Section 4

1. In a group of n men and n women, Bob, one of the men, gets tipped off that he is the second-highest preference on every womans list. Bob is pretty happy to hear this. Assuming we use the traditional (male-optimal) algorithm, we can guarantee that at worst he will be matched the kth highest woman on his list for some $k \leq n$. What is k? Give a bad example where Bob is matched to the kth woman on his list.

2. (a) What is the inverse of 5 modulo 7?

(b) Do the following numbers have inverses modulo 3580225?

$$5, \ 16, \ 29$$

Give a short explanation for each.

3. (a) Solve the following system of equations:

$$5x \equiv 8y \pmod{13}$$

$$x \equiv 9y - 11 \pmod{13}$$

(b) Does the following equation have a solution?

$$18x \equiv 19 \pmod{29}$$

Prove your answer.

4. (a) Find all values x such that $x^2 \equiv 4 \pmod 5$.

(b) Compute

$$mod\left(2010^{2009^{2008}}, 2009\right)$$

(c) Show that if $n \equiv 3 \pmod 4$ then n is not the sum of 2 perfect squares.

5. (a) Compute $3^{19} \mod 23$.

(b) Find the last digit of $9^{2938} - 5^{8460}$.

(c) Show that if a is an odd natural number, then $a^2 \equiv 1 \pmod 8$.

6. (a) Let $F(n)$ denote the nth Fibonacci number. Show that for all $n \geq 1$, $\gcd(F(n+1), F(n)) = 1$. (Recall that the Fibonacci numbers are generated by the recursive relation $F(1) = 1$, $F(2) = 1$, and $F(n) = F(n-1) + F(n-2)$ for $n \geq 3$.)

(b) Prove that an integer is divisible by 3 if and only if the sum of its digits in base 10 is divisible by 3.