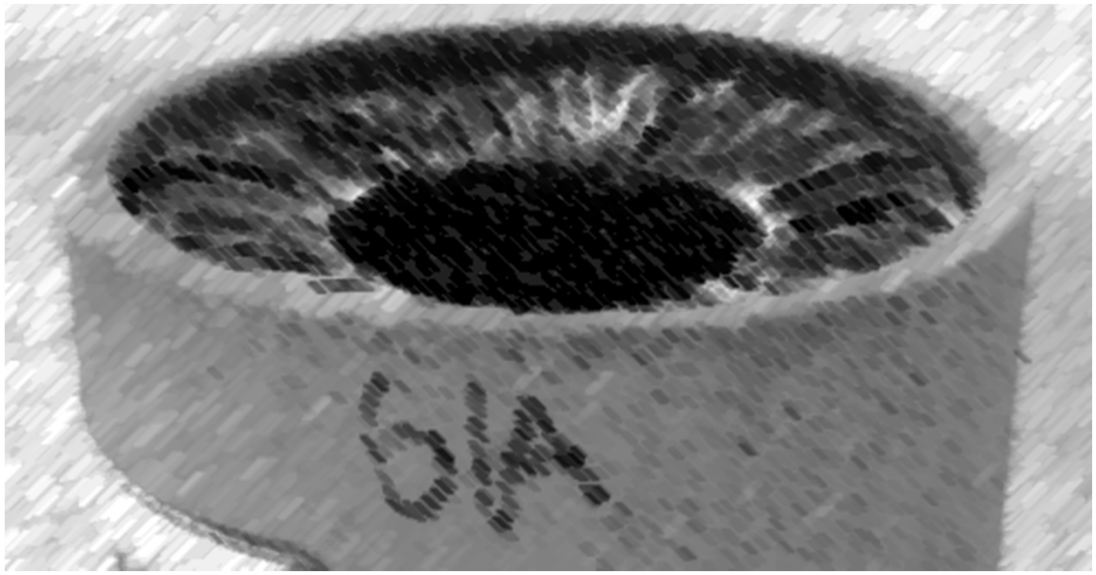

CS61A Lecture 5

Amir Kamil
UC Berkeley
February 1, 2013

Announcements

- Quiz today!
  - Only worth two points, so don't worry!
- Hog project
  - Get started early!
  - If you still don't have a partner (and want one), find one on Piazza
    - Use existing post; don't make a new one

The Art of the Function

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- Define functions generally

Generalizing Patterns with Parameters

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Regular geometric shapes relate length and area.

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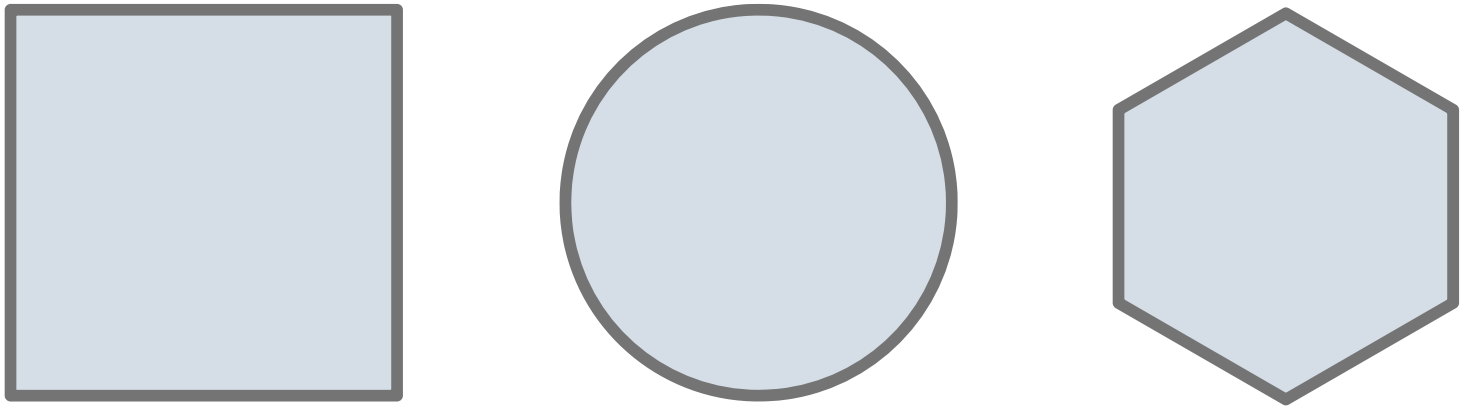

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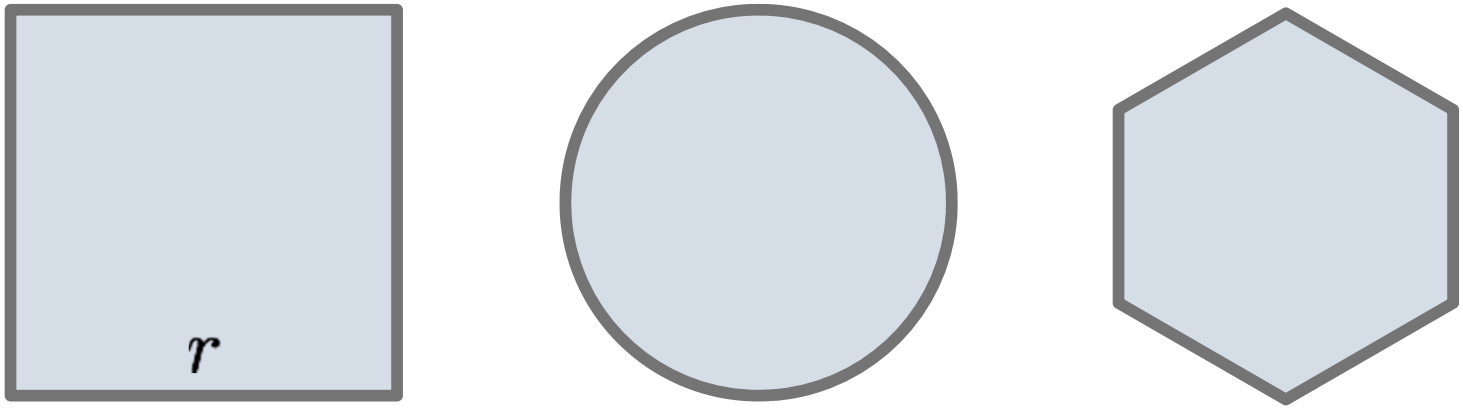

