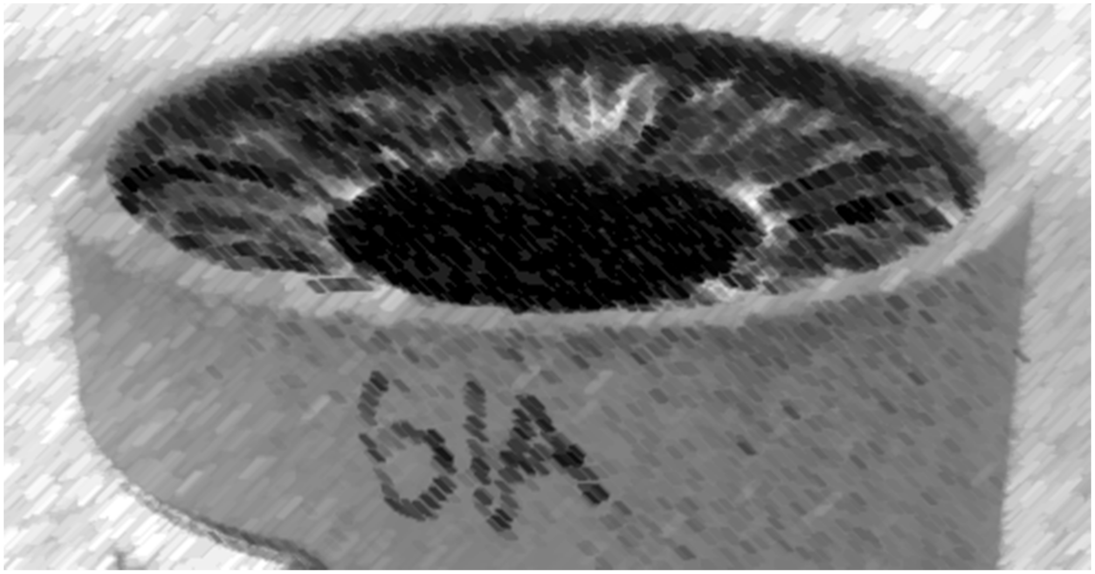

CS61A Lecture 31

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UC Berkeley

April 3, 2013

Announcements

- HW9 due tonight
- Ants extra credit due tonight
    - See Piazza for submission instructions
- Hog revisions out, due Monday
- HW10 out tonight

Pairs

Scheme has built-in pairs that use weird names:

- **`cons`**: Two-argument procedure that **creates a pair**
- **`car`**: Procedure that returns the **first element** of a pair
- **`cdr`**: Procedure that returns the **second element** of a pair

A pair is represented by a dot between the elements, enclosed in parentheses

```
> (cons 1 2)
(1 . 2)
> (car (cons 1 2))
1
> (cdr (cons 1 2))
2
```

Recursive Lists

A recursive list can be represented as a pair in which the second element is a recursive list or the empty list

Scheme lists are recursive lists:

- **`nil`** is the empty list
- A non-empty Scheme list is a pair in which the second element is **`nil`** or a Scheme list

Scheme lists are written as space-separated combinations

```
> (define x (cons 1 (cons 2 (cons 3 (cons 4 nil)))))
> x
(1 2 3 4)
> (cdr x)
(2 3 4)
> (cons 1 (cons 2 (cons 3 4)))
(1 2 3 . 4)
```

Not a well-formed list!

Symbolic Programming

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Symbols are normally evaluated to produce values; how do we refer to symbols?

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> (define a 1)
> (define b 2)
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> (define a 1)
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> (list a b)
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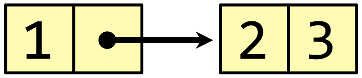

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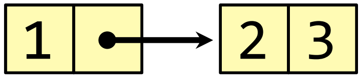

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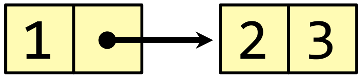

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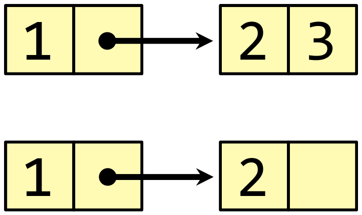

