Section 11

1. **Monkey writing Shakespeare**
   A monkey types on a 26-letter keyboard, with all lowercase letters. Assume that the monkey chooses each character independently and uniformly at random.

   (a) The monkey types a million six-letter words at random. What is the expected number of times the word "hamlet" is typed?
   Let H_i be the event that the ith word typed is "hamlet." Are H_i and H_j independent for $i \neq j$?

   (b) Now the monkey types a million characters at random. What is the expected number of times the sequence "hamlet" appears?
   Letting H_i denote the event that the six-letter sequence that starts at the ith character is "hamlet," are H_i and H_j independent for $i \neq j$?

   (c) Finally the monkey types a six-letter word at random. The monkey copies this word a million times to make a million-word text (meaning spaces between words are retained). What is the expected number of times the word "hamlet" appears in the text?
   Letting H_i be the event that the ith word is "hamlet," are H_i and H_j independent for $i \neq j$?

   (d) Let random variable X be the number of times "hamlet" appears. Think about what the distribution of X looks like in each of the three cases (a)-(c) above. Are any of the three distributions the same?

2. A drawer contains 10 socks, where 6 of them have holes and 4 of them do not. Suppose you pull two random socks out of the drawer, look at them, and then put them back. If you do this 5 times, what is the probability that you pull out a pair with no holes precisely 4 out of 5 times?

3. Suppose you are at a casino and betting on a game in which you have probability p of winning in each round. Your strategy is to bet k in the first round and then in each subsequent round, double the amount of money you bet in the previous round. So you would bet k, $2k$, $4k$, $8k$, and so on. You stop as soon as you earn a profit.

   (a) Suppose you have unlimited money. What is the expected amount of money you will earn?

   (b) What is the expected number of rounds you will play?

   (c) What is the expected amout of money lost before your first win?

4. You just rented a large house and the realtor gave you five keys, one for the front door and the other four for each of the four side and back doors of the house. Unfortunately, all keys look identical, so to open the front door, you are forced to try them at random.

   Find the distribution and the expectation of the number of trials you will need to open the front door, under the following alternative assumptions:

   (a) after an unsuccessful trial, you mark the corresponding key so that you never try it again, or

   (b) at each trial, you are equally likely to choose any key.

5. Suppose a box has 6 brown balls and 4 purple balls. A random sample of size n is selected with replacement. Let X= "number of brown balls selected." Write the distribution of X. Can this be closely approximated with a Poisson distribution? If so, write the approximate distribution. If not, explain why not. If so, but only under certain conditions, explain these conditions and write the approximate distribution.

6. Now suppose the box has 6 brown balls and 4,000 purple balls. As in the previous exercise a random sample of size n is selected with replacement and X= "number of brown balls selected." Write the distribution of X. Can this be closely approximated with a Poisson distribution? If so, write the approximate distribution. If not, explain why not. If so, but only under certain conditions, explain these conditions and write the approximate distribution.