

Section 16

1. [Normal Practice]

The average jump of a certain frog is 3 inches. However, because of the wind, the frog does not always go exactly 3 inches. A zoologist tells you that the distance the frog travels is normally distributed with mean 3 and variance $\frac{1}{4}$.

As in the homework, we will use the following fact: given a r.v. X distributed normally with mean μ and variance σ^2 , the random variable $Z = \frac{X-\mu}{\sigma}$ is normally distributed with mean 0 and variance 1. Observe that this means $\Pr[Z \leq z] = \Pr[X \leq \sigma z + \mu]$.

For the following problems, express your answer two ways. First, define the function $\Phi(z)$ as the CDF of a standard normal random variable Z (i.e. $\Phi(z) = \Pr[Z \leq z]$ where Z is distributed normally with mean 0 and variance 1), and write your answer in terms of $\Phi(z)$. Then use the table on the back to get a numerical answer.

- (a) What is the probability that the frog jumps more than 4 inches?
- (b) What is the probability that distance the frog jumps is between 2 and 4 inches?

2. [Coin flipping with the CLT]

Your friend challenges you to a coin flipping contest. However you know this friend to be of questionable moral character. In fact you know that he usually carries two coins: one is a weighted coin that comes up heads with probability 0.55, and the other is a fair coin. Before the contest, you demand that he flip one coin 10000 times, and if it comes up heads more than 5250 times, that he use the other coin for the contest. Suppose your friend flips the fair coin. What is the probability that you will force him to switch even though he is actually flipping the fair coin?

3. [Illustrating the CLT]

Consider the random variable $Z = X_1 + \dots + X_n$ where the variables X_i are independent and identically distributed according to a discrete uniform distribution over the values 1,2, and 3, i.e.

$$\Pr[X_i = x] = \begin{cases} \frac{1}{3} & x \in \{1, 2, 3\} \\ 0 & \text{otherwise.} \end{cases}$$

Compute and plot the distribution of Z for $n = 1$, $n = 2$, and $n = 3$.

4. [Using the CLT]

The Cal basketball team plays 100 independent games, each of which they have probability 0.8 of winning. Whats the probability that they win at least 90?

5. [Countability]

Consider degree-one polynomials, i.e. polynomials of the form $P(x) = a \times x + b$. Determine if the set of all degree-one polynomials is countable under the following conditions:

- (a) a and b must be integers.
- (b) a and b must be rational numbers.
- (c) a and b are real numbers.

Z-Score Table

Because Z-scores are nasty to compute, they are commonly presented in a table similar to the one below. To look up a particular Z-score (i.e. to look up the probability that a standard normal distribution is at most x), you look up the row of the first two digits and then the column of the third. For example, to look up 1.34, first find the row labeled 1.3 and then the column 0.04. The result is the entry in that cell, i.e. 0.9099.

Data Source: Wikipedia

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990