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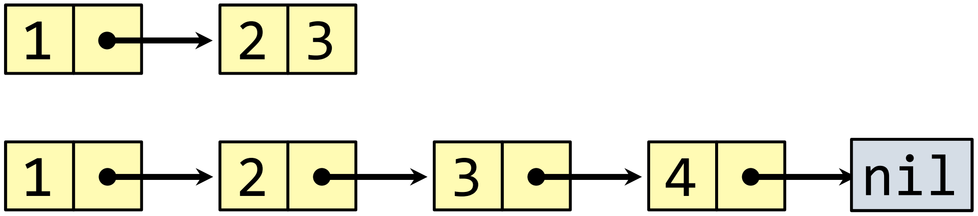

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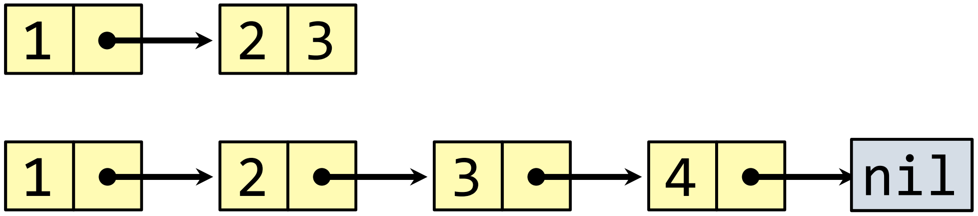

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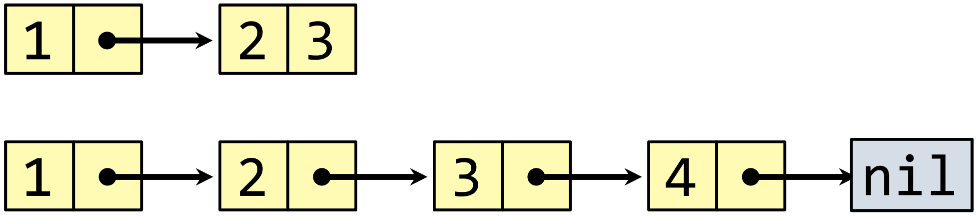

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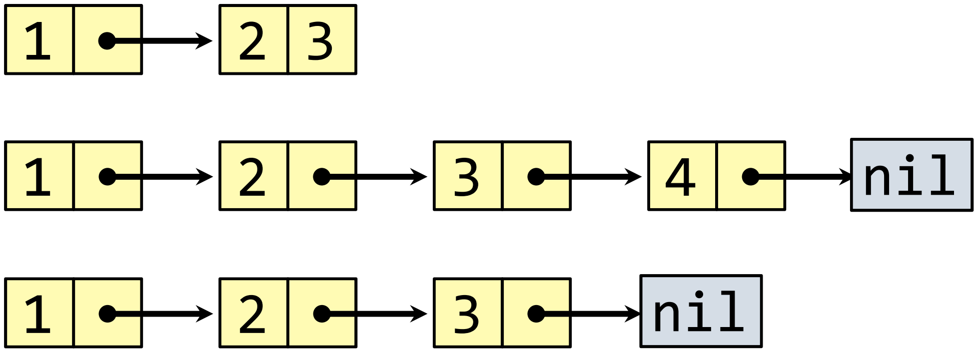

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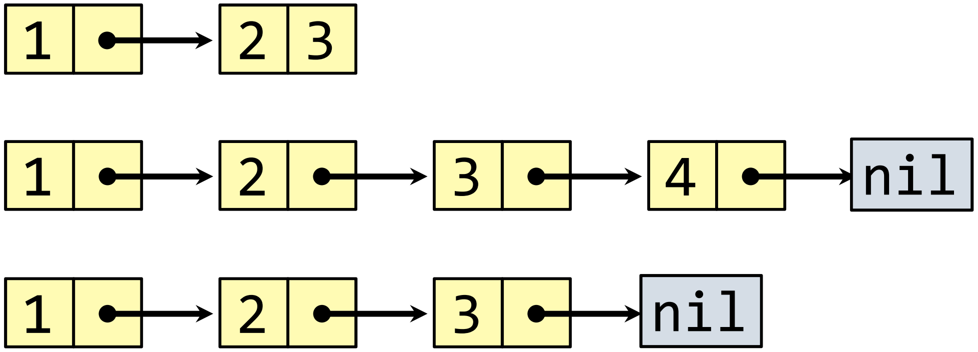