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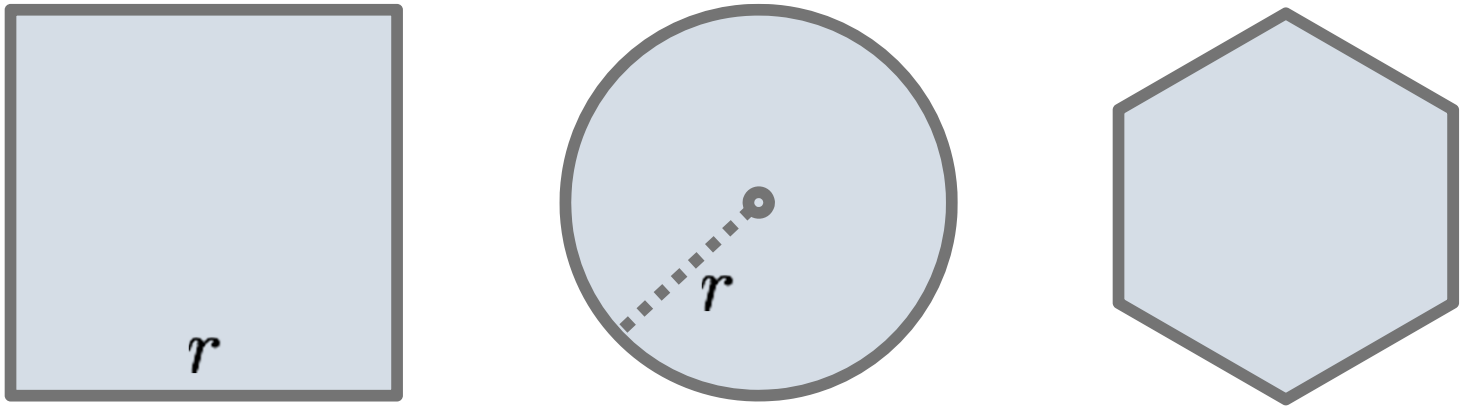

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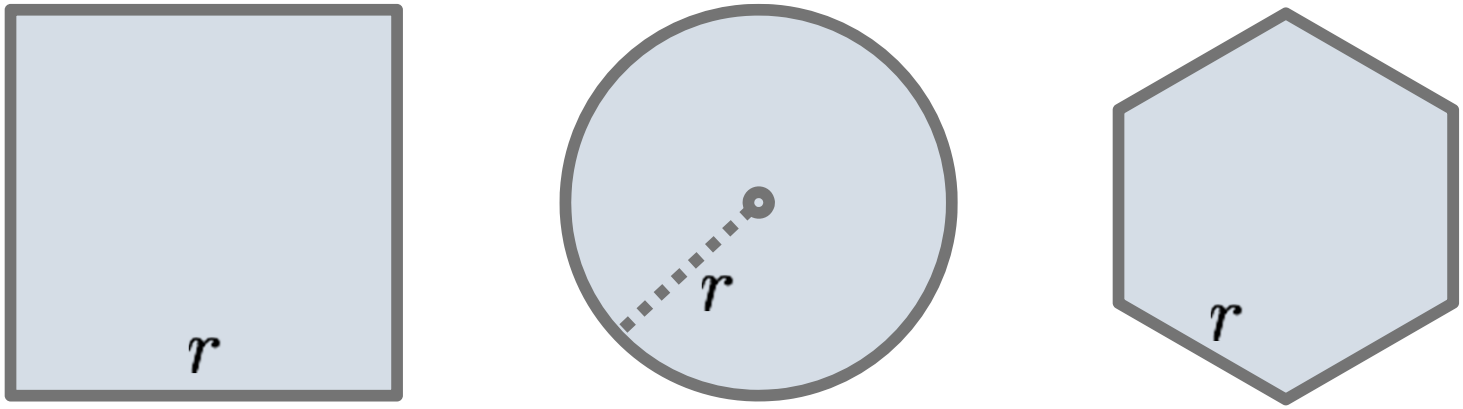

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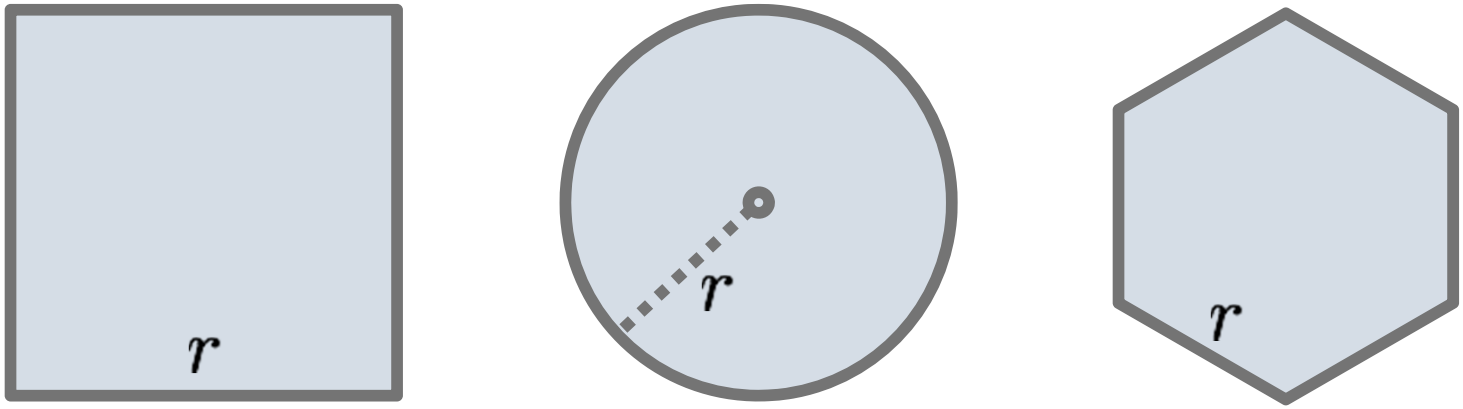

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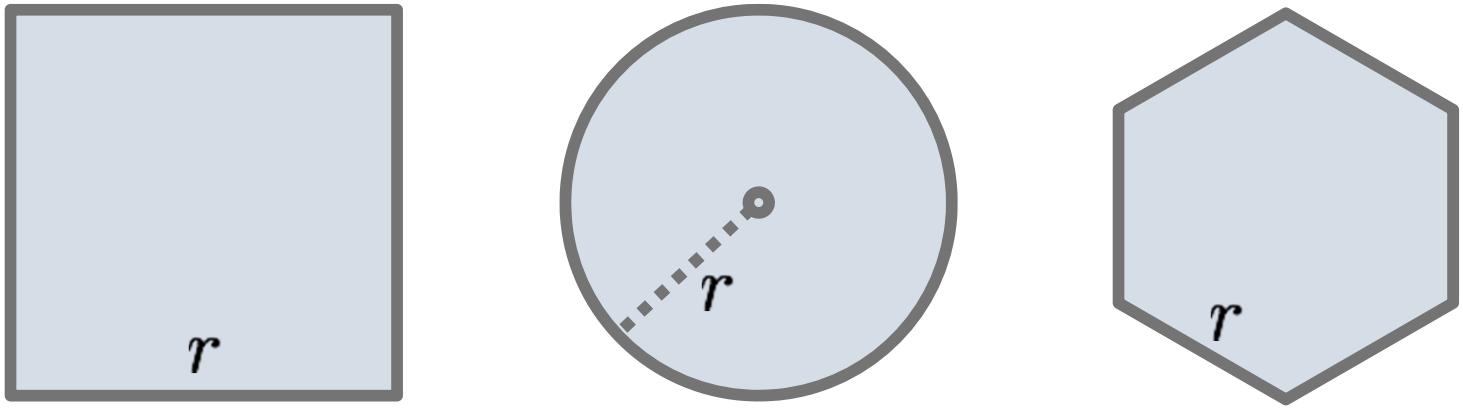

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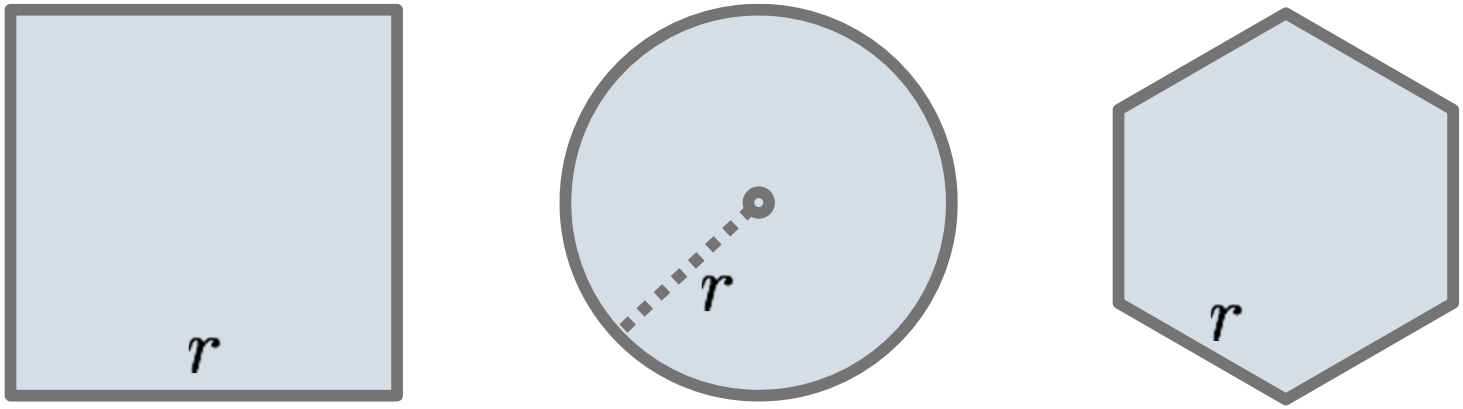

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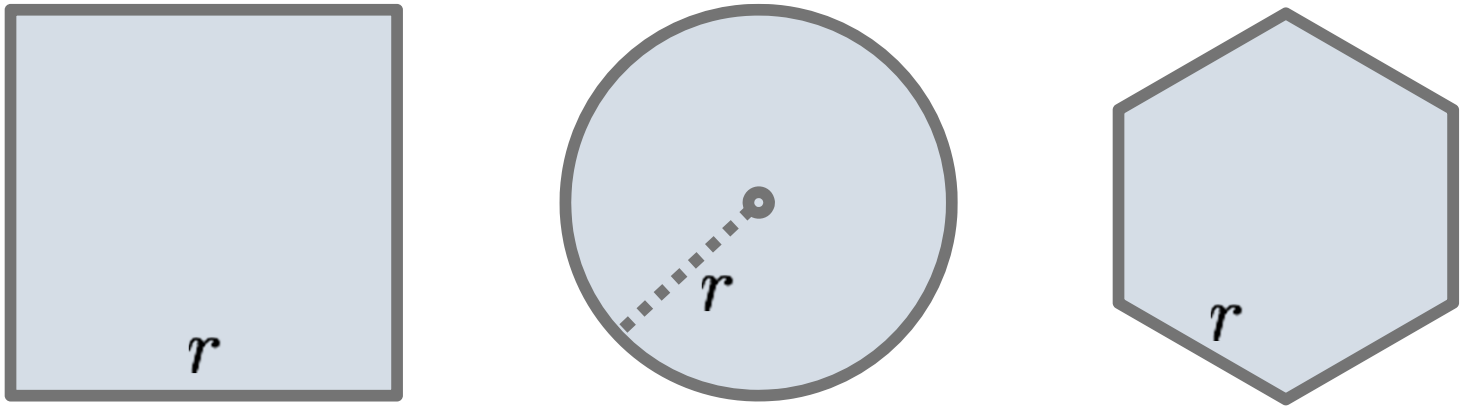

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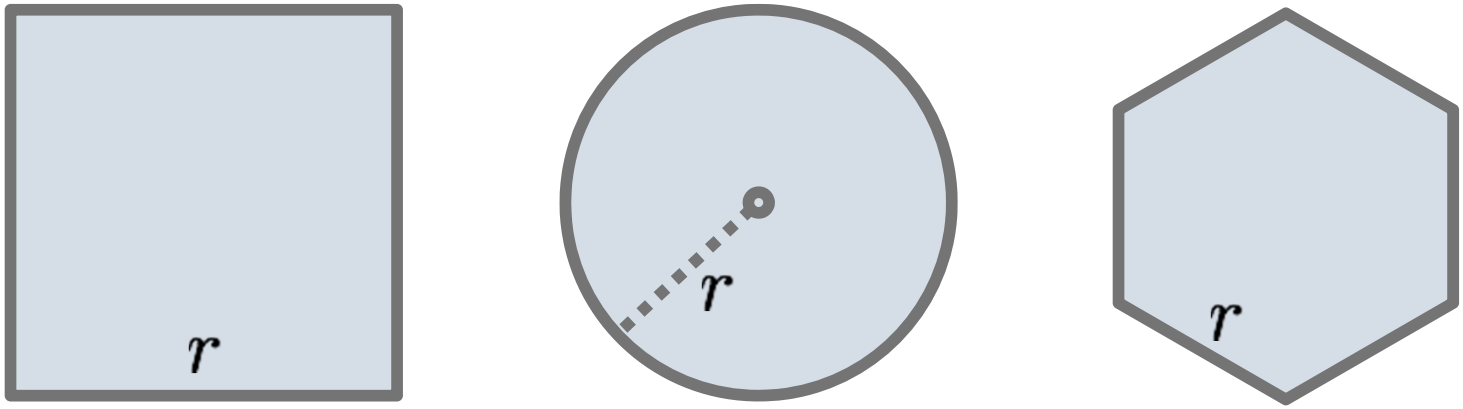

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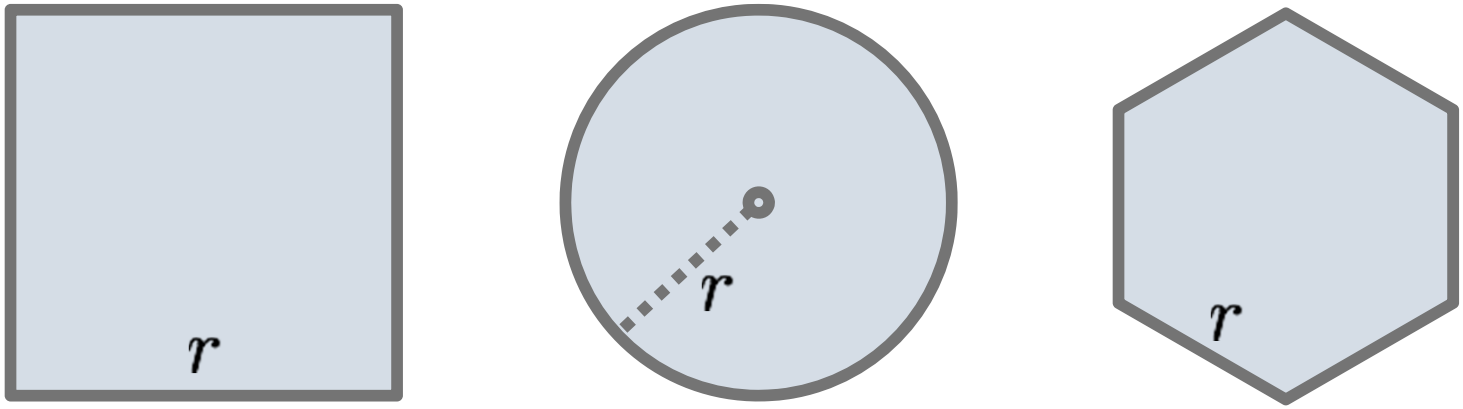

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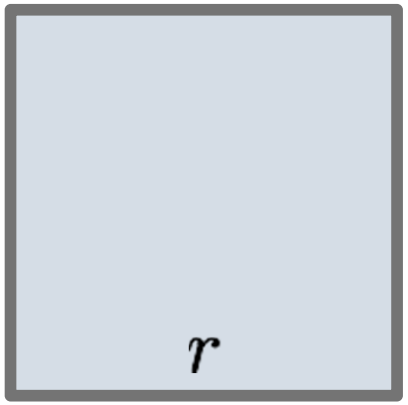
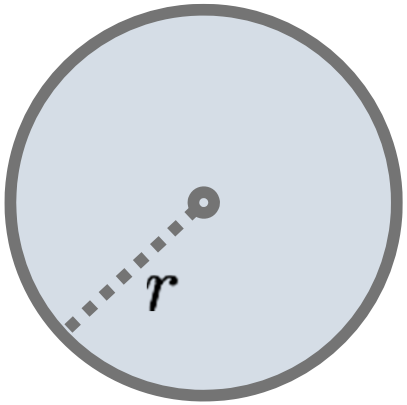
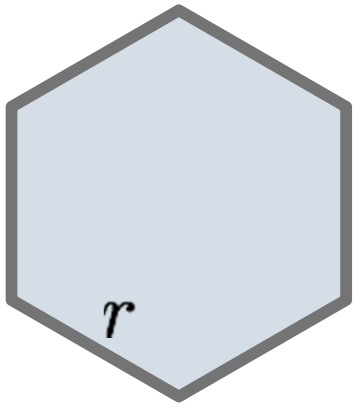
Shape:			
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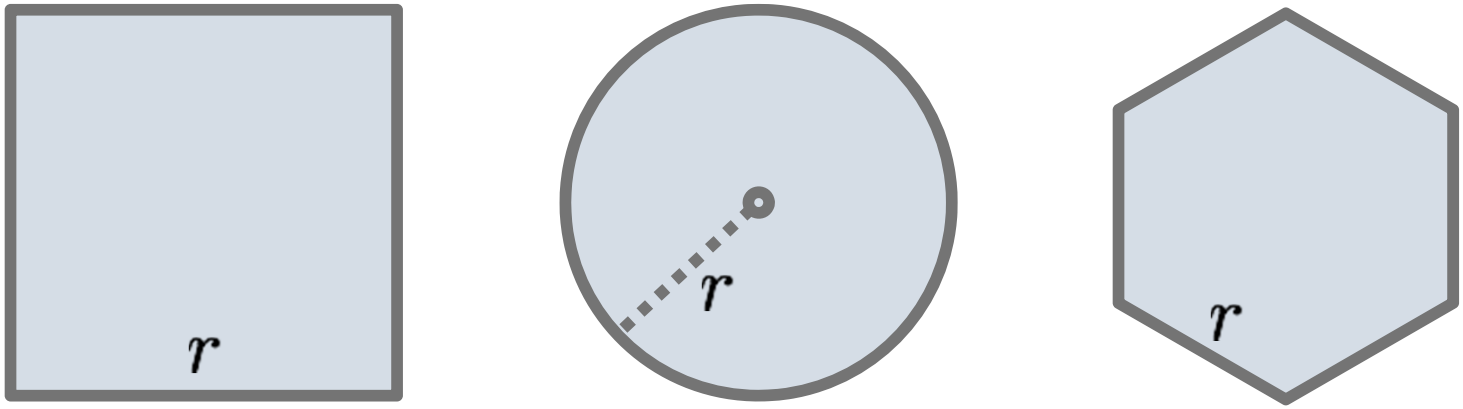

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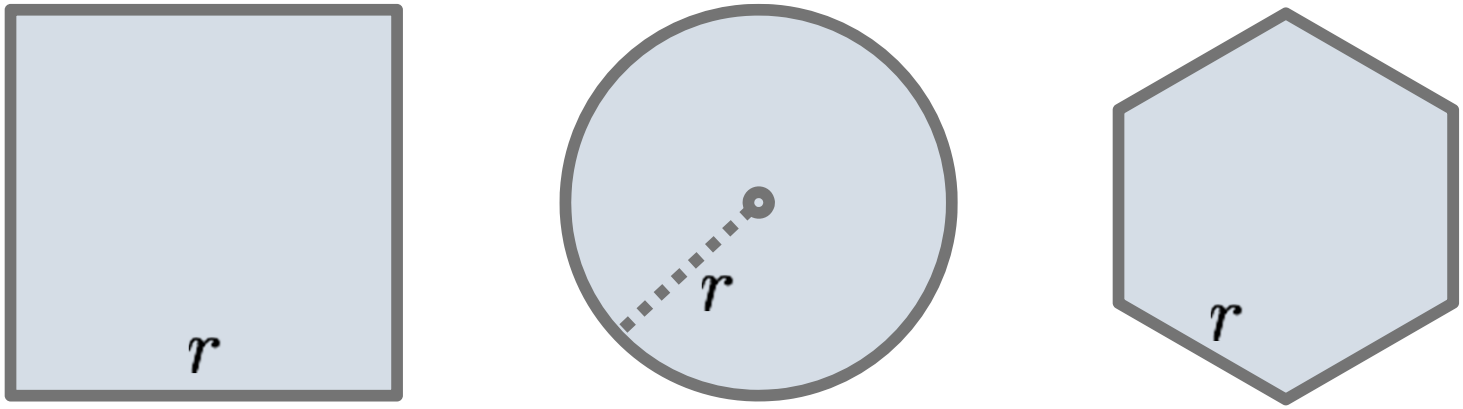

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Finding common structure allows for shared implementation

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$$\sum_{k=1}^{5} \frac{8}{(4k-3)\cdot(4k-1)} = \frac{8}{3} + \frac{8}{35} + \frac{8}{99} + \frac{8}{195} + \frac{8}{323} \qquad = 3.04$$

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def cube(k):
    return pow(k, 3)

def summation(n, term):
    """Sum the first n terms of a sequence.

    >>> summation(5, cube)
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    """
    total, k = 0, 1
    while k <= n:
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Example: http://goo.gl/e4YBH

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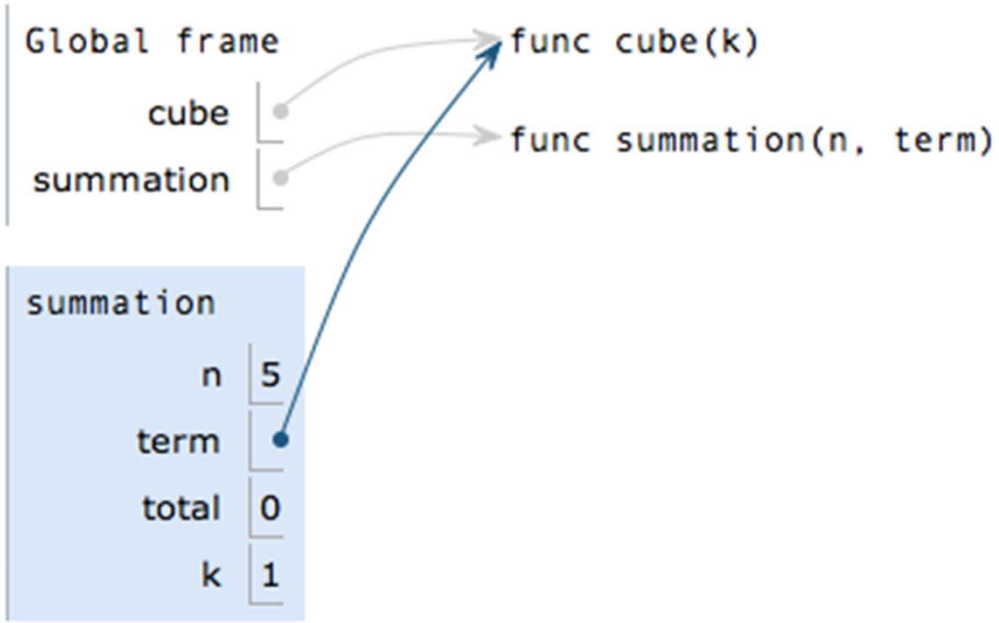


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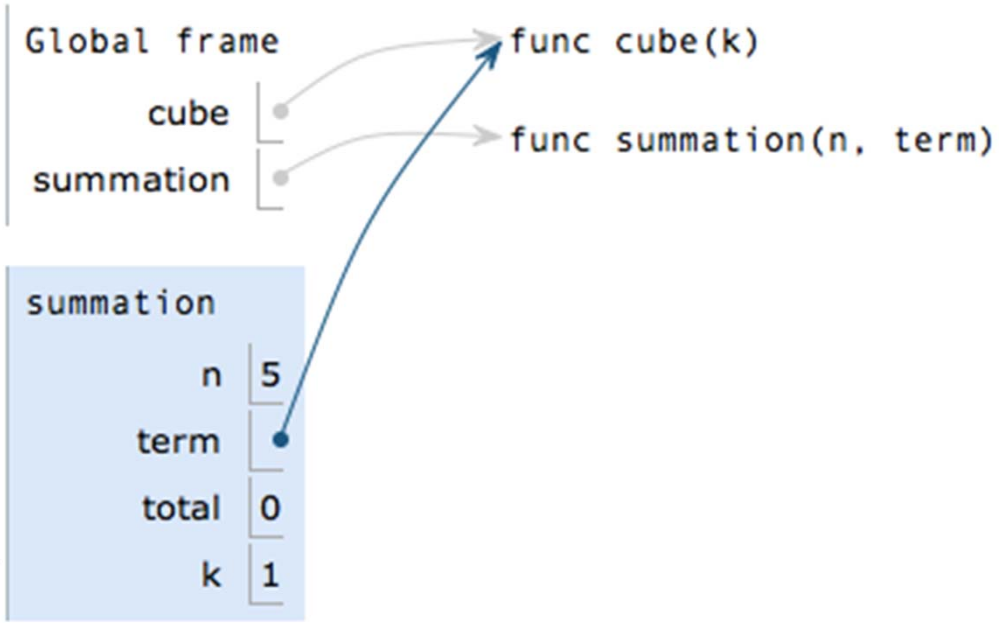


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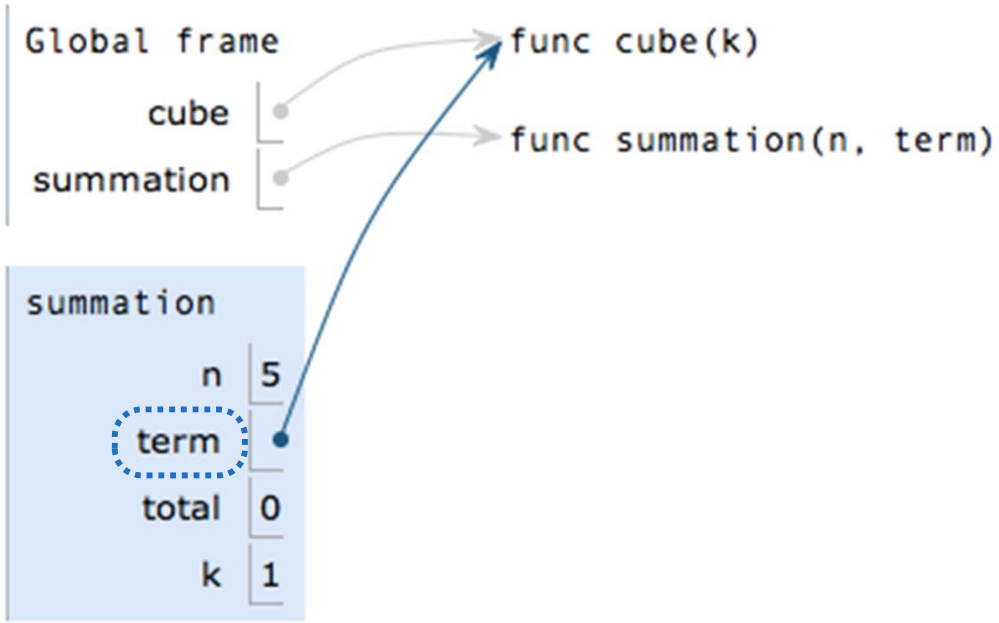


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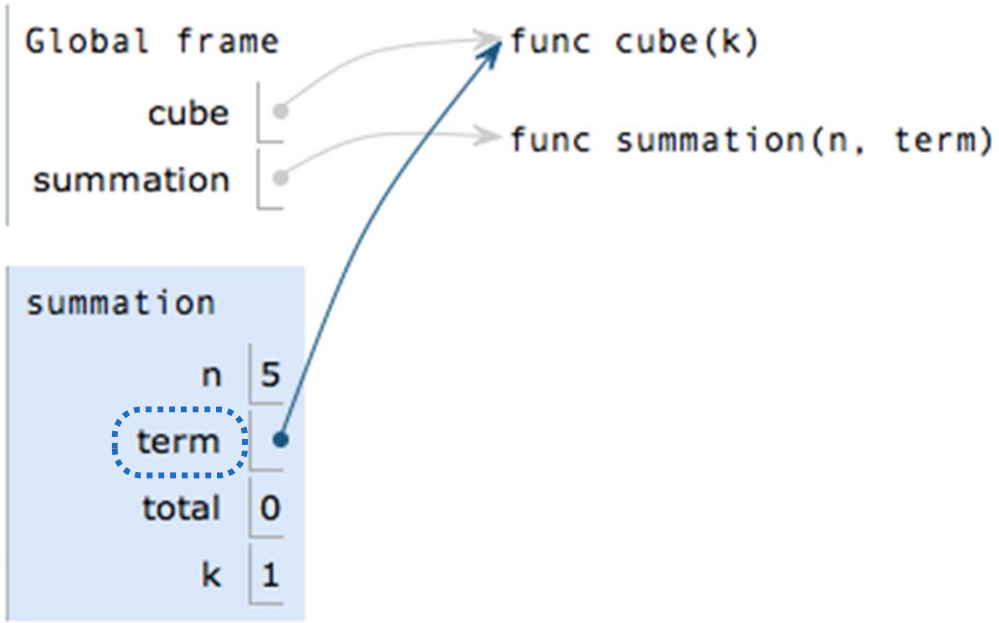


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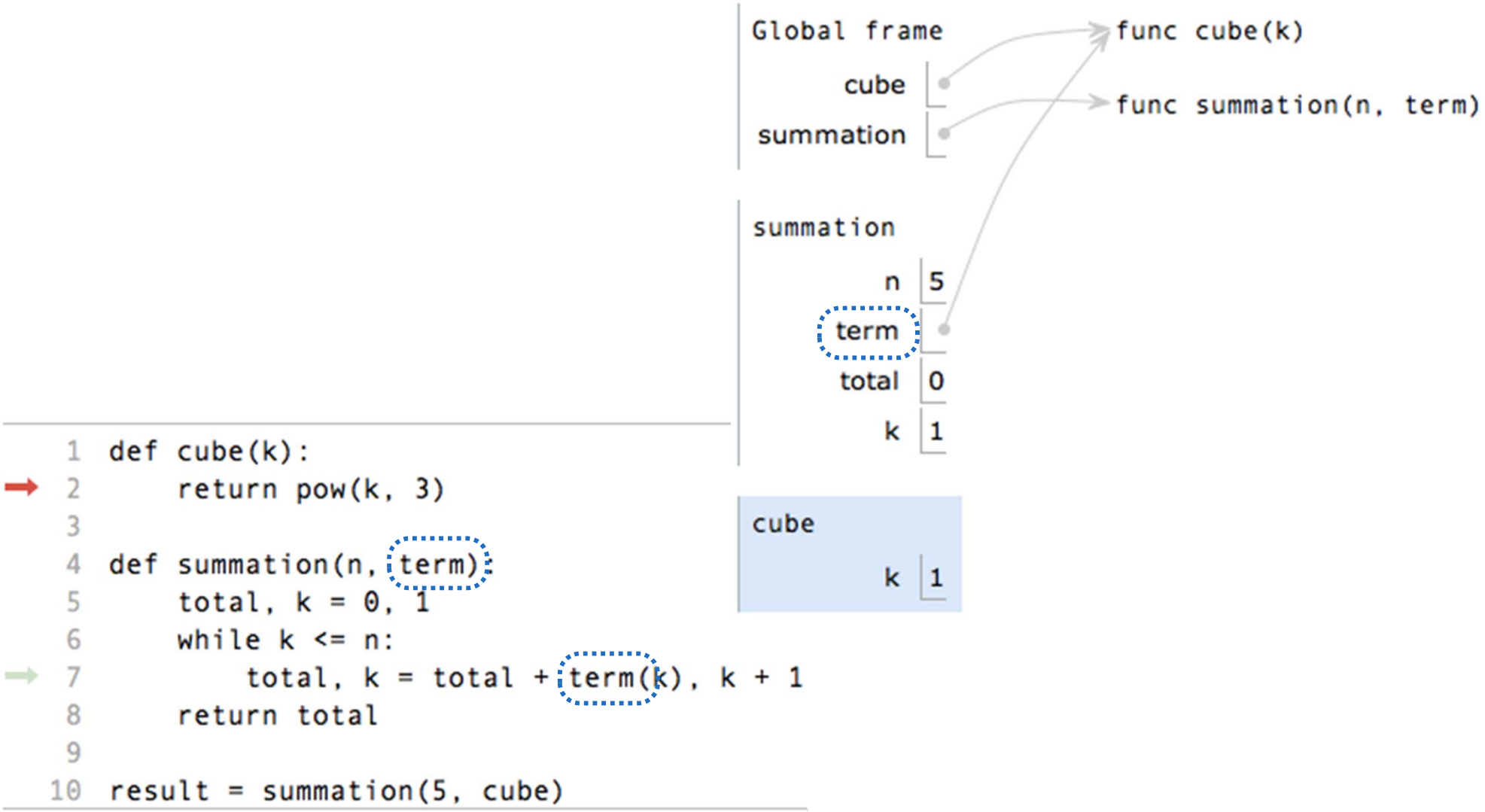

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Functions as Return Values

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def make_adder(n):
    """Return a function that adds n to its argument.

    >>> add_three = make_adder(3)
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A local def statement

Can refer to names in the enclosing function

Call Expressions as Operators

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make_adder(1)(2)
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Call Expressions as Operators

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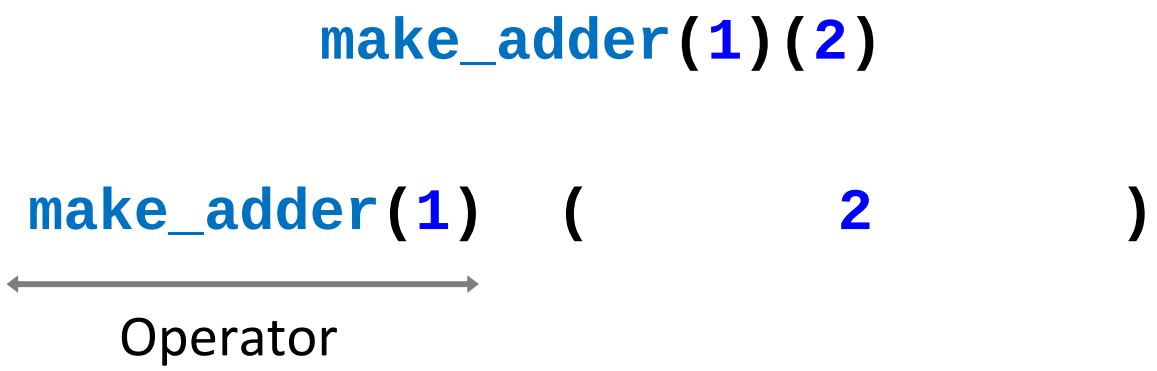


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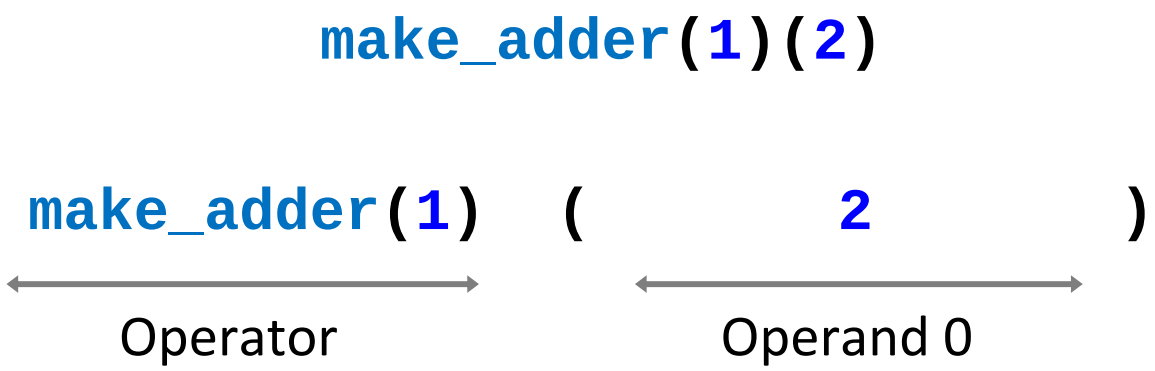


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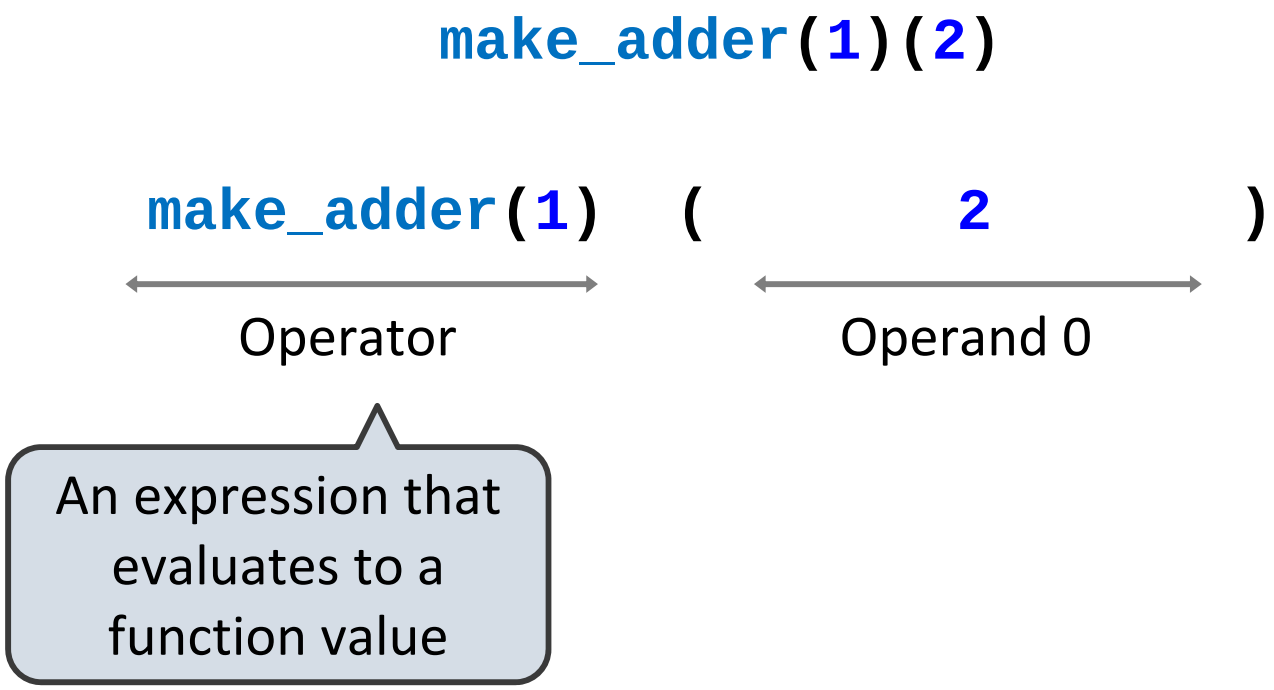


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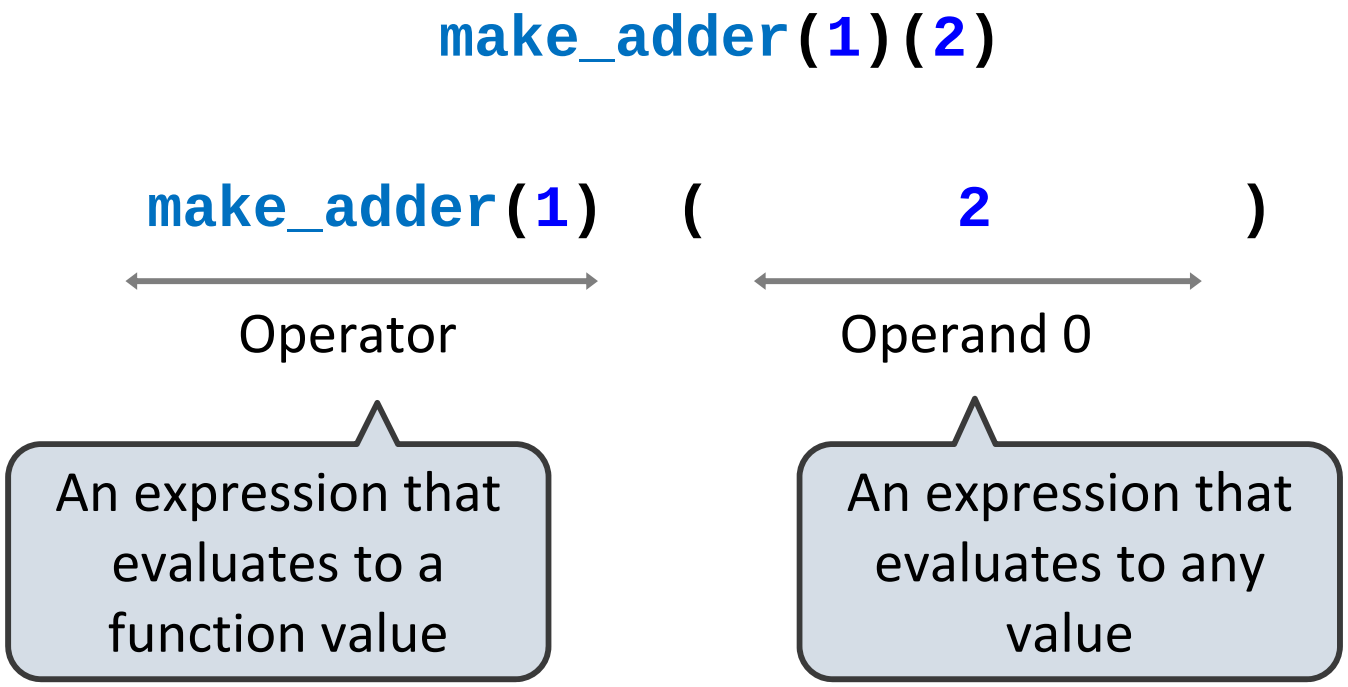


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Higher order functions:

- Express general methods of computation
- Remove repetition from programs
- Separate concerns among functions