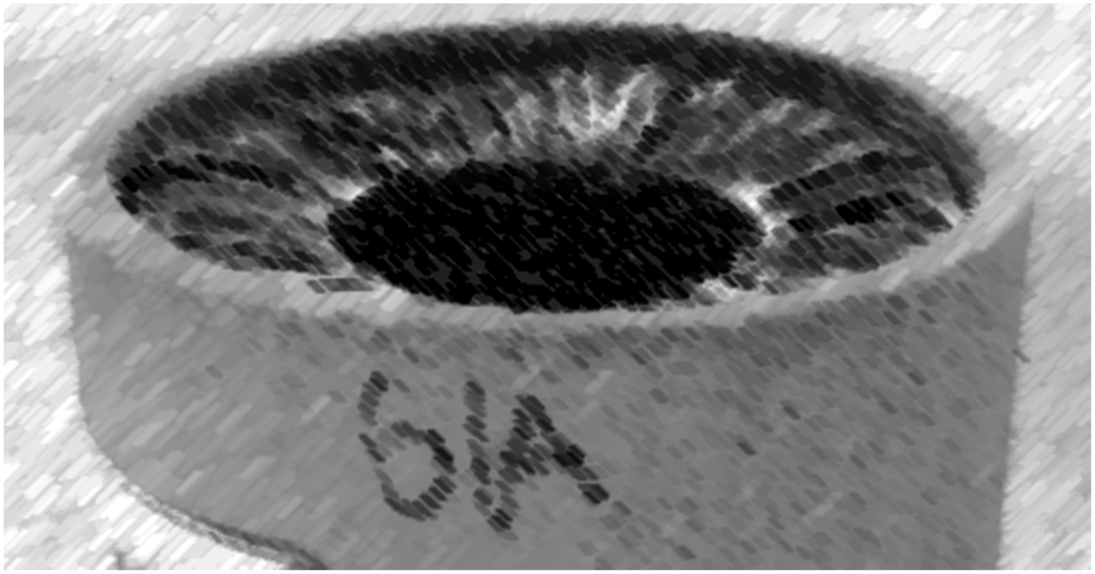

CS61A Lecture 6

Amir Kamil
UC Berkeley
February 4, 2013

Locally Defined Functions

```
from operator import mul
def square_inside():
    def square(x):
        return mul(x, x)
square_inside()
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Example: http://goo.gl/pnU8f

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The inner definition is executed each time the outer function is called

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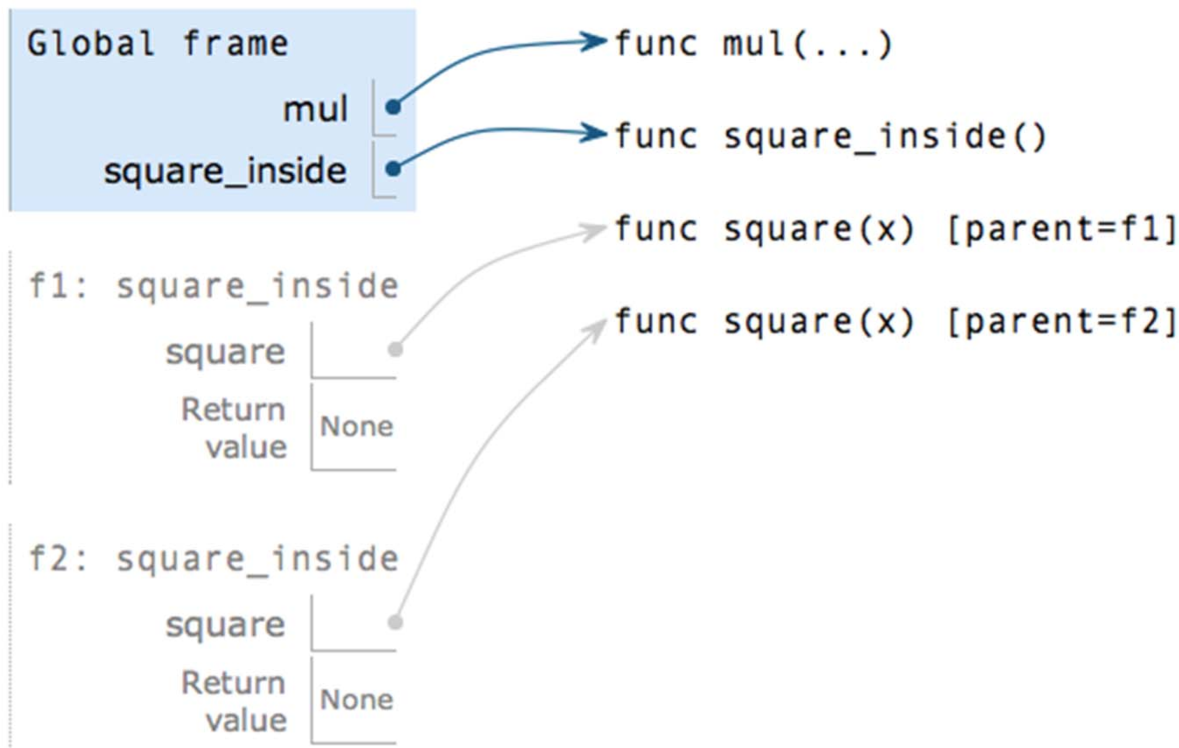

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Functions as Return Values

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def make_adder(n):
    """Return a function that adds n to its argument.

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Can refer to names in the enclosing function

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  - For a locally-defined function, this includes all local frames in the definition environment, plus the global frame

Environment for Non-Nested Function

The environment of a function application is a new local frame plus the environment in which the function was *defined*

Example: http://goo.gl/73anC

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def f(x, y):
    return g(x)

def g(a):
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result = f(1, 2)
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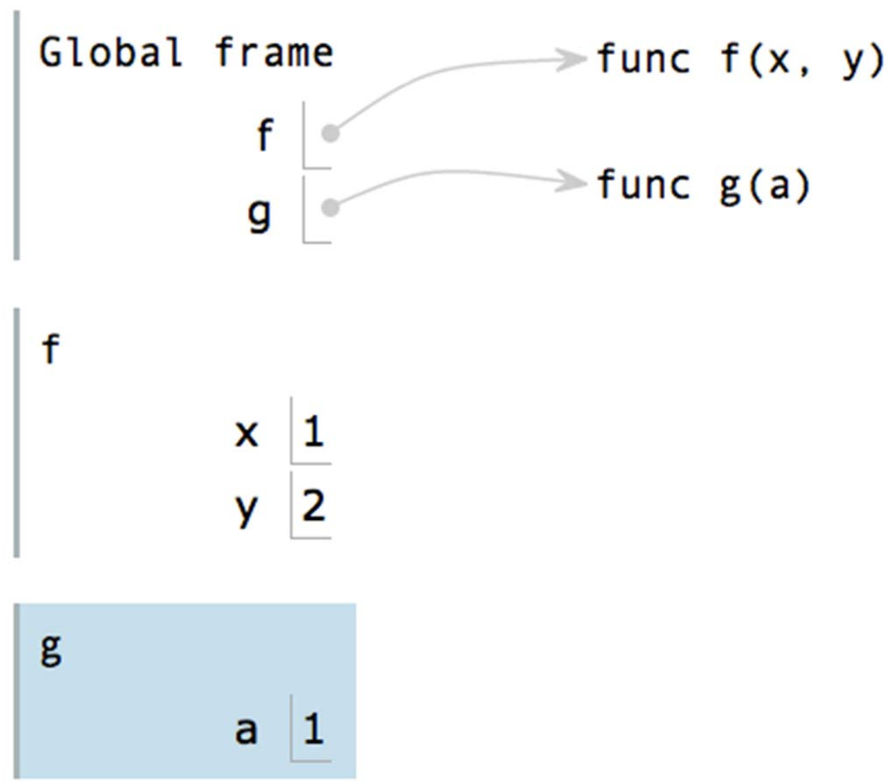

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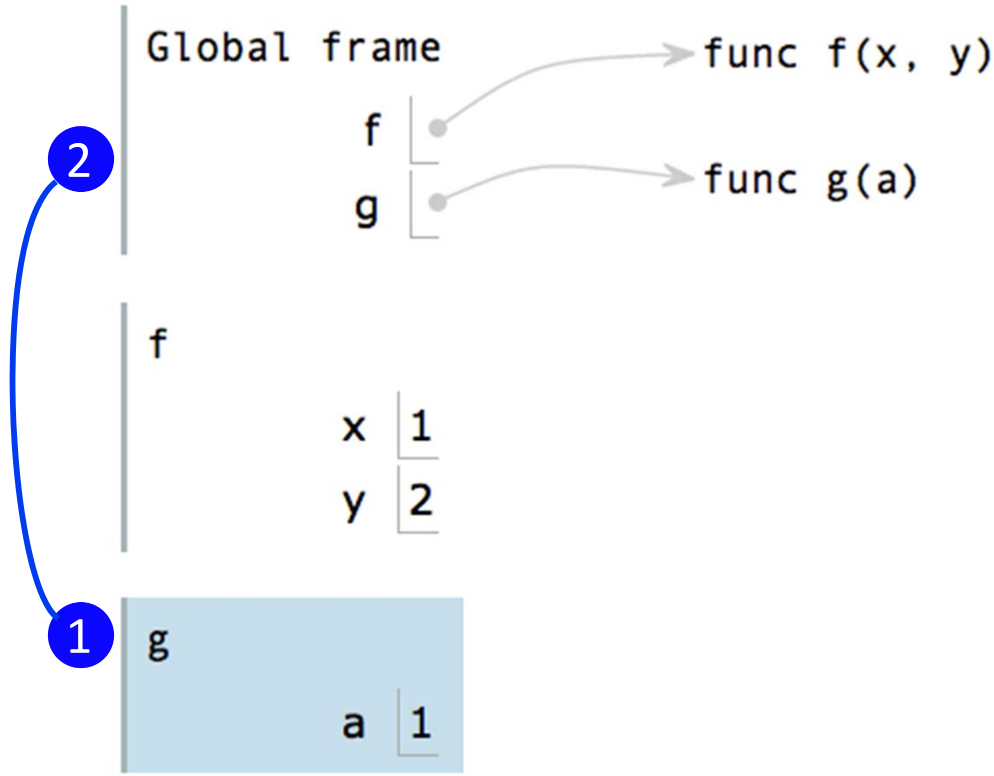

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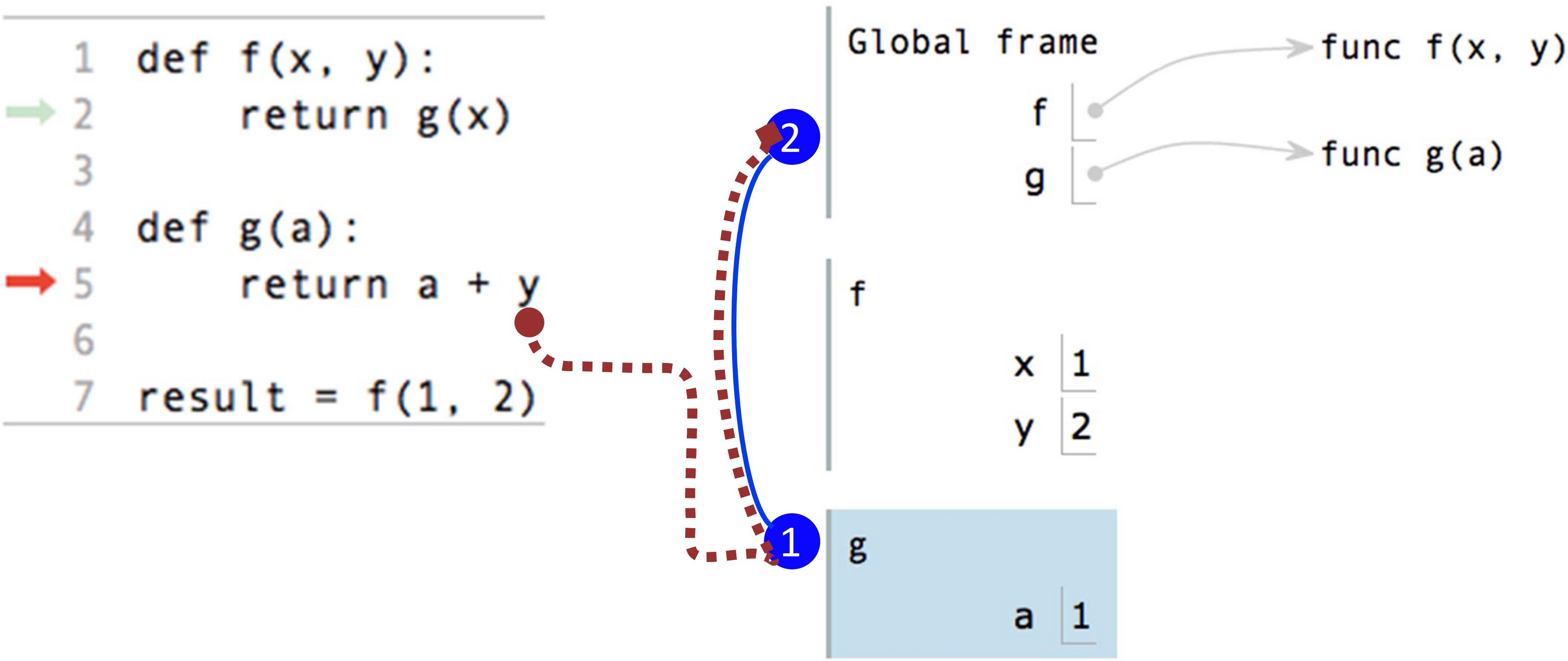

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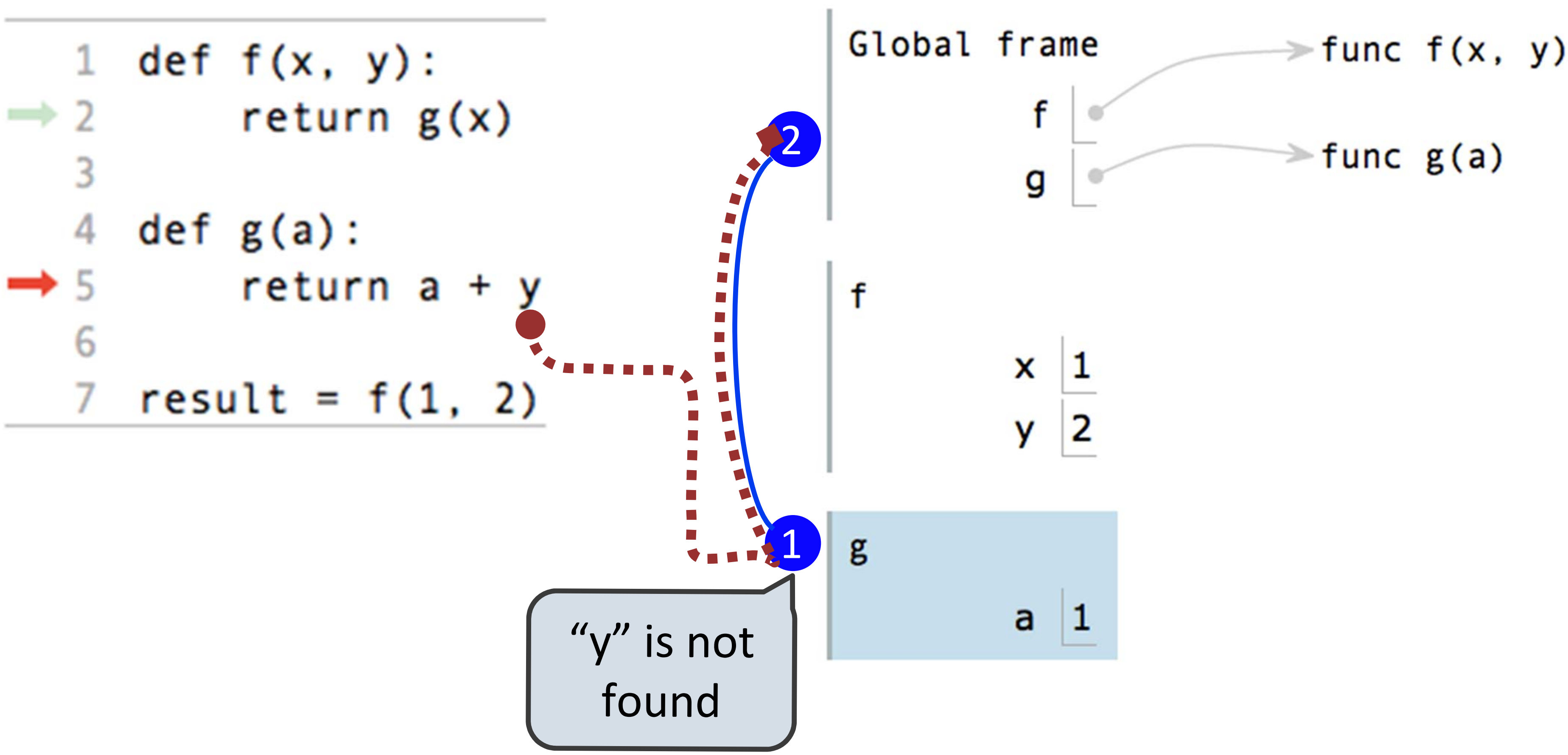

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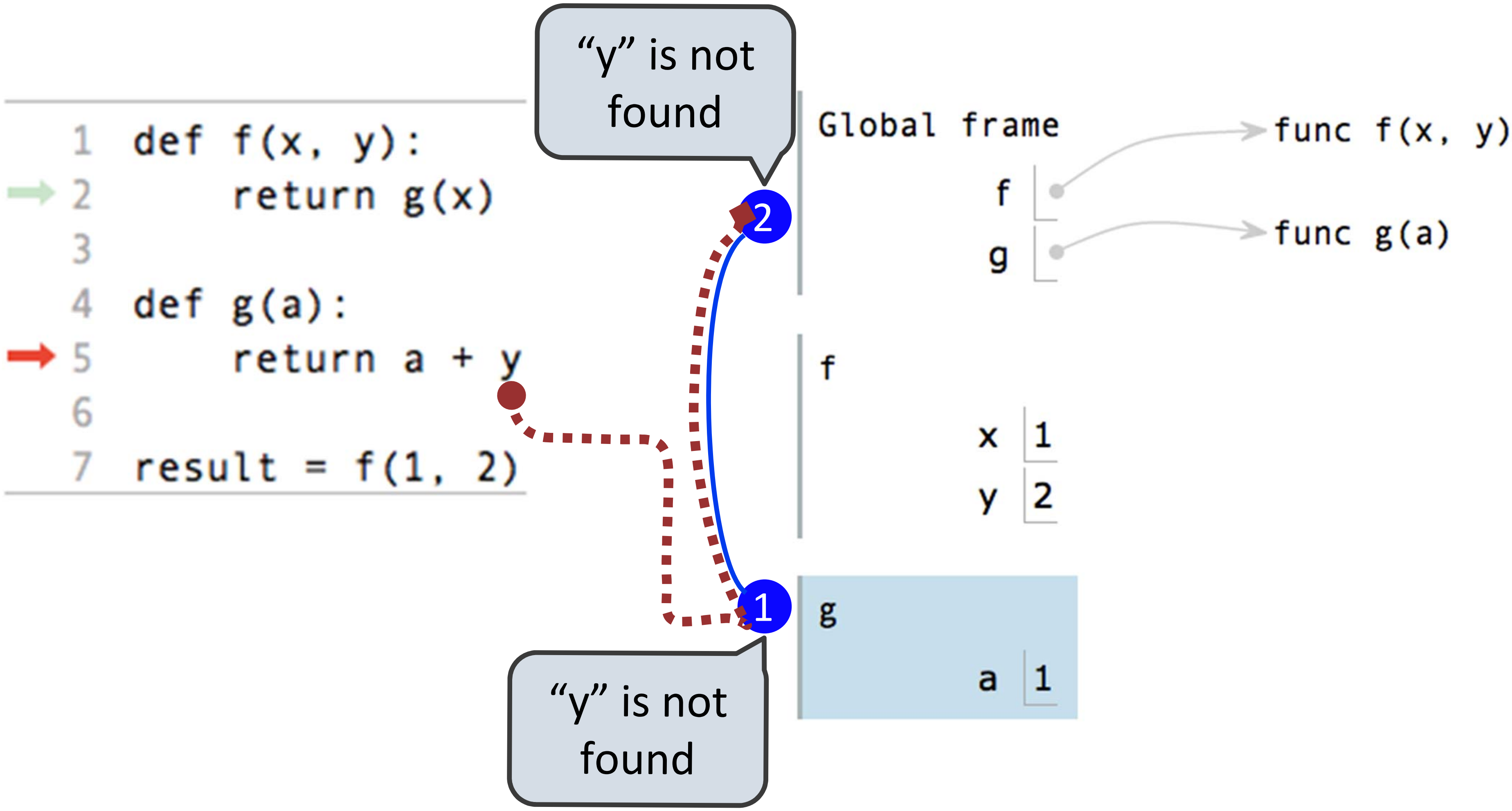

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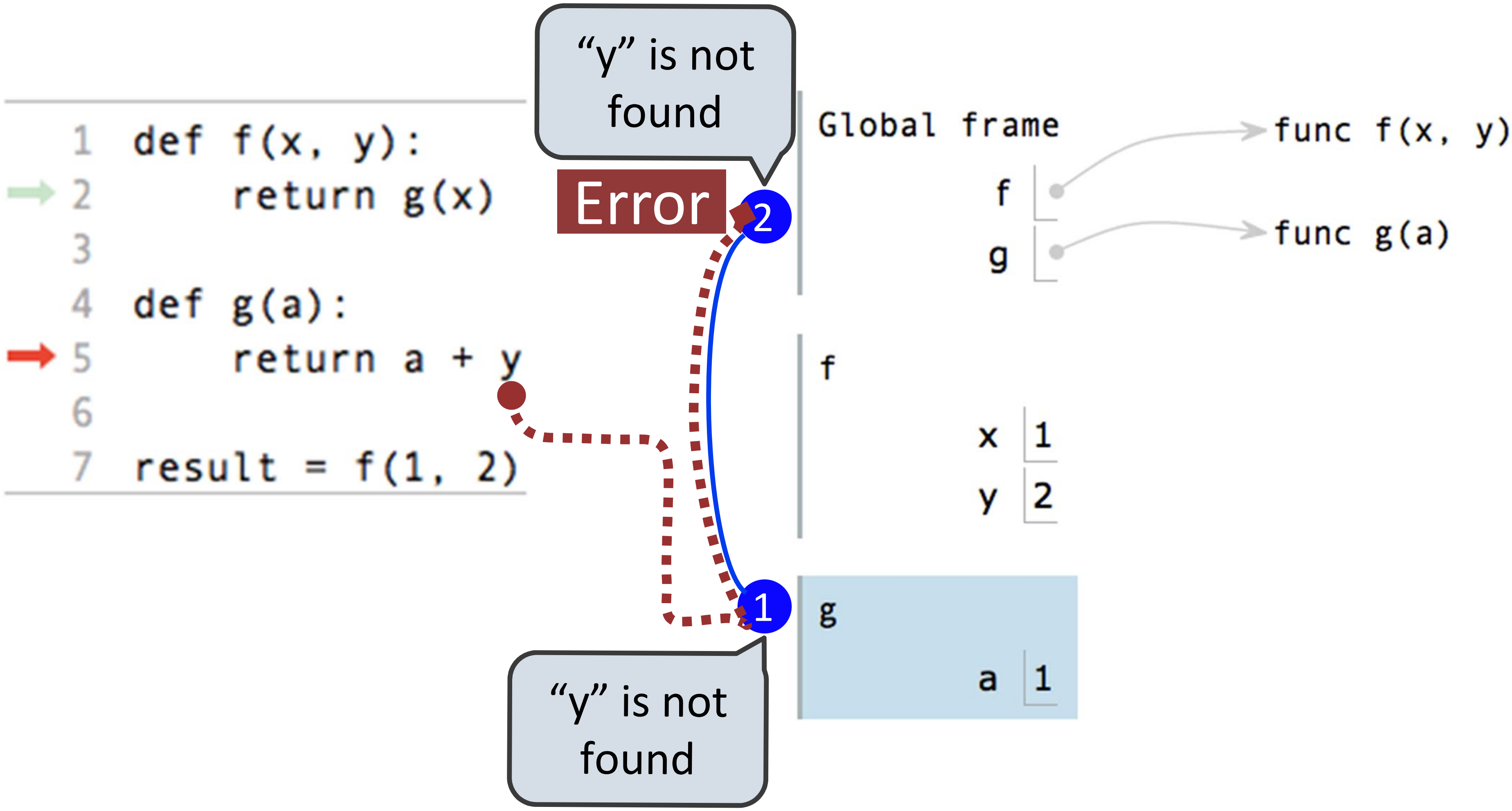

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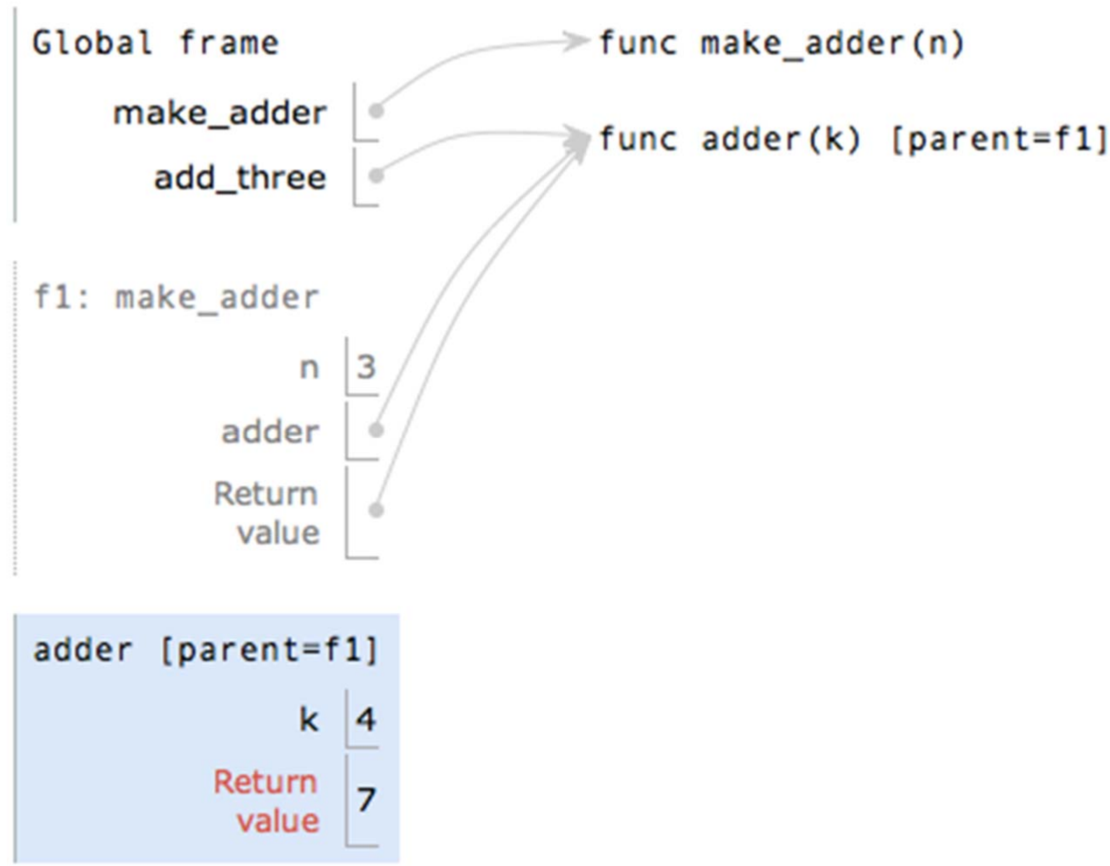

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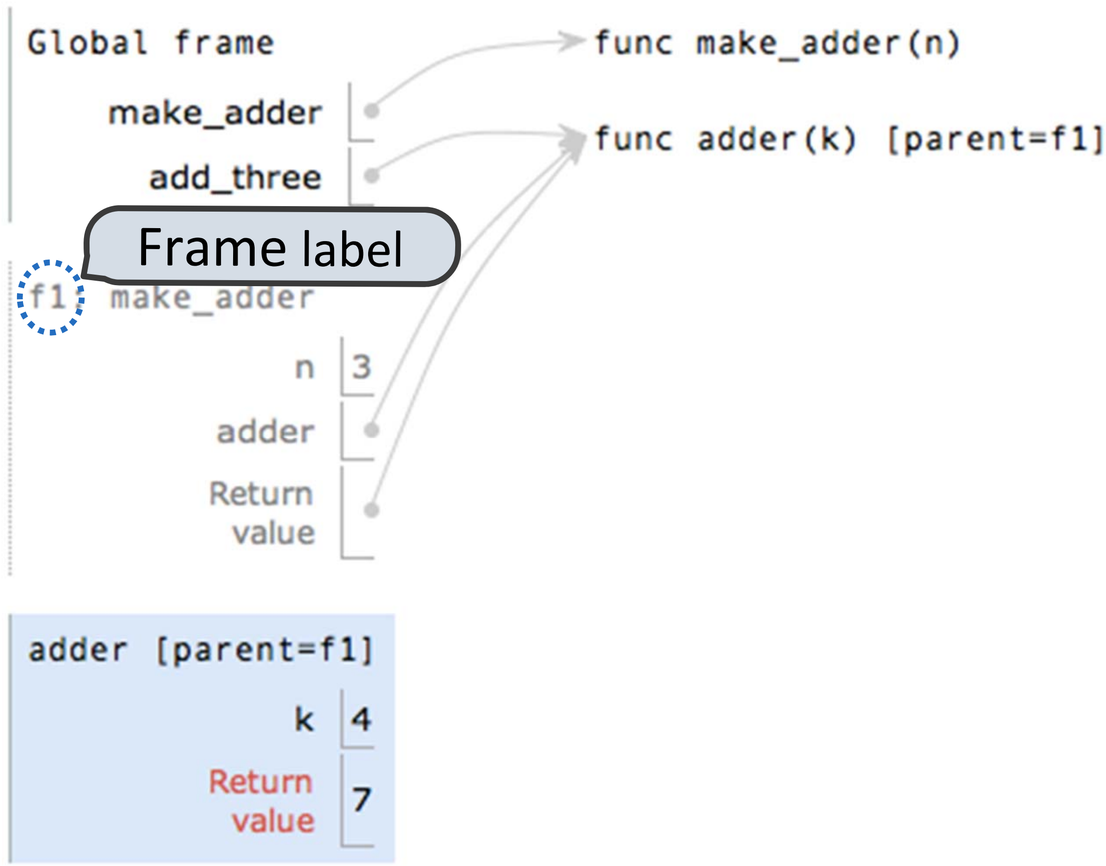

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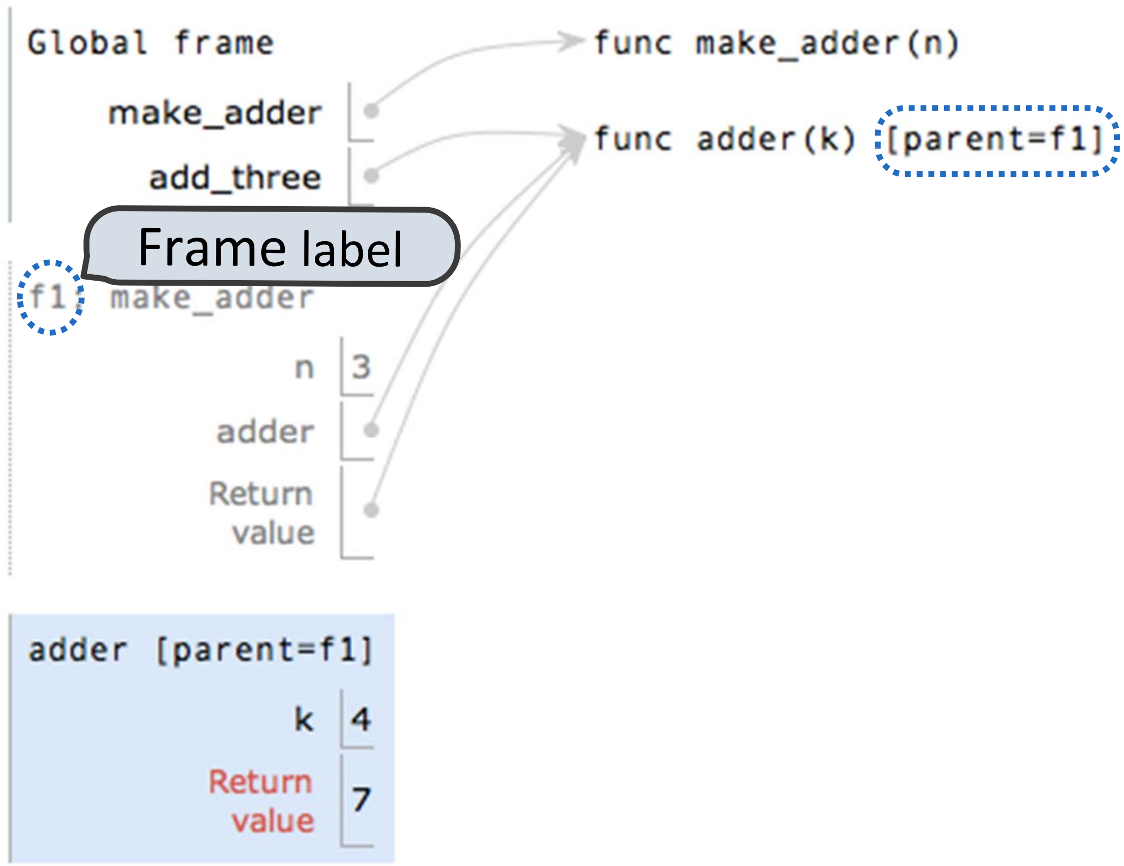

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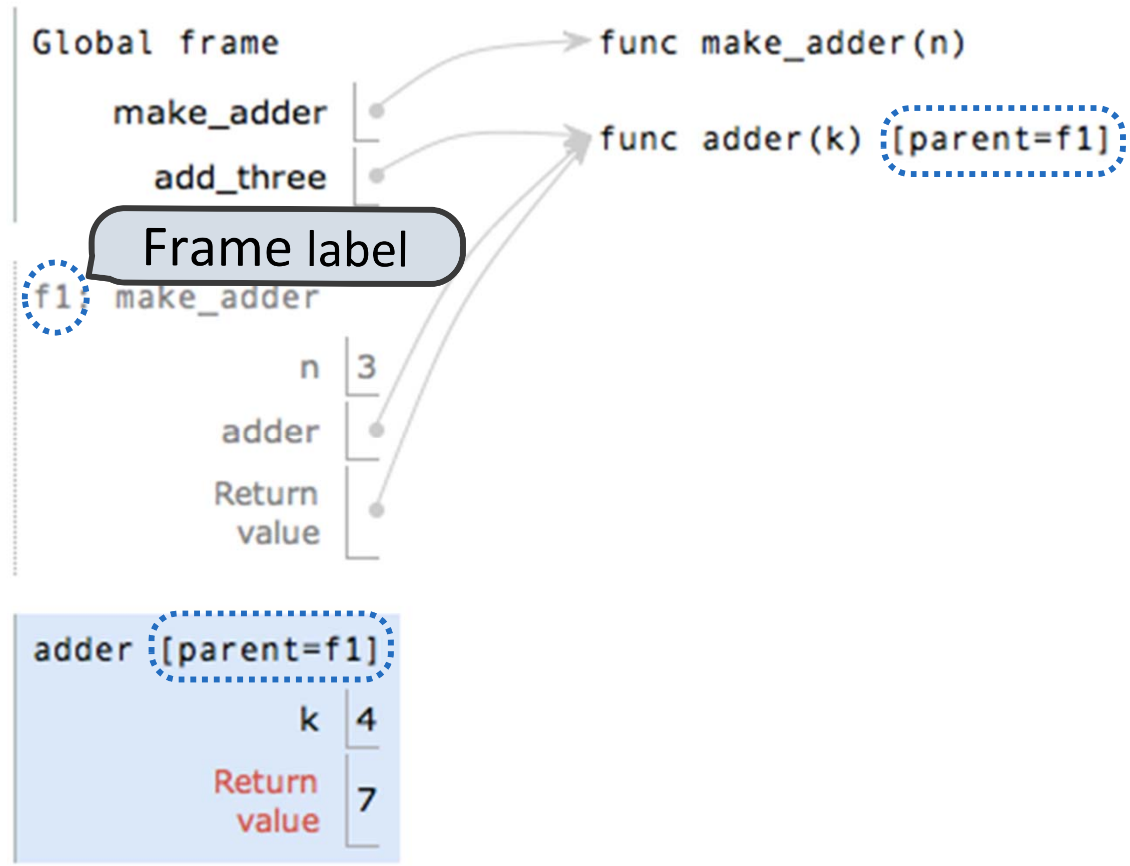

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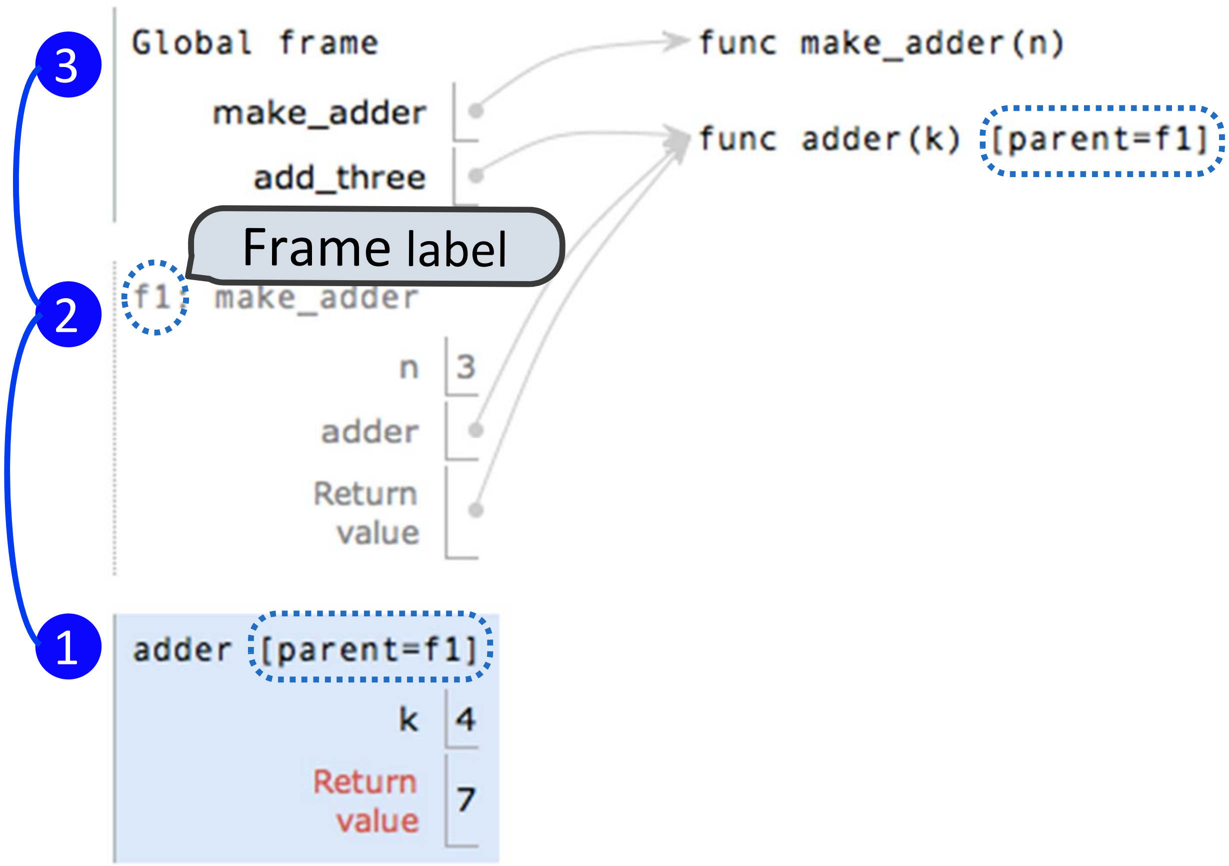

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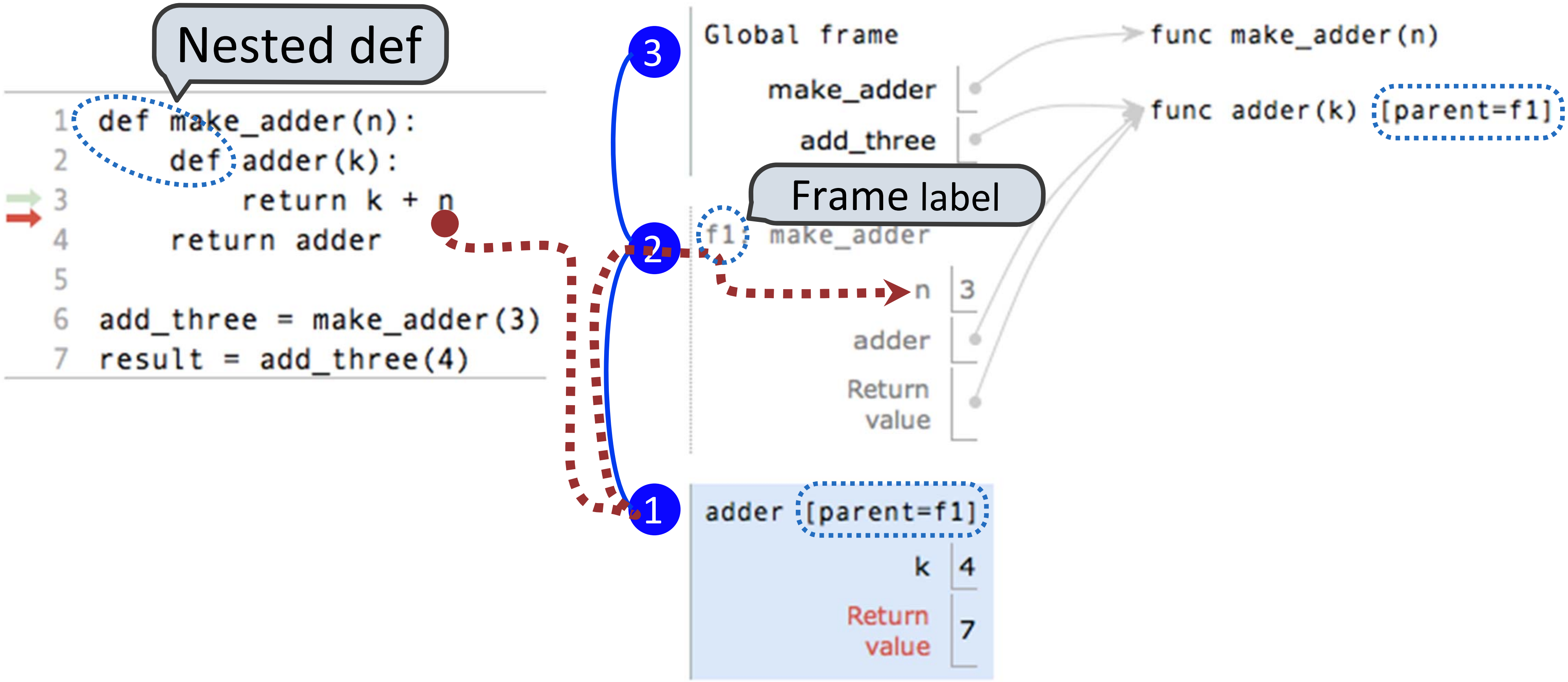

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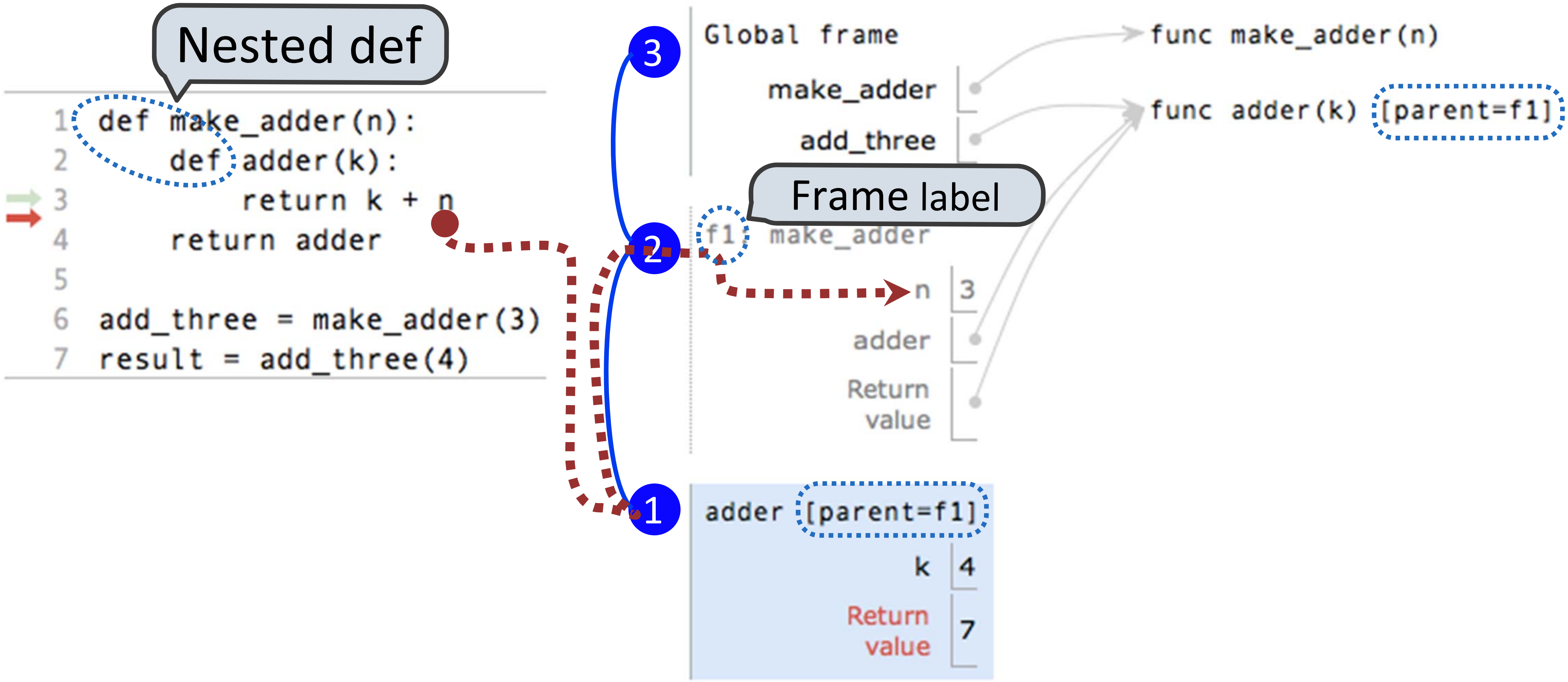

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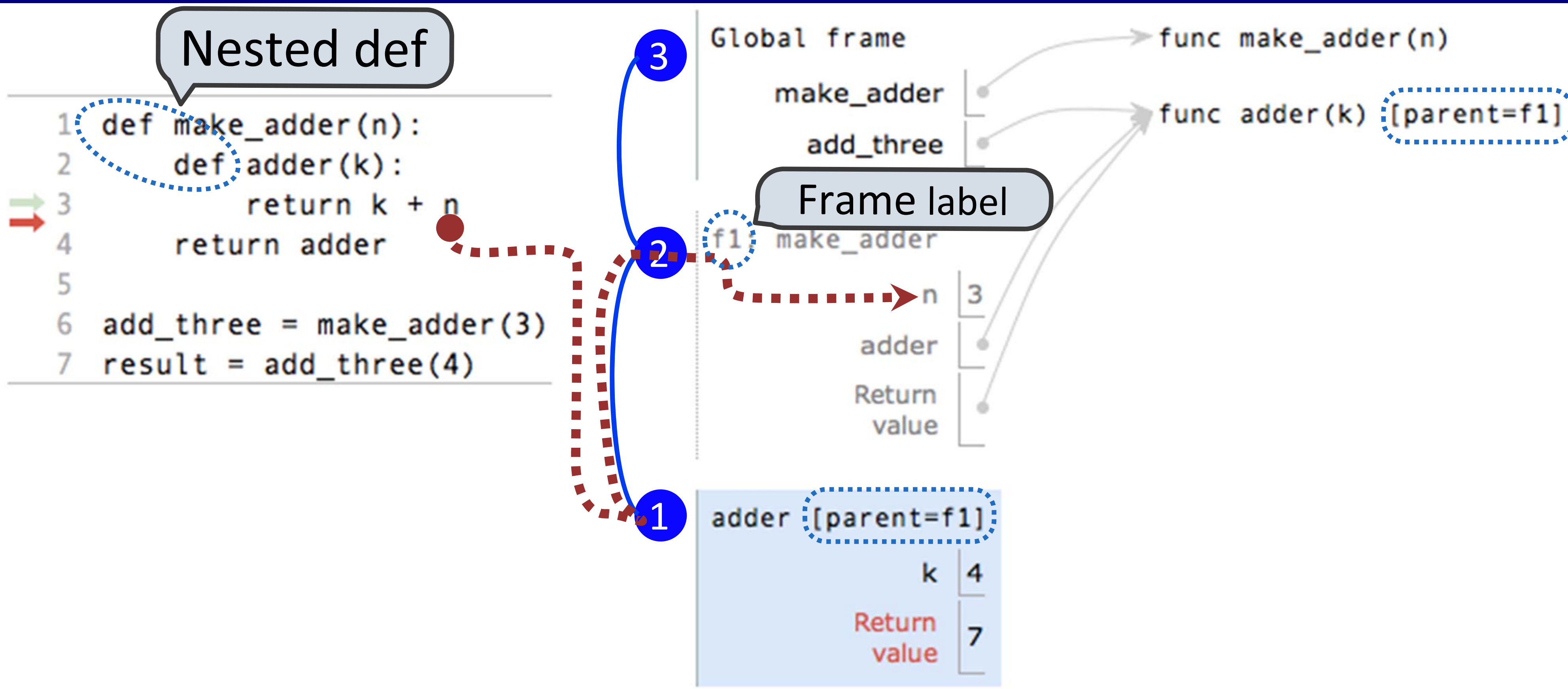

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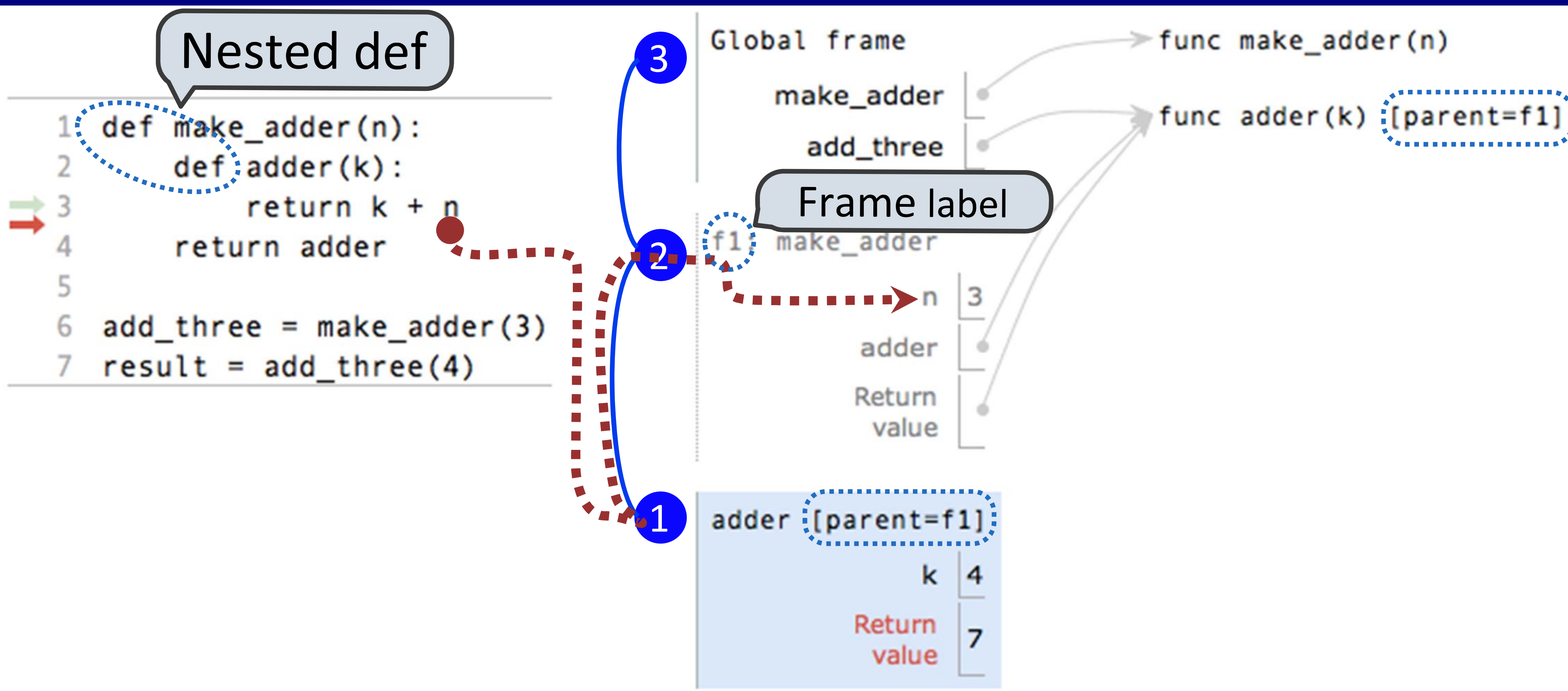

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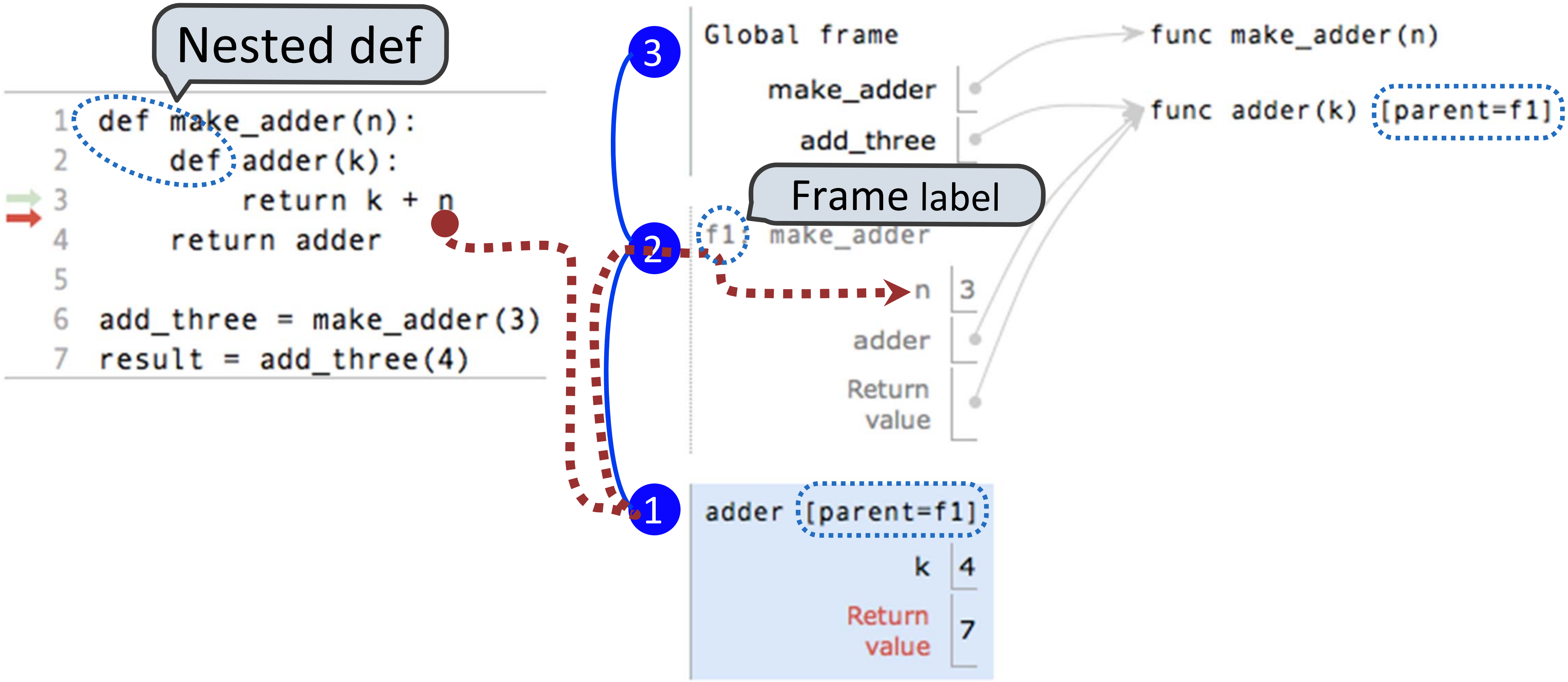

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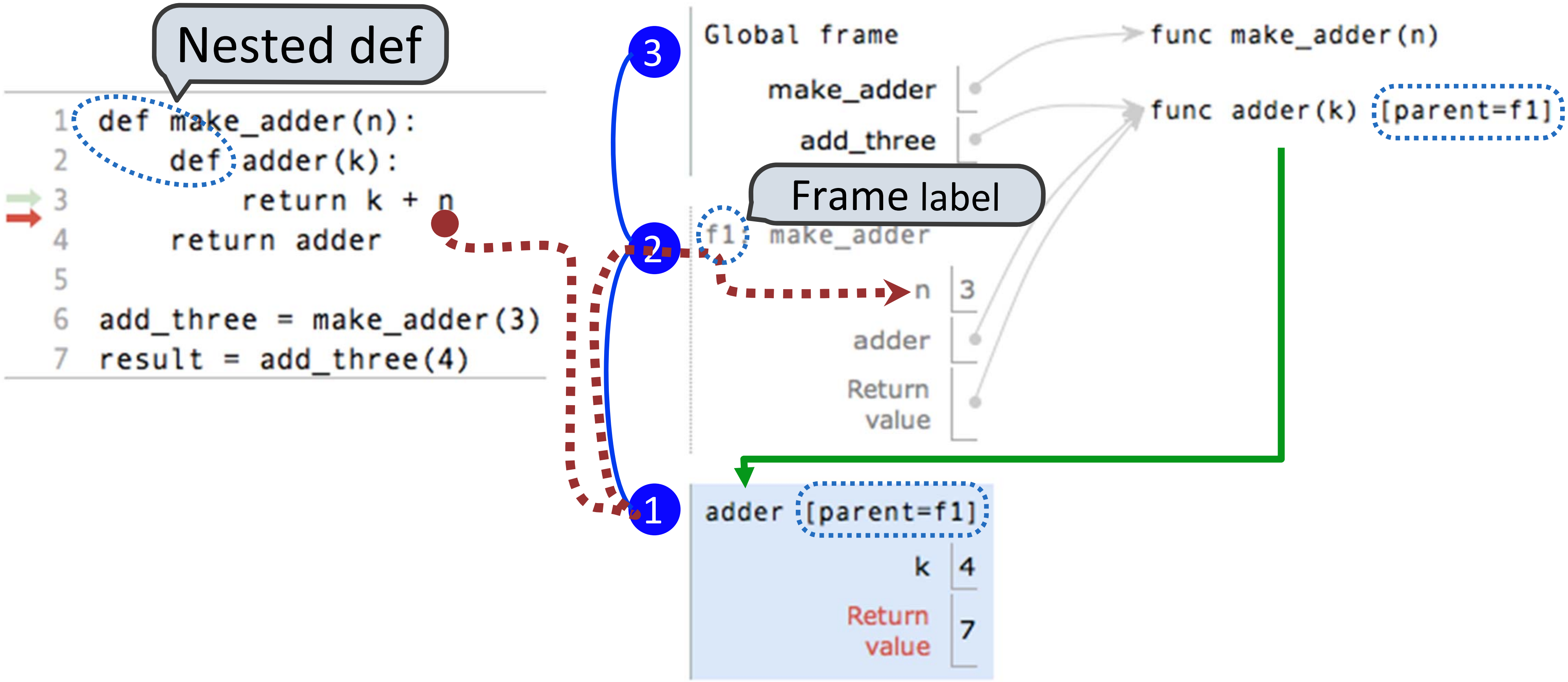

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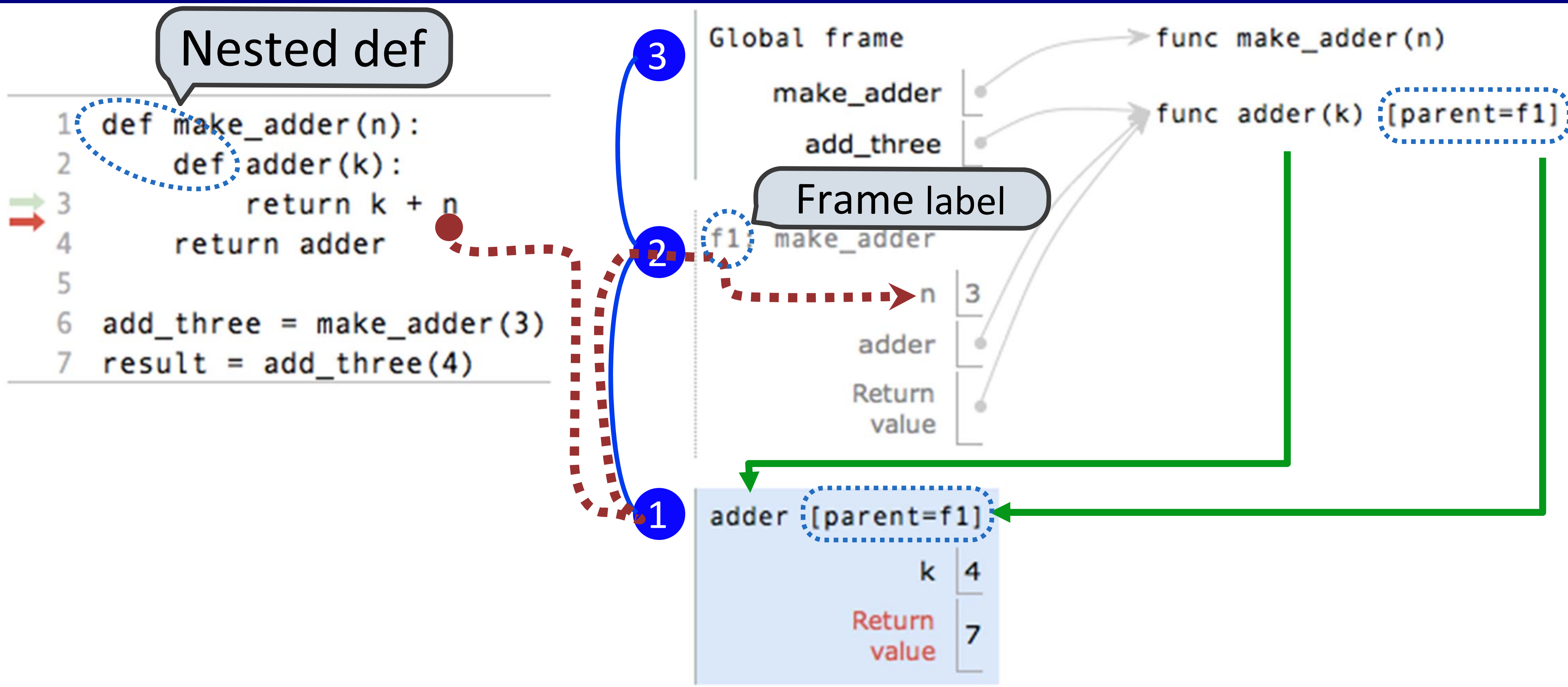

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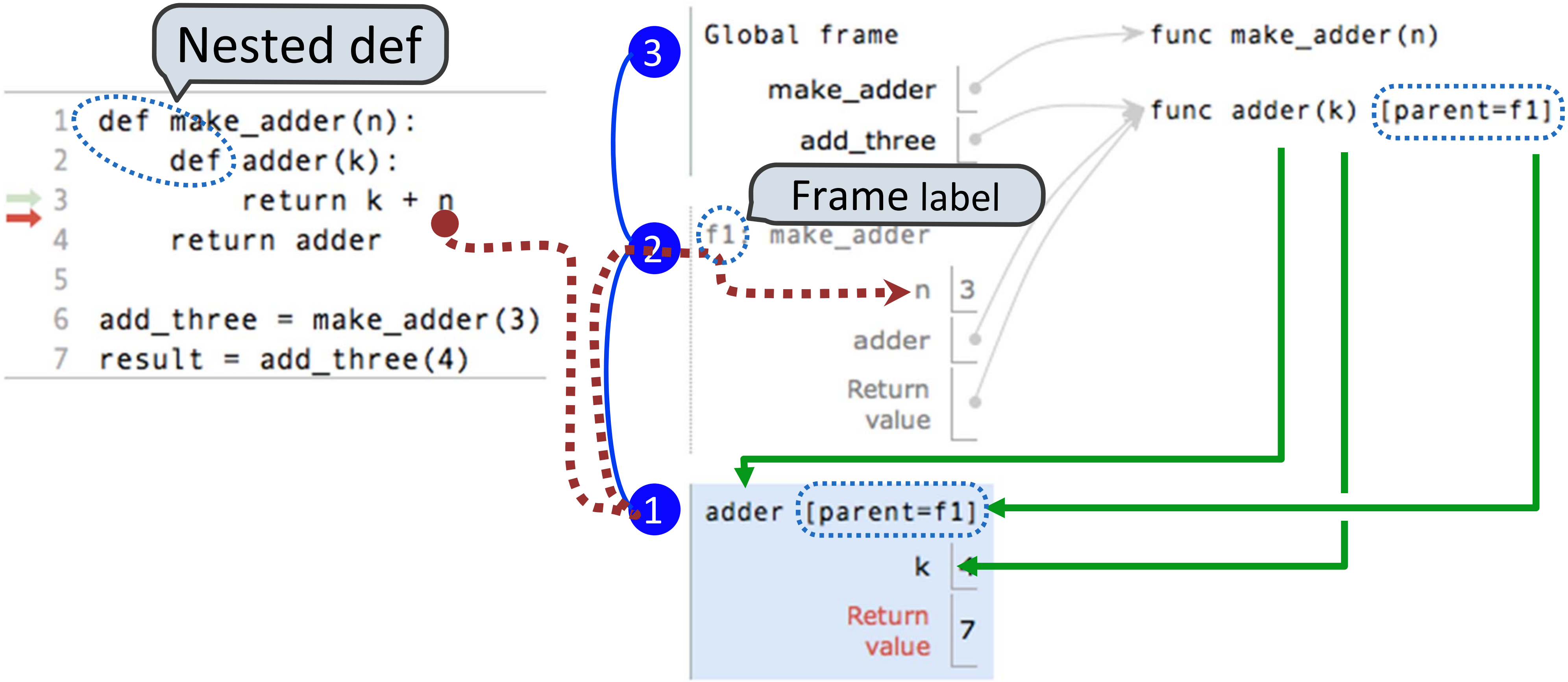

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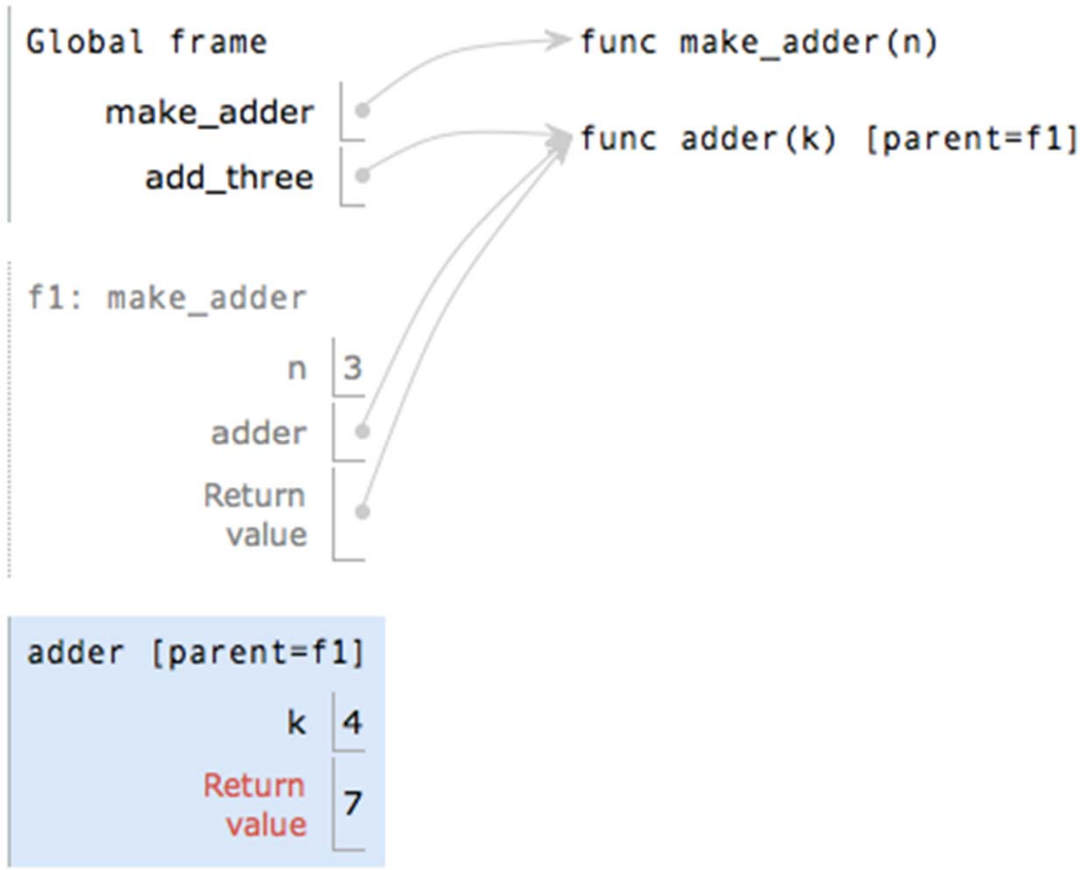

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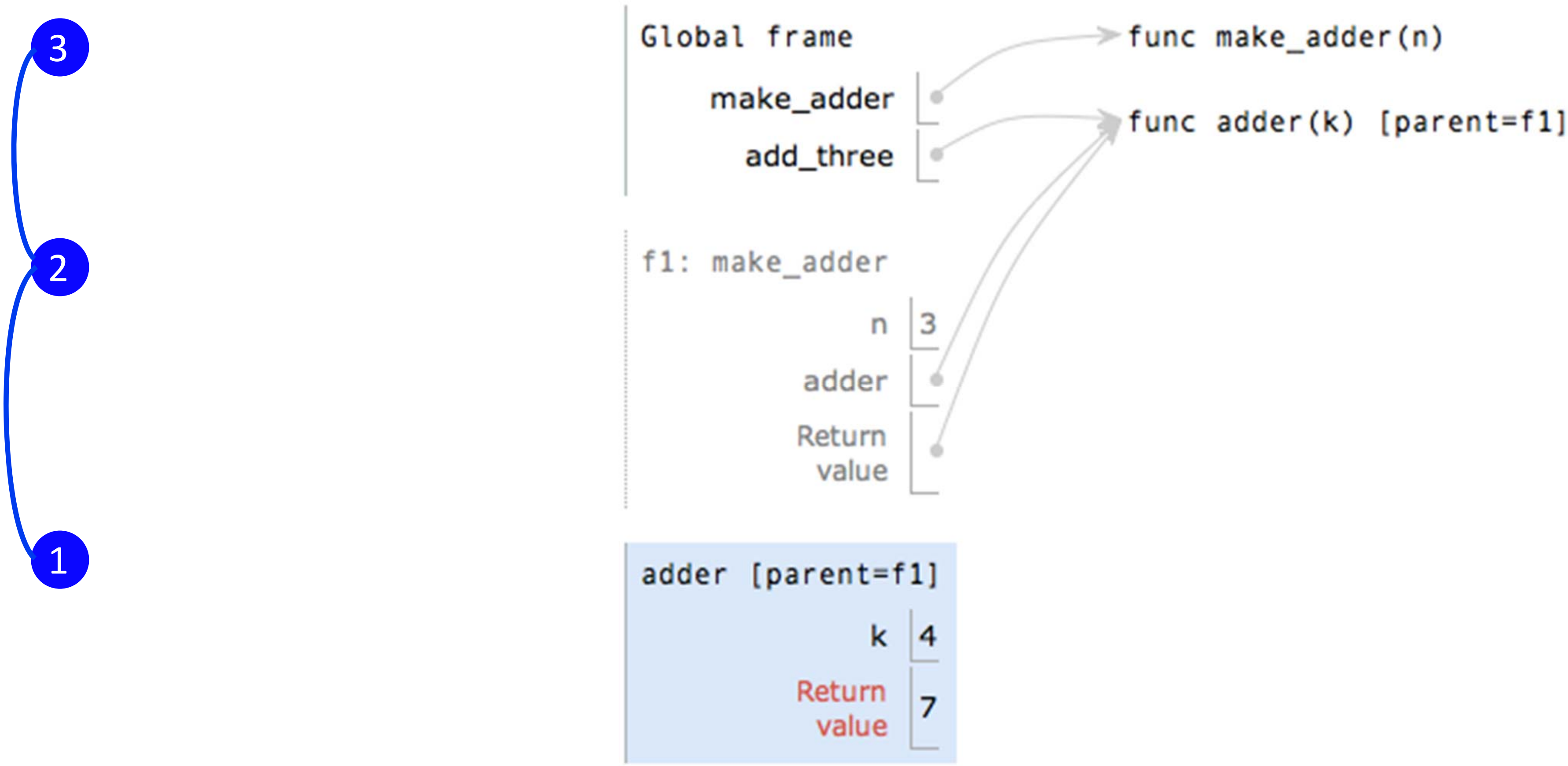

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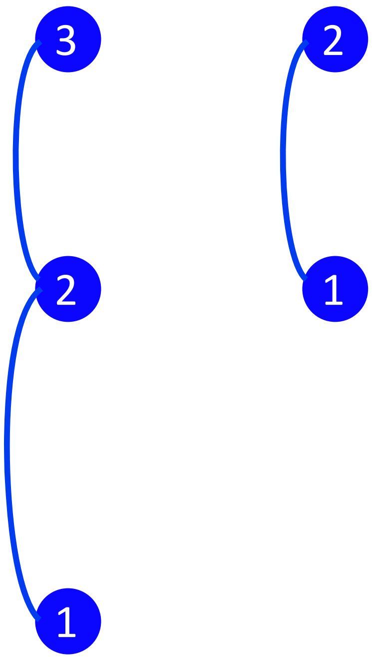

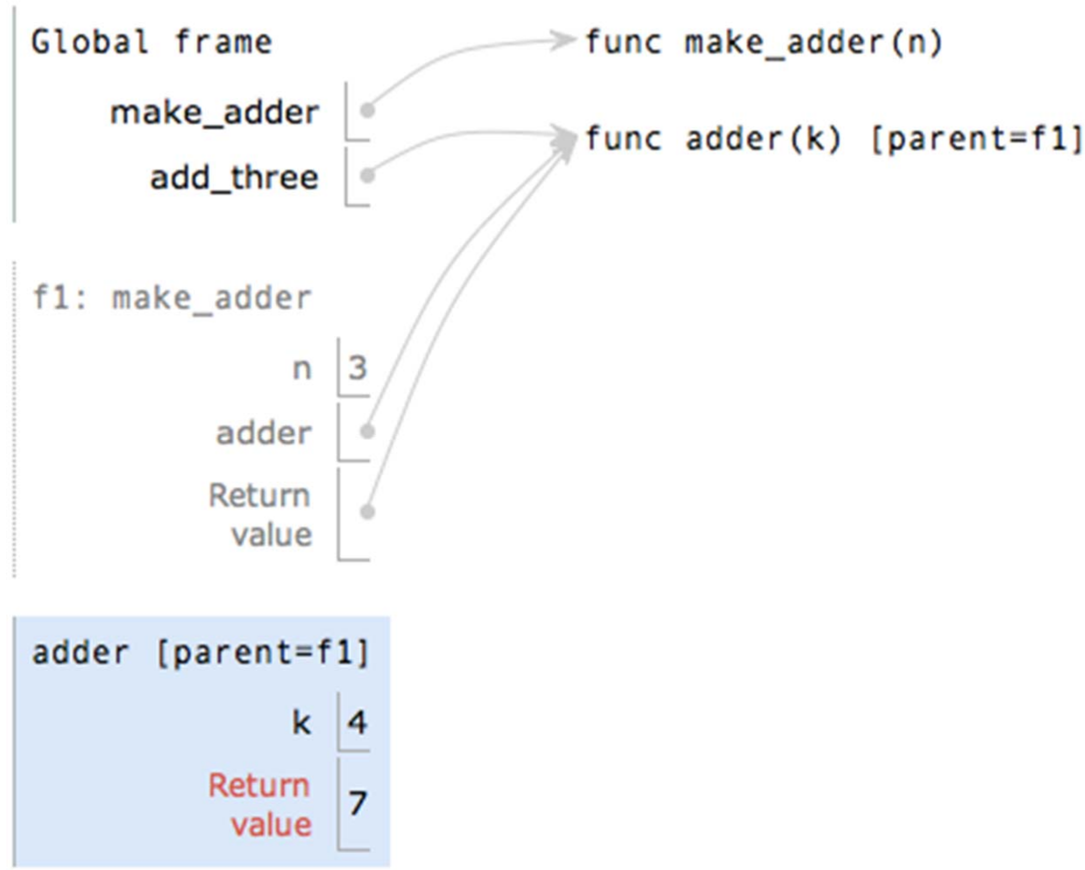

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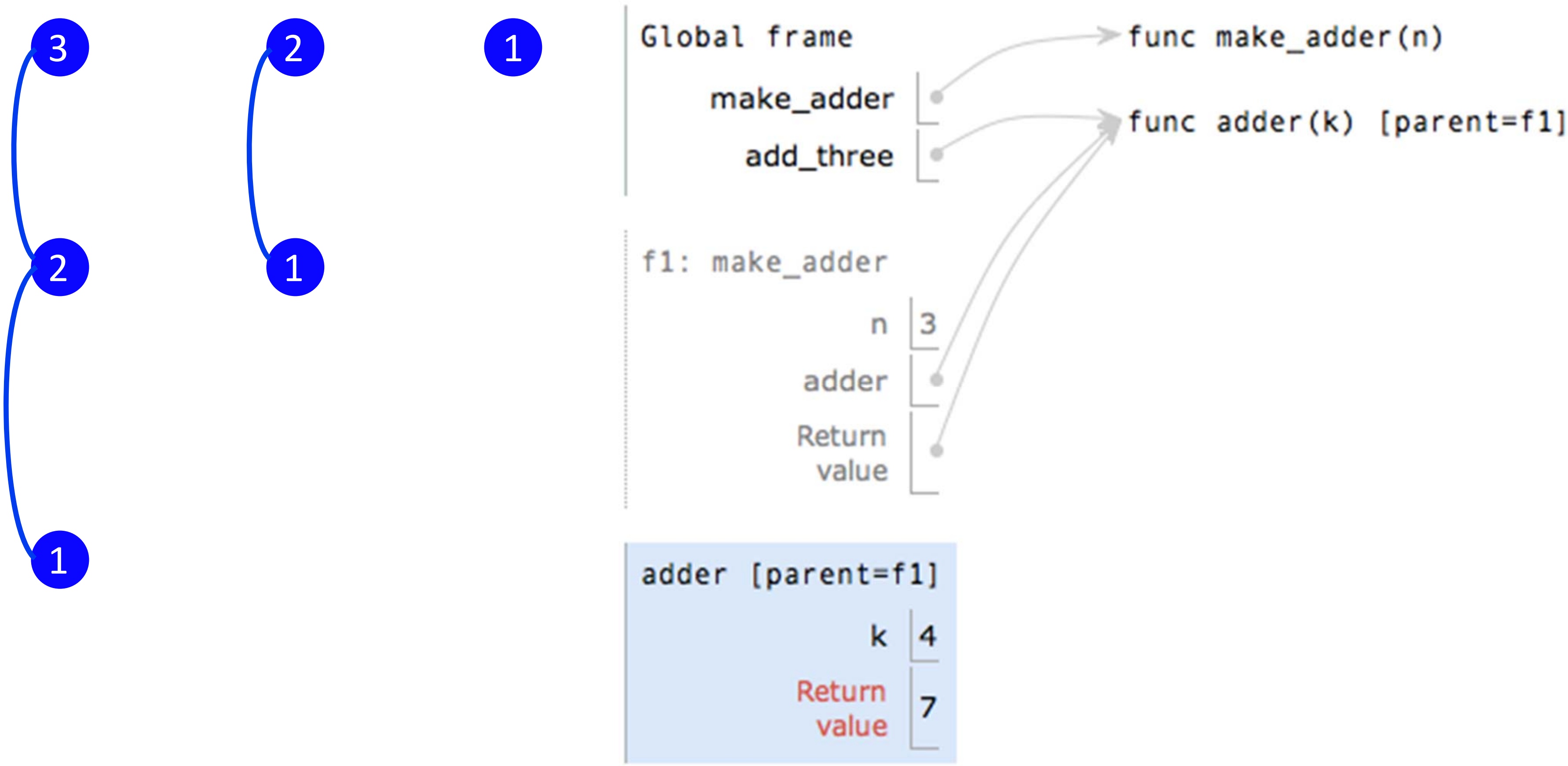

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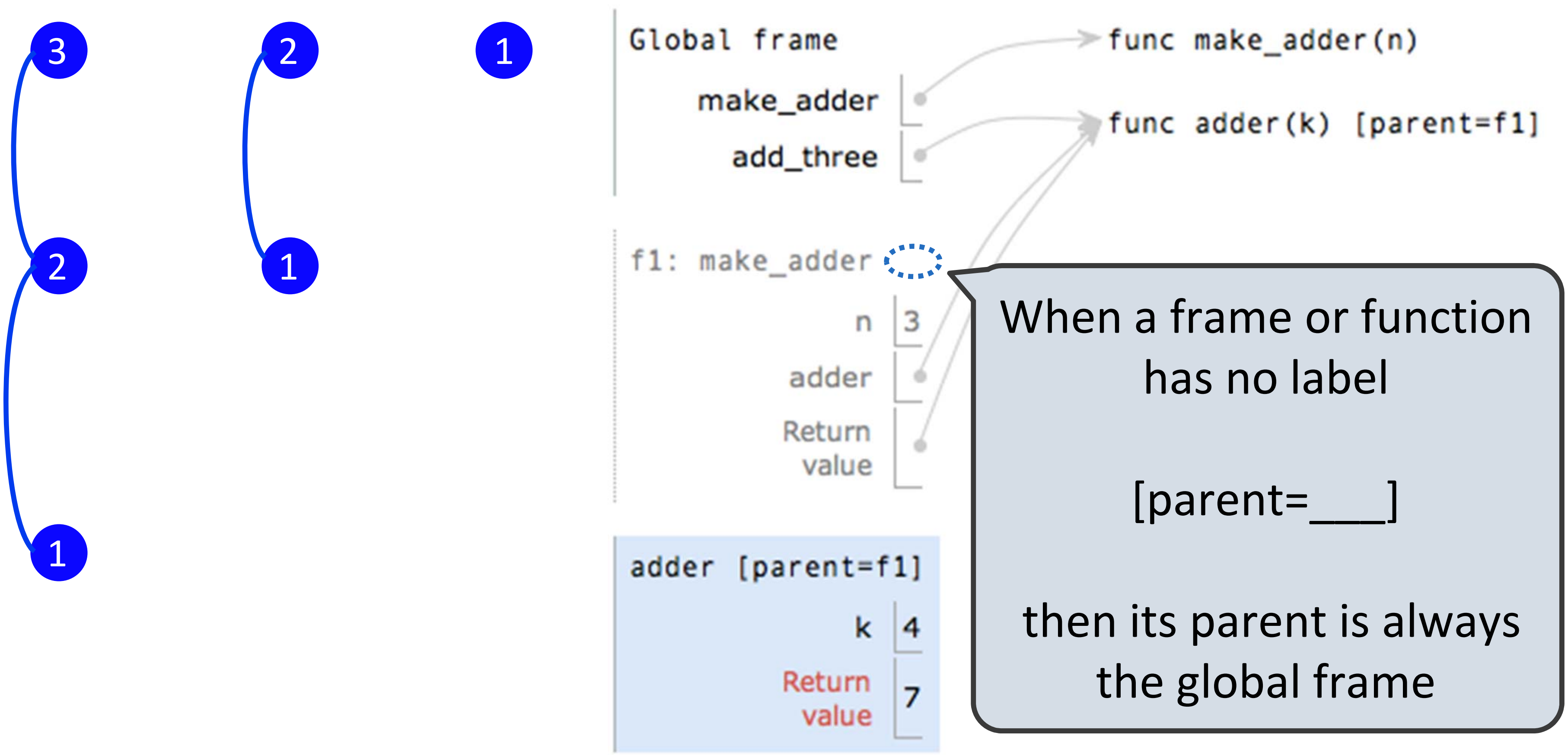

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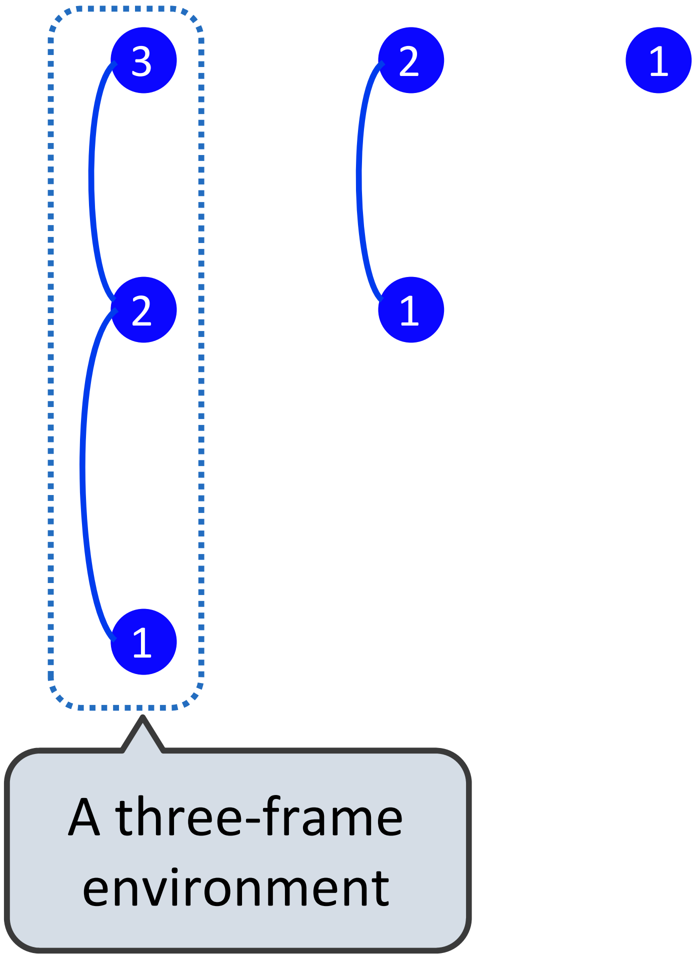

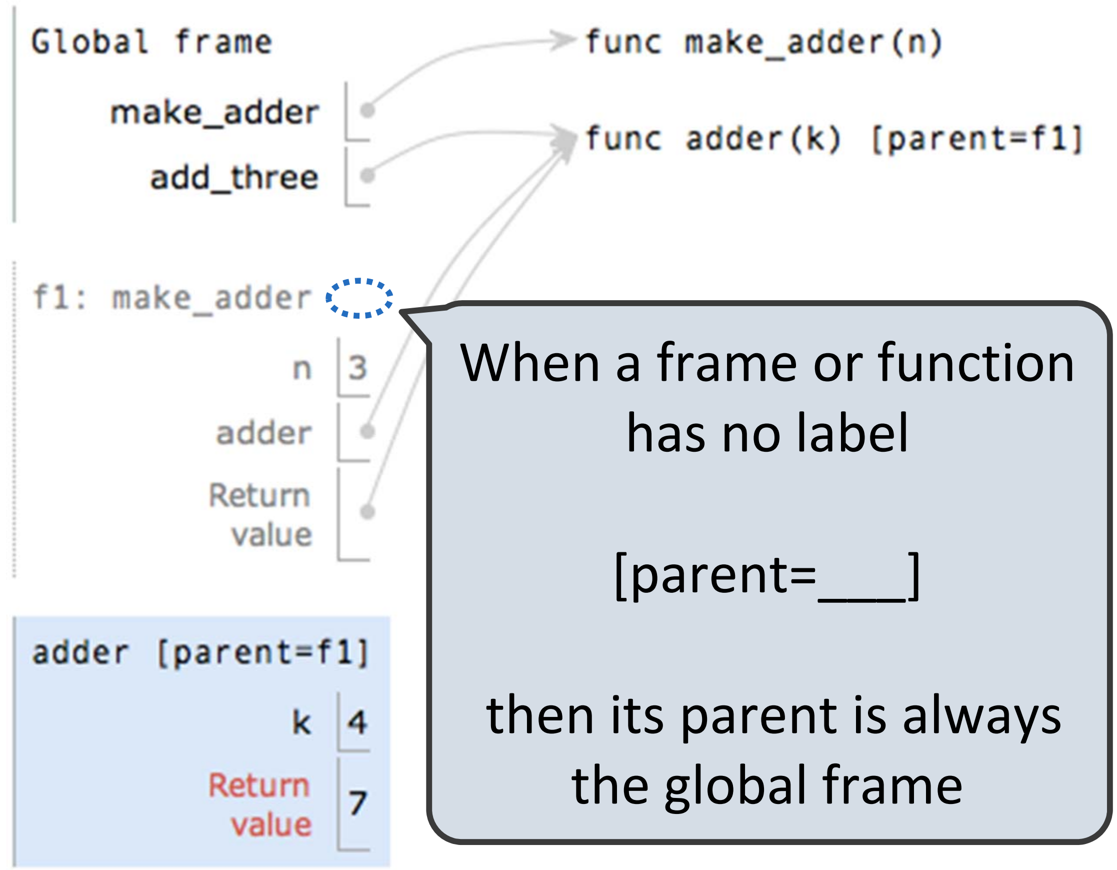

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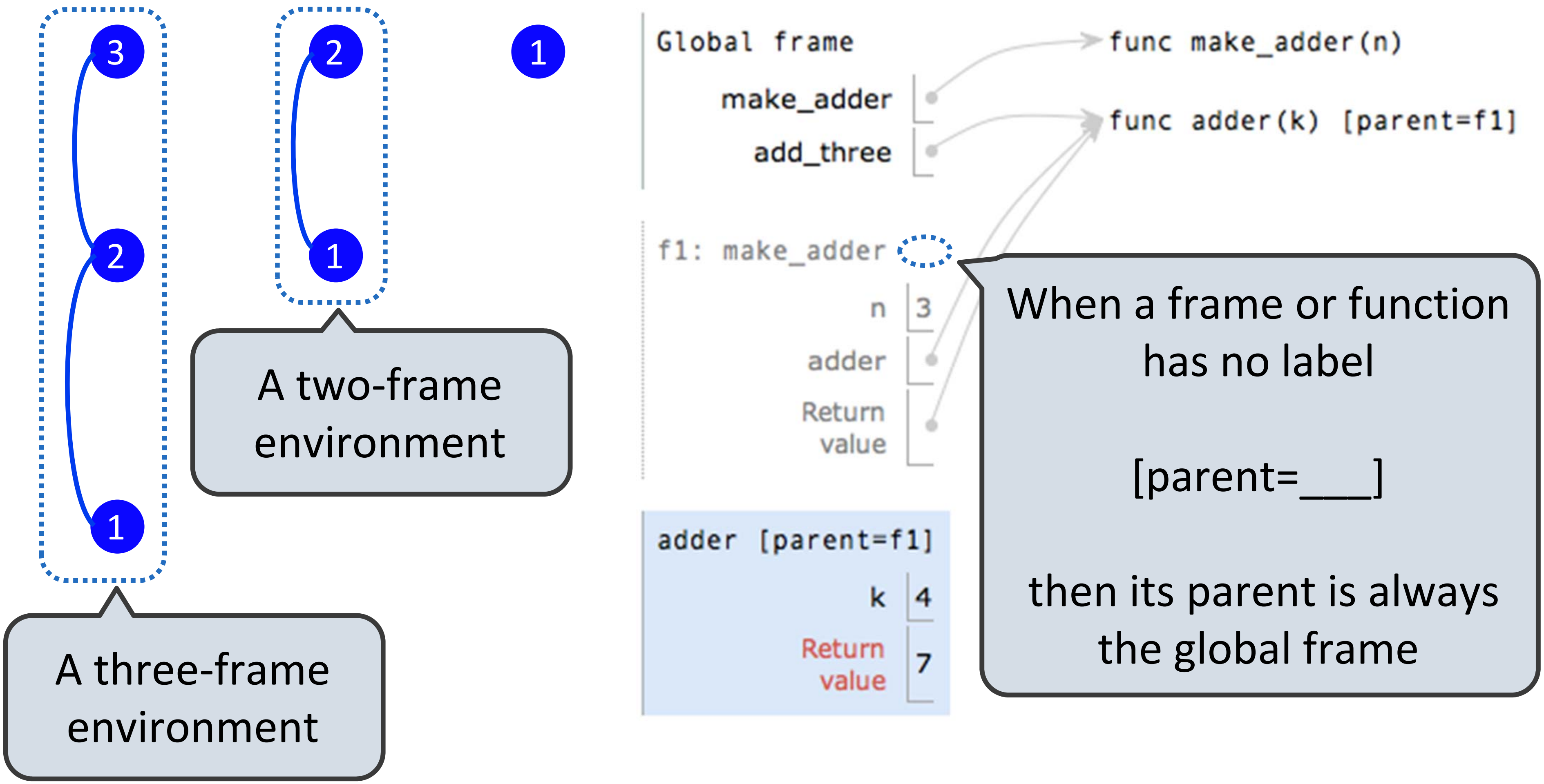

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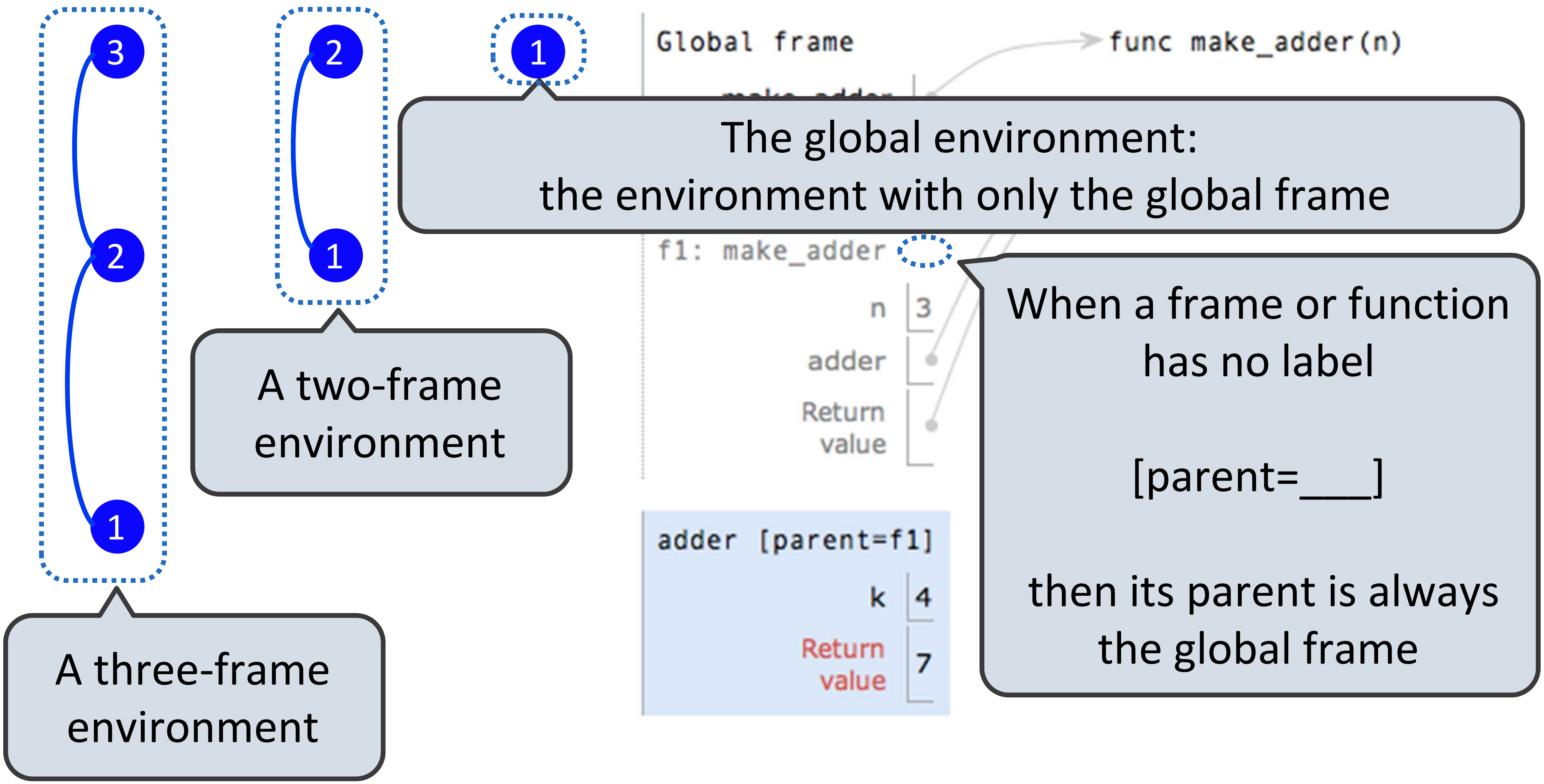

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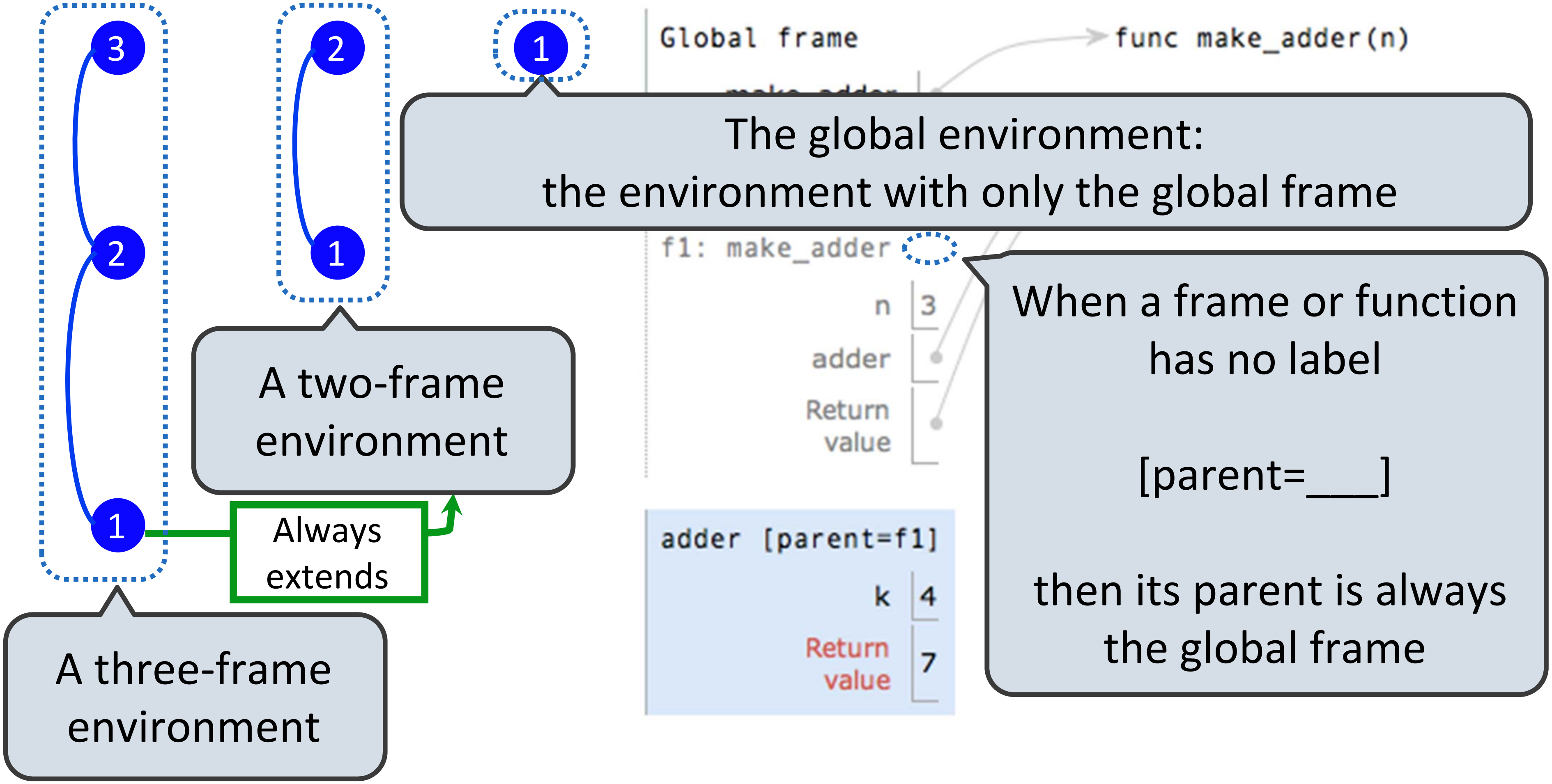

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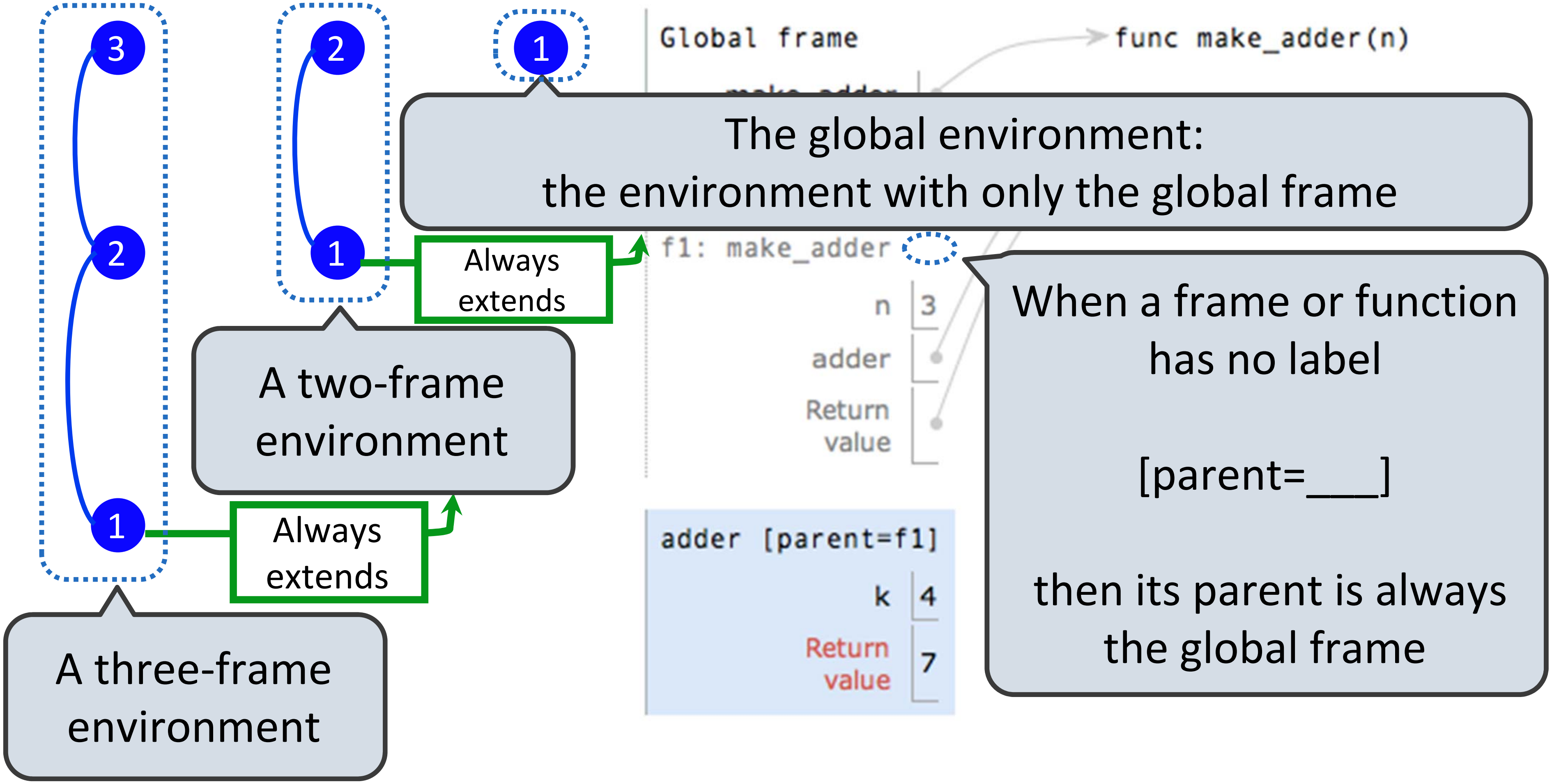

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4. Execute the body of the function in the environment that starts with this frame

Environment for Function Composition

Example: http://goo.gl/5zcug

Environment for Function Composition

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def square(x):
    return x * x

def make_adder(n):
    def adder(k):
        return n + k
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def compose1(f, g):
    def h(x):
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compose1(square, make_adder(2))(3)
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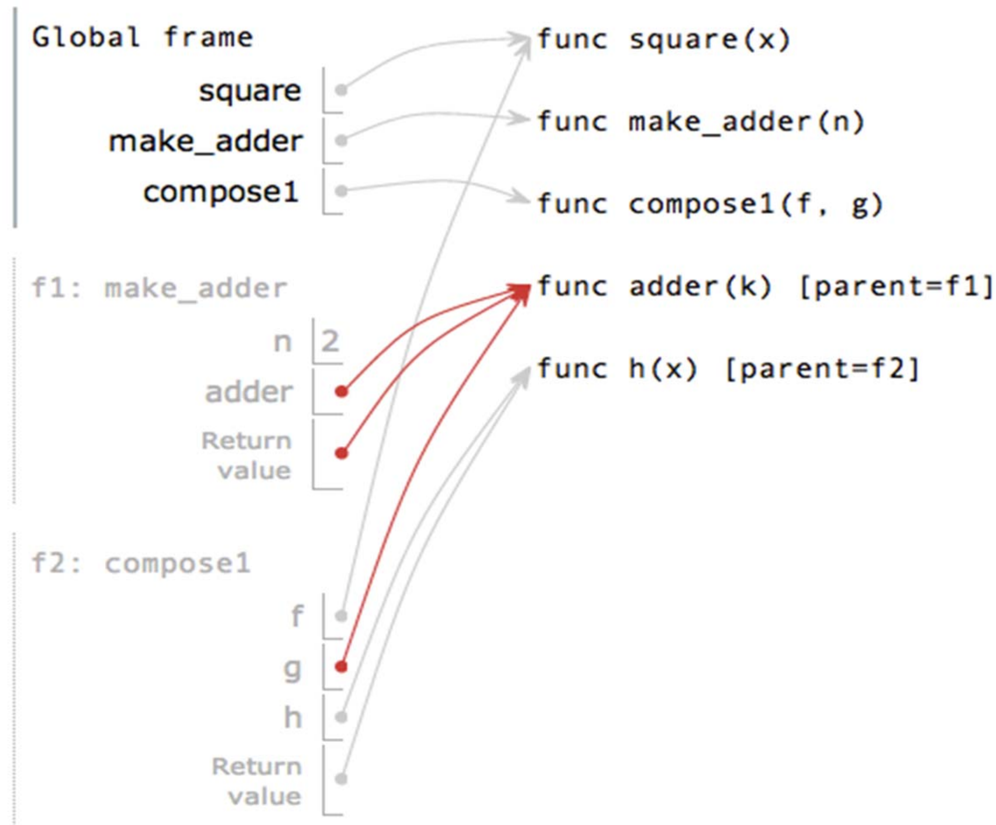

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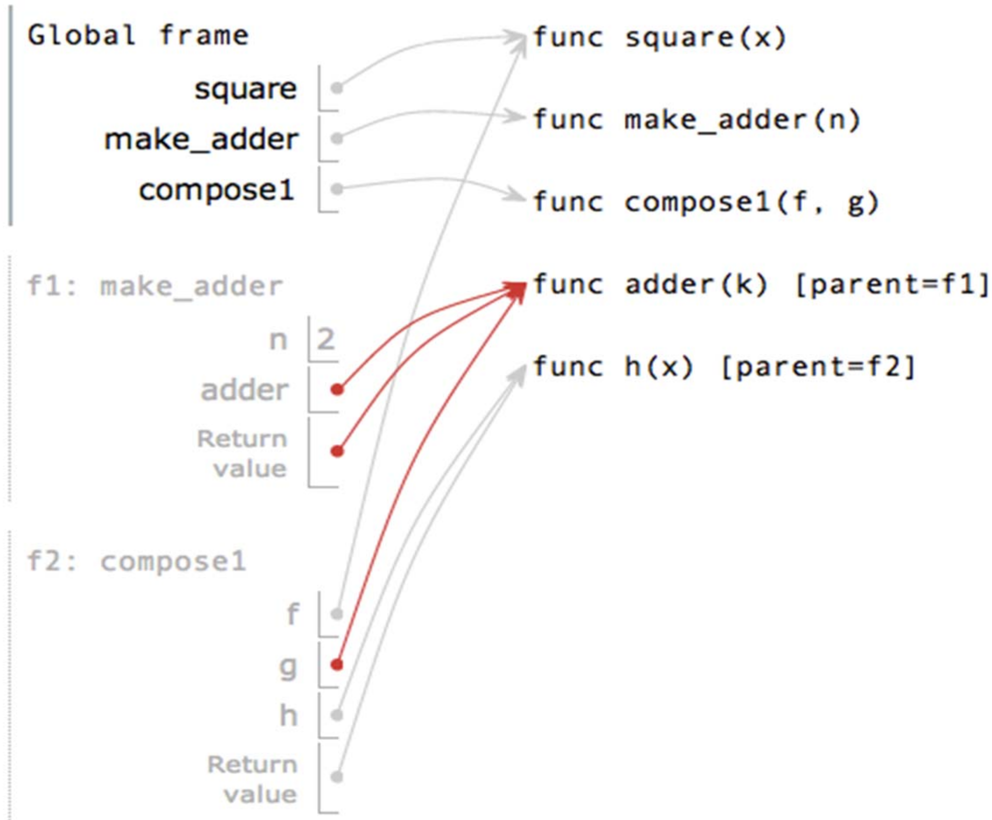

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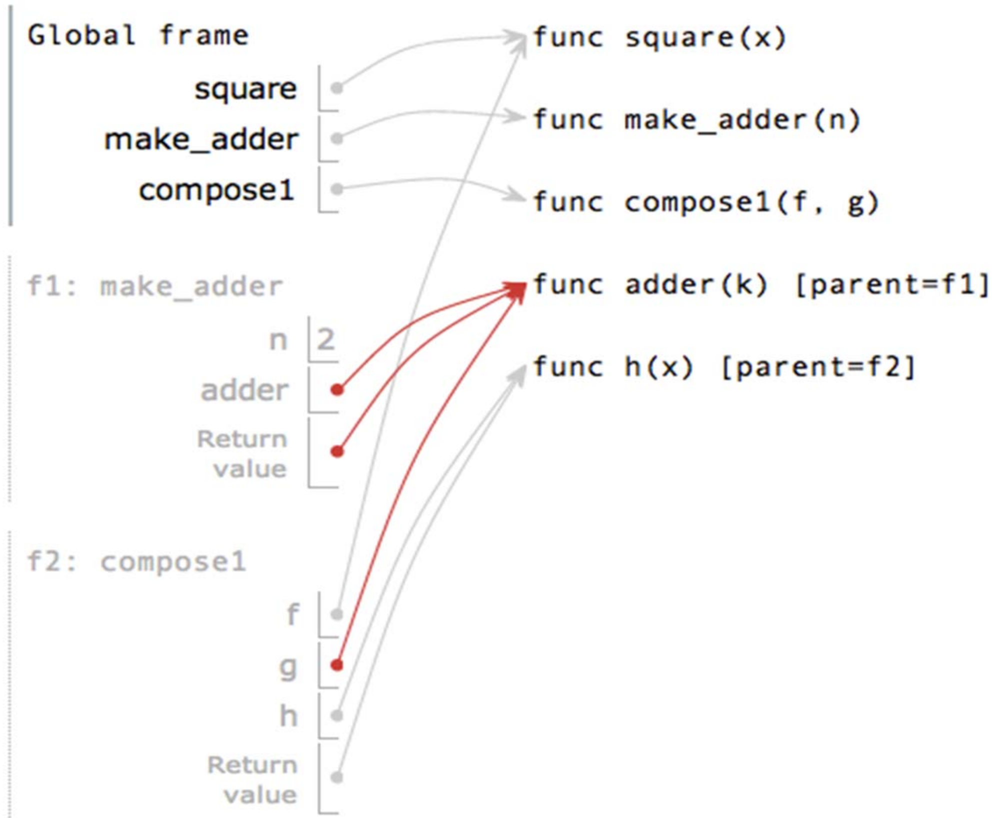

Example: http://goo.gl/5zcug

Environment for Function Composition

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def square(x):
    return x * x

def make_adder(n):
    def adder(k):
        return n + k
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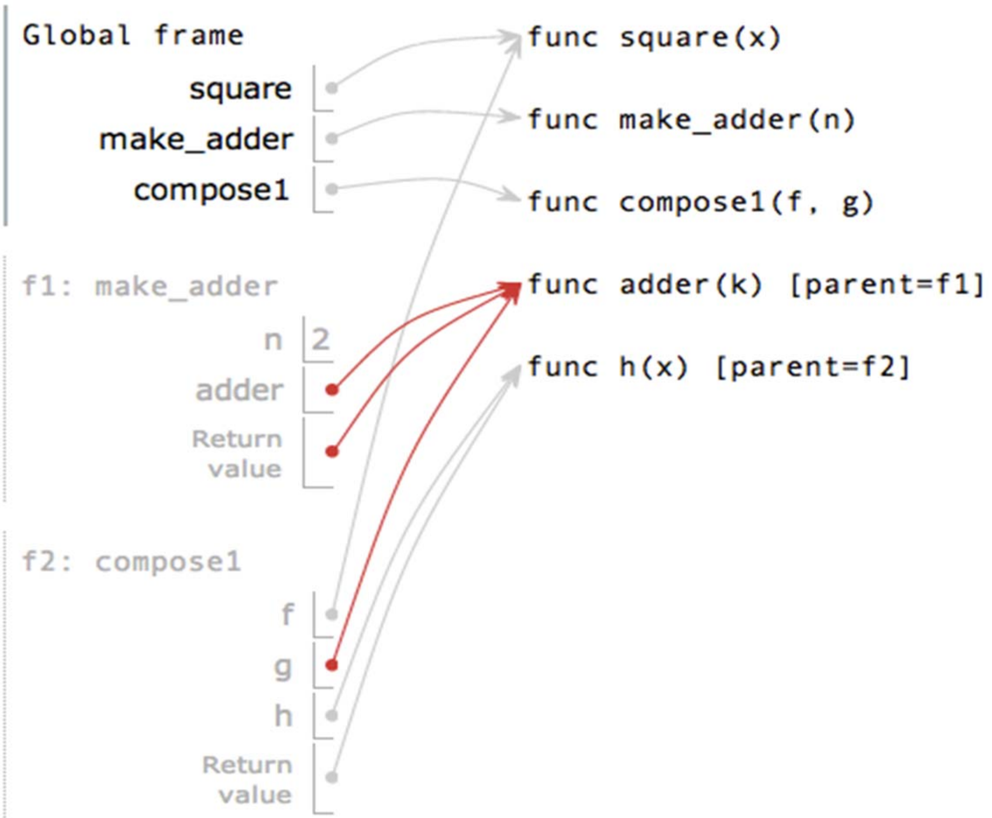
Return value of make_adder is an argument to compose1



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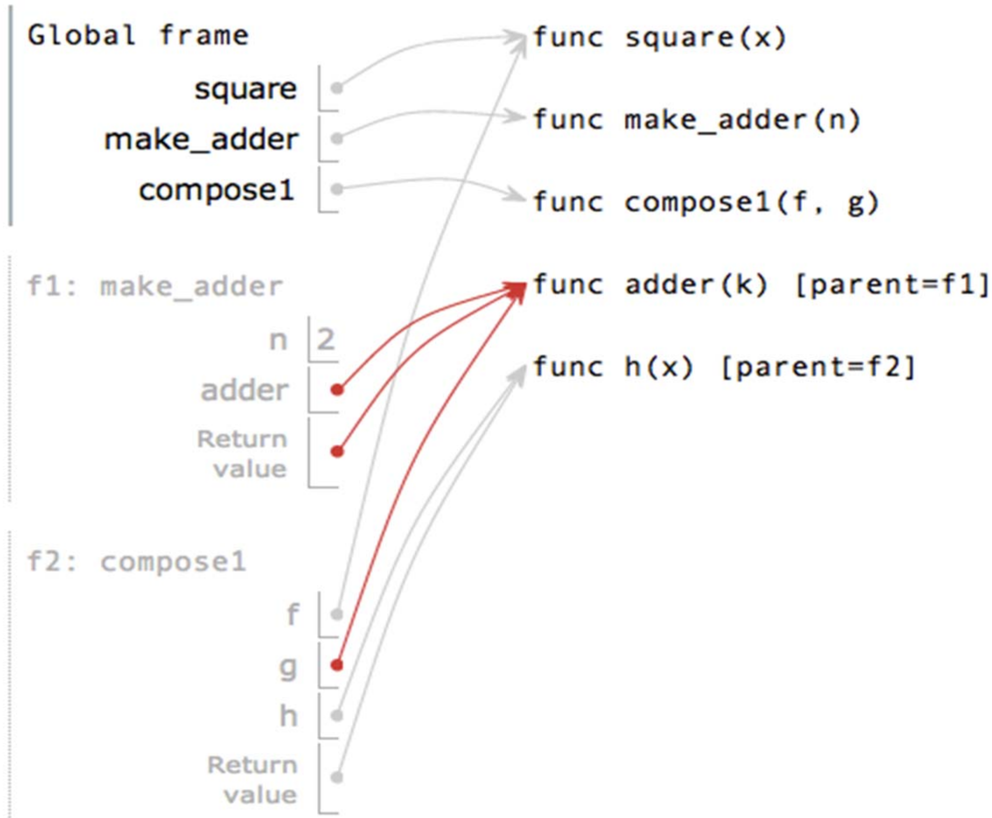

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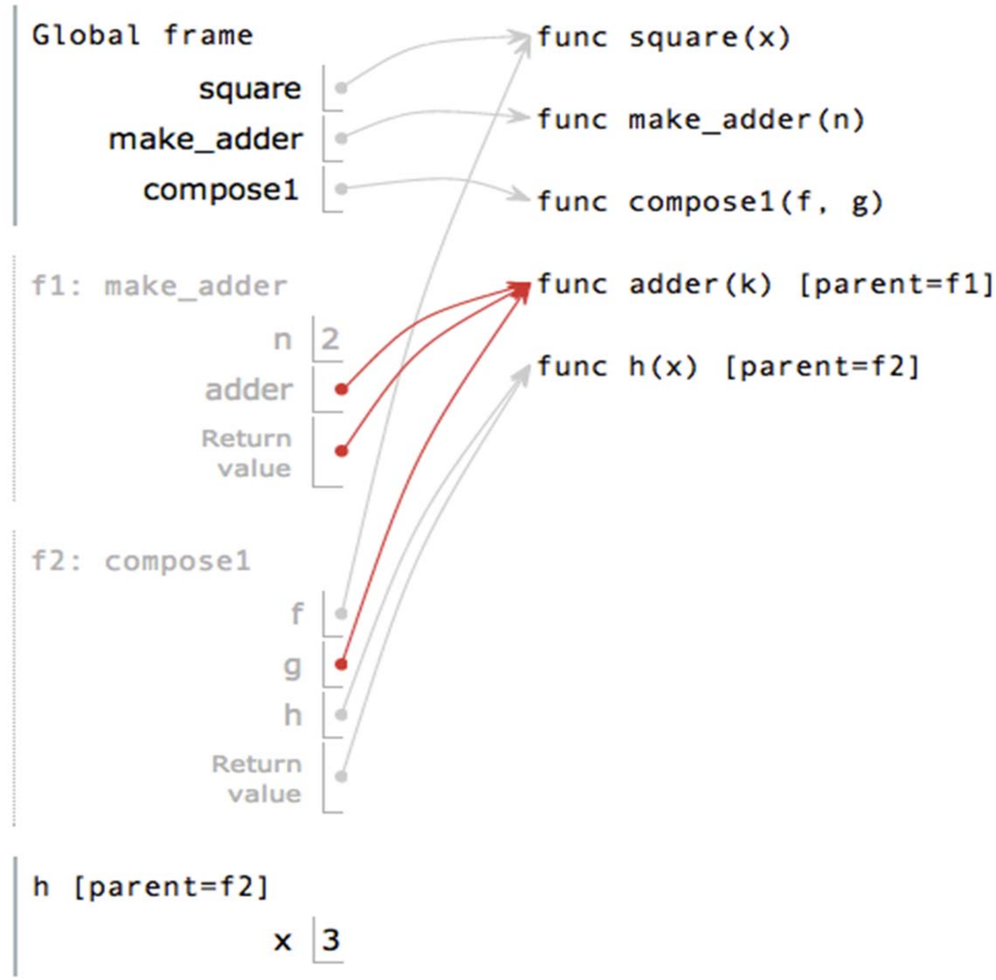

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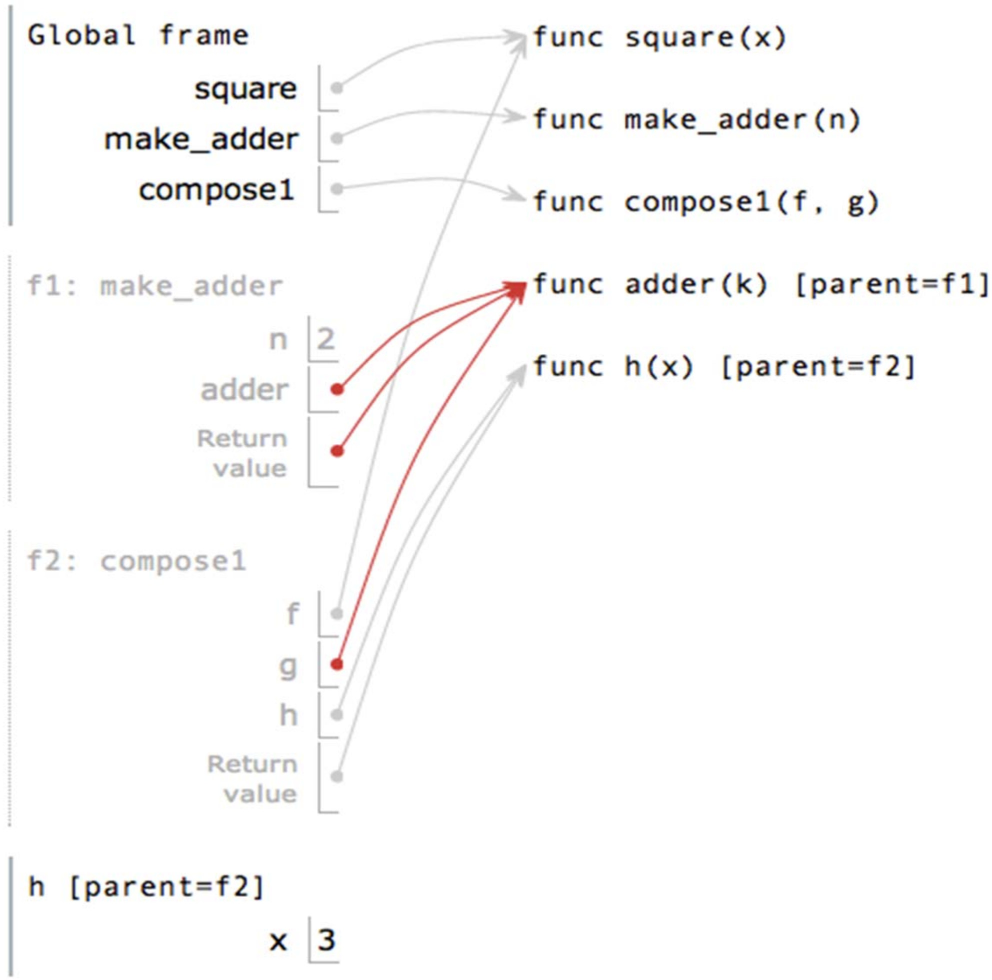

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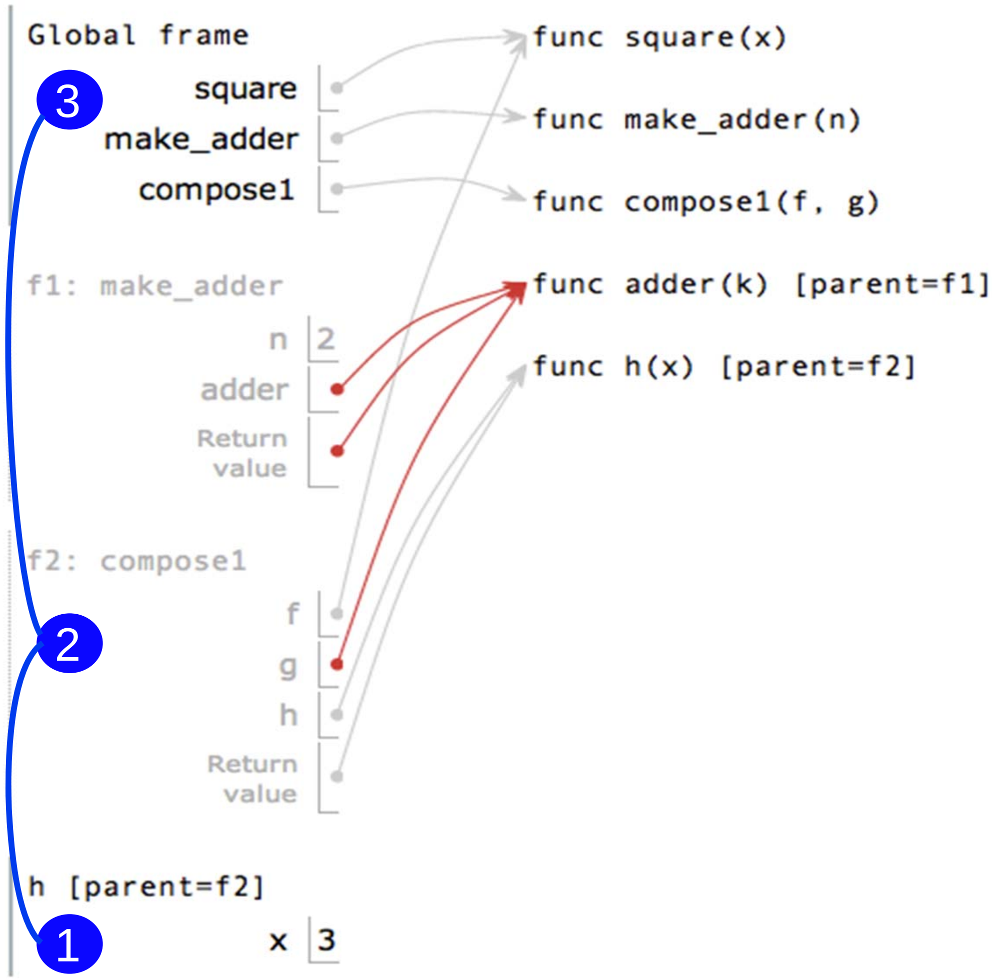

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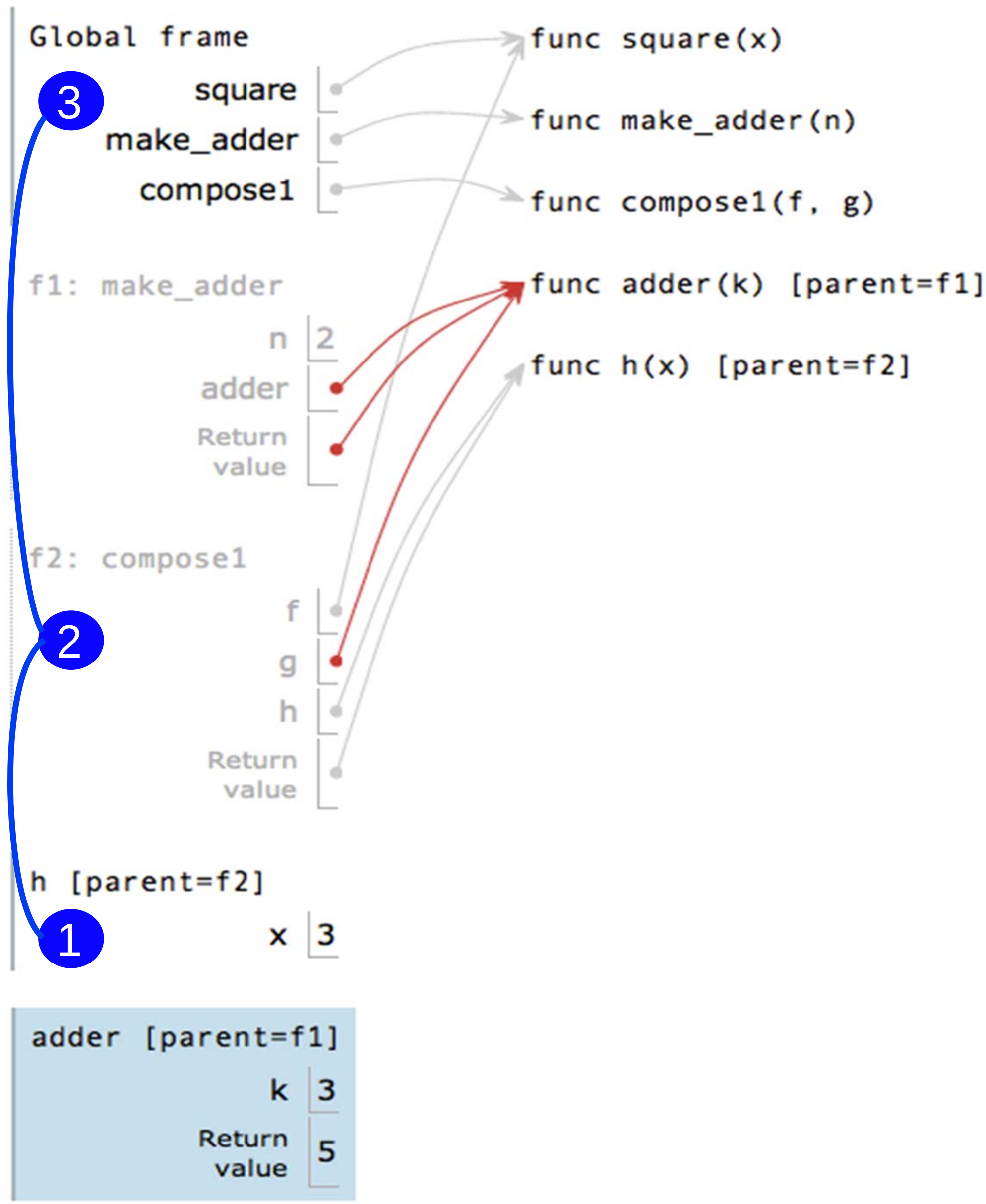

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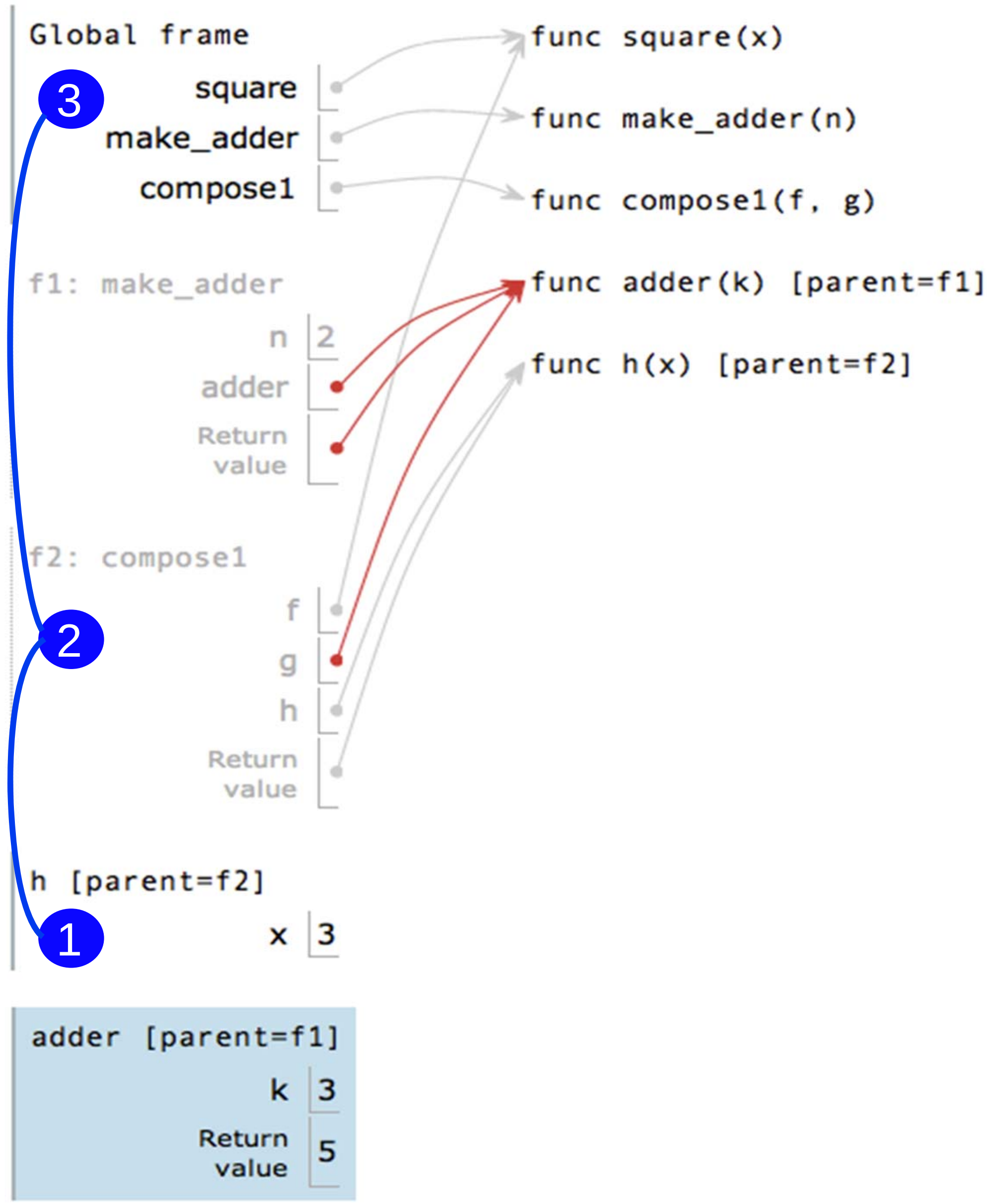

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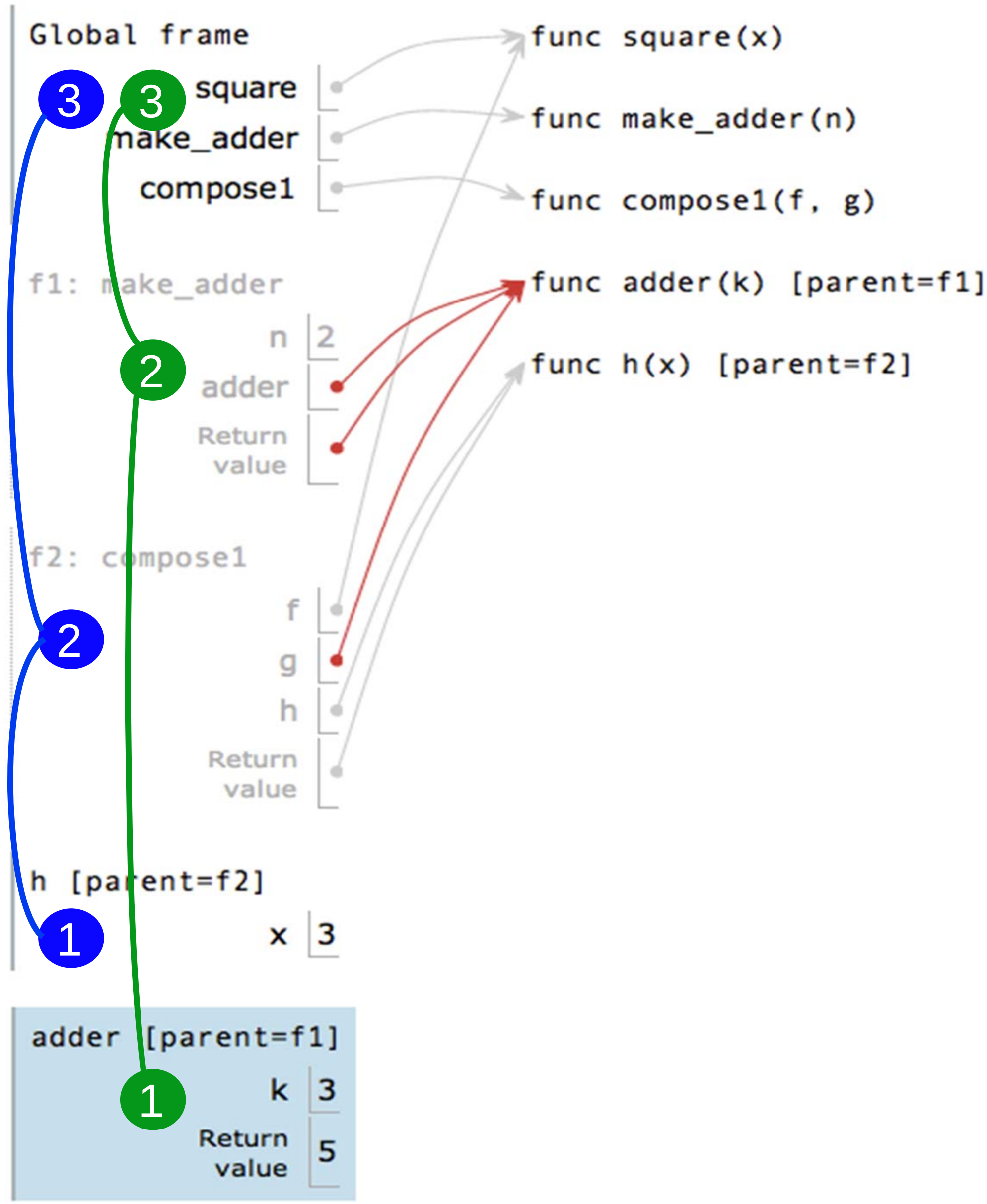

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Lambda Expressions

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Lambda expressions are rare in Python, but important in general

Evaluation of Lambda vs. Def

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`lambda x: x * x`   VS

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Execution procedure for def *statements*:

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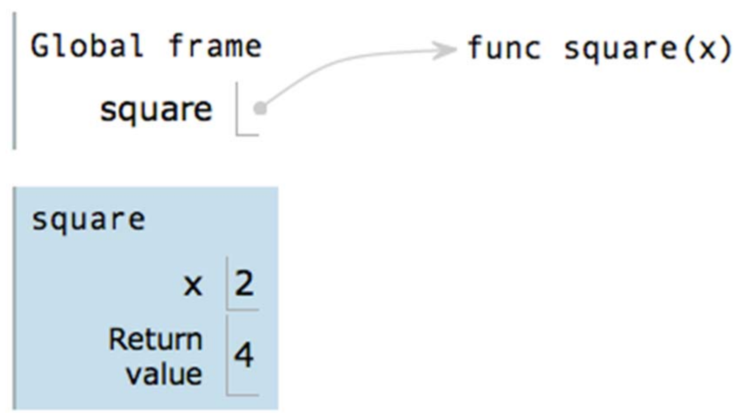

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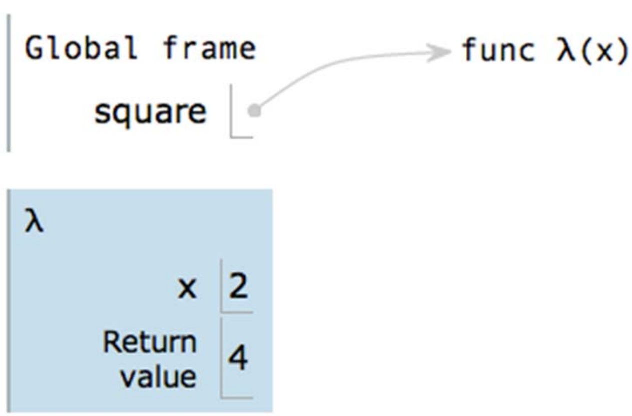

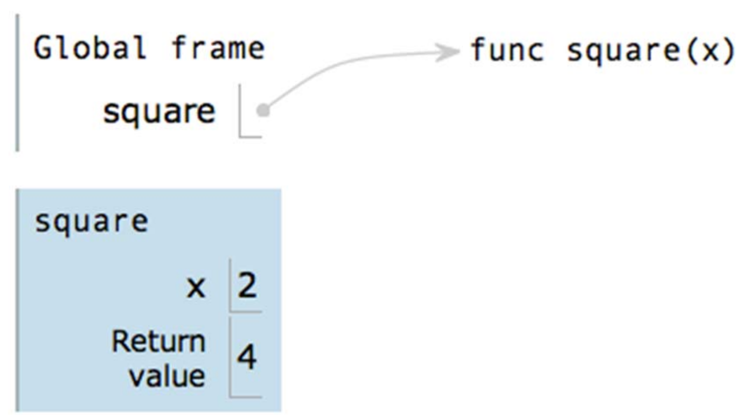

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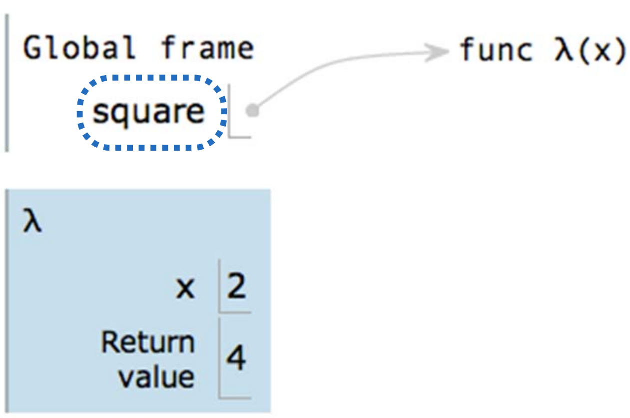

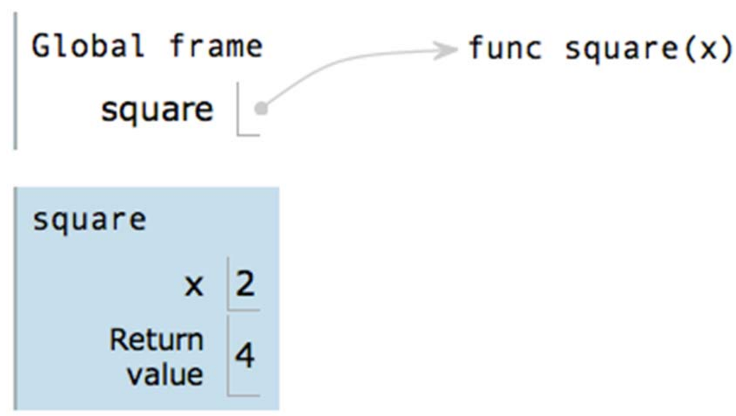

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