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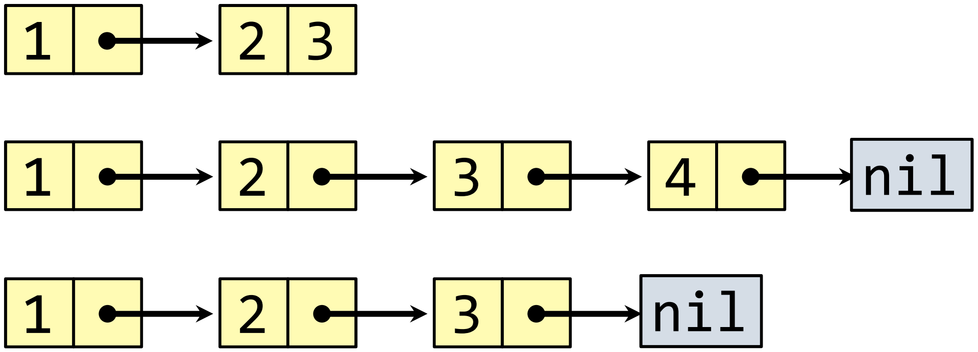

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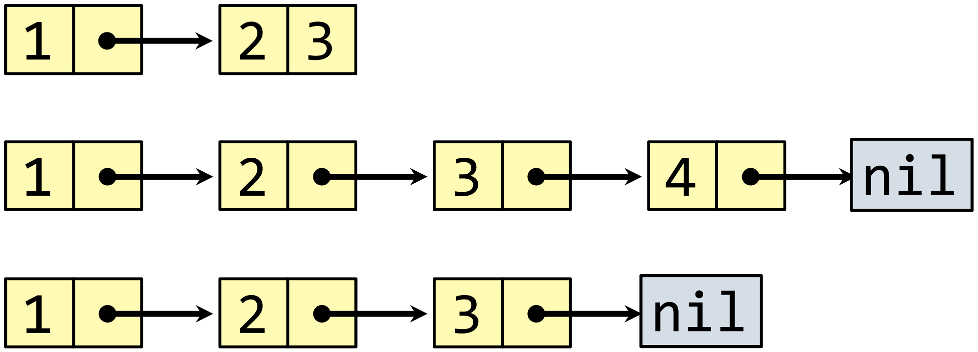

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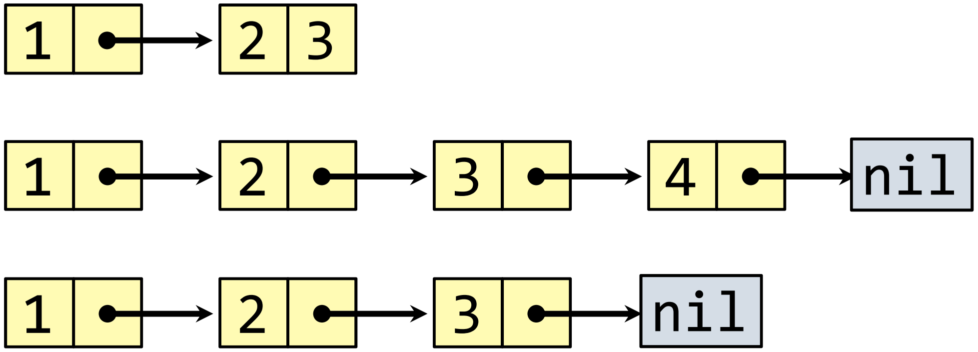

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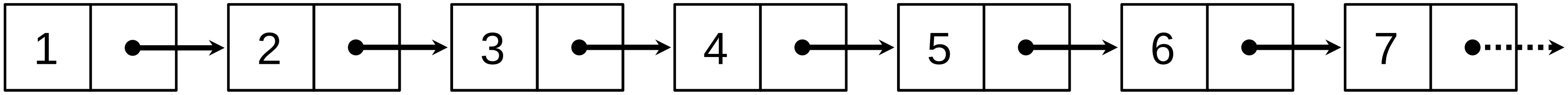

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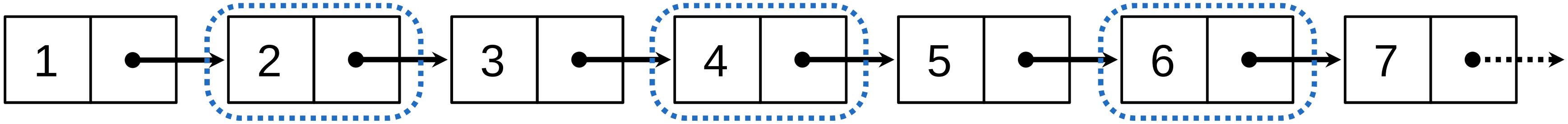

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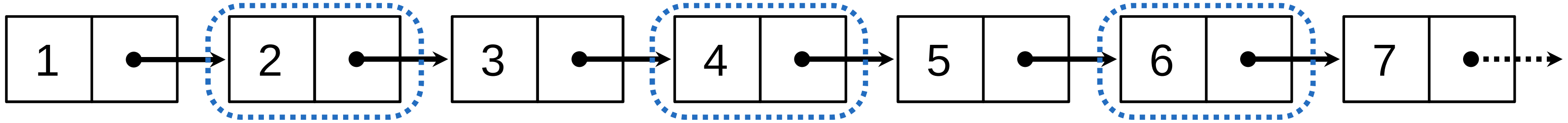

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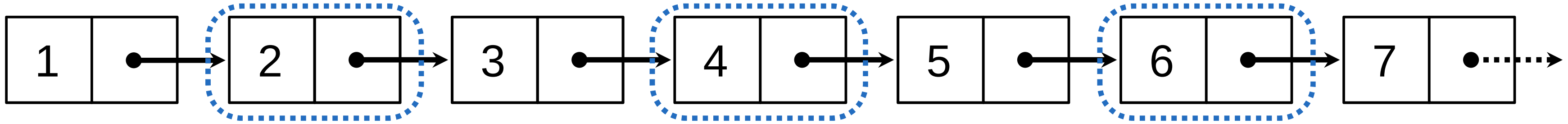

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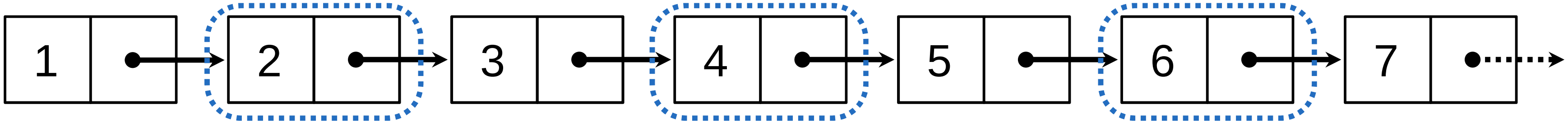

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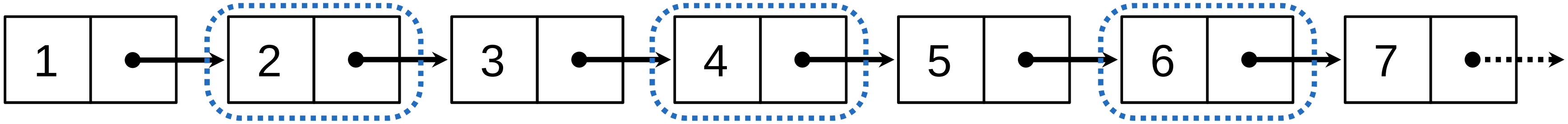

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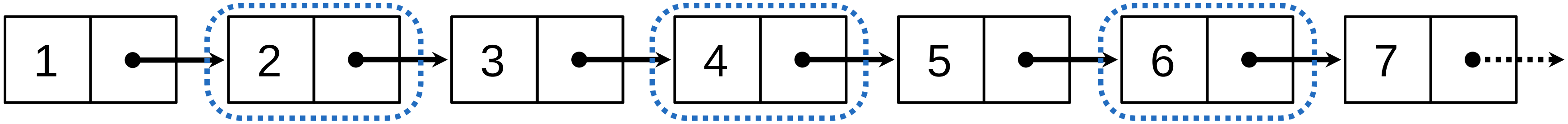

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(define (filter fn s)
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      (let ((first (car s))
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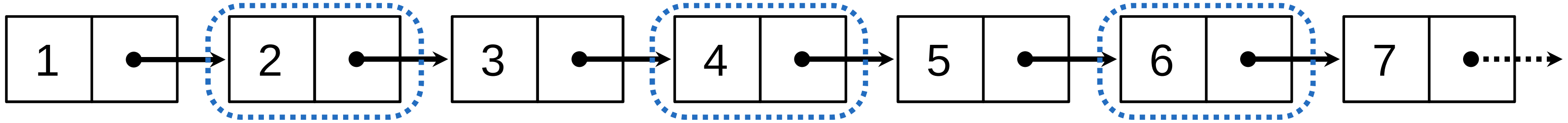

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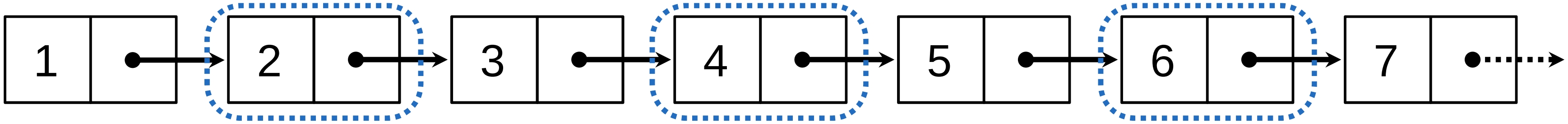

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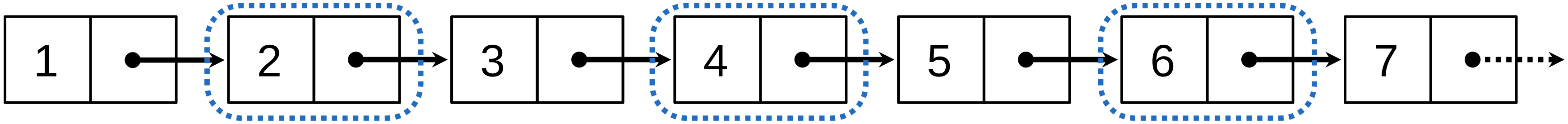

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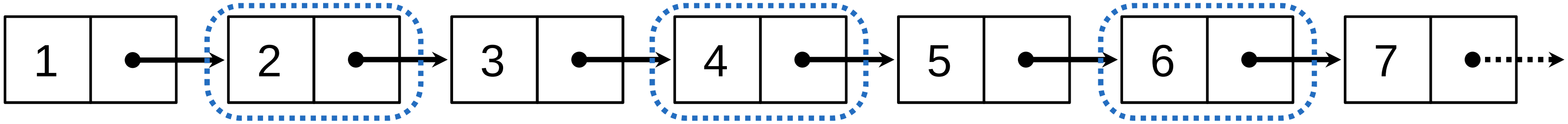

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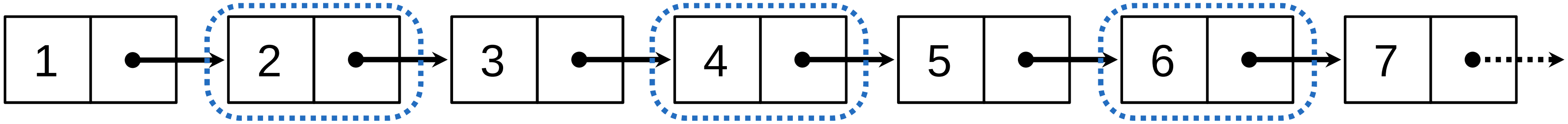

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9	3	1	6	5	8	7

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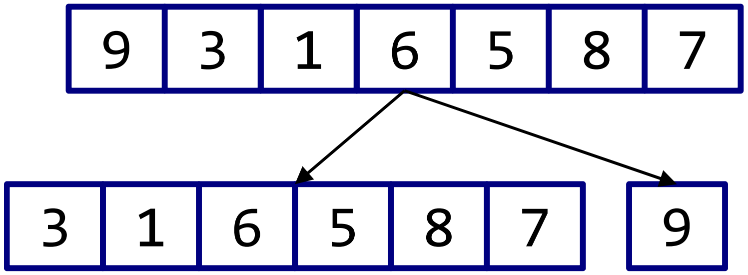

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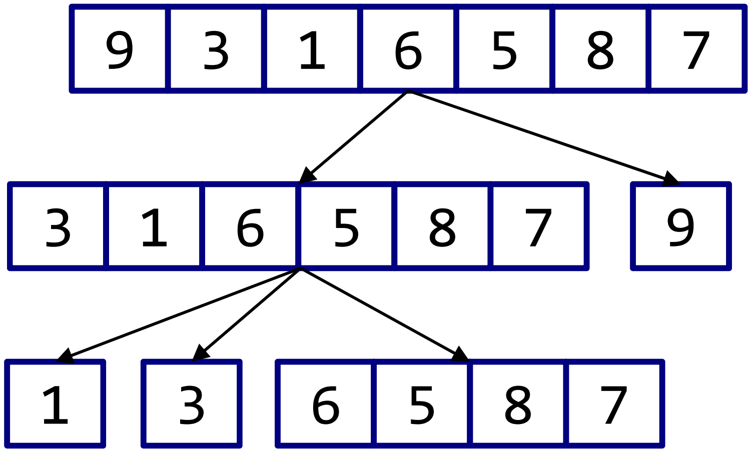

Quick Sort

Quick sort algorithm:

1. Choose a pivot (e.g. first element)
2. Partition into three pieces:
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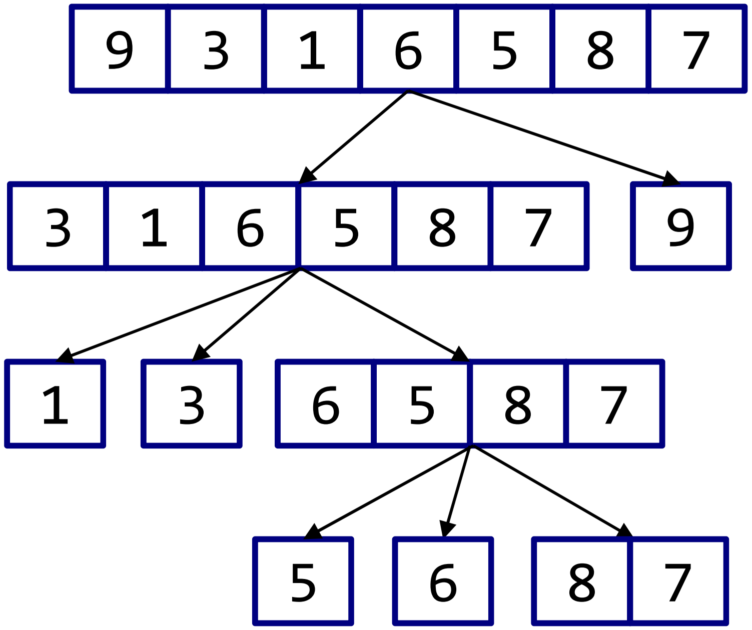

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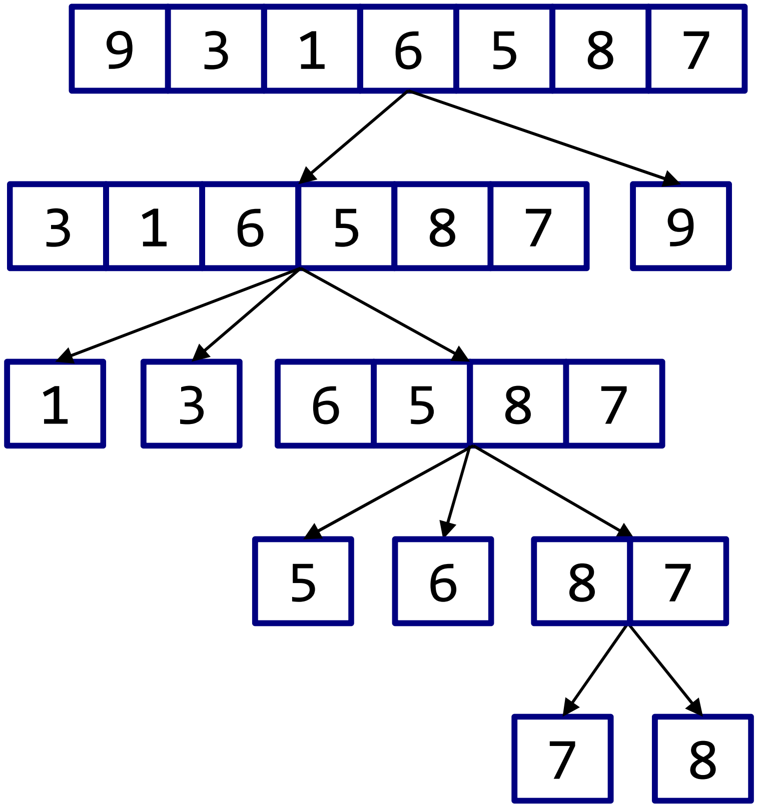

Quick Sort

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1. Choose a pivot (e.g. first element)
2. Partition into three pieces:
   < pivot, = pivot, > pivot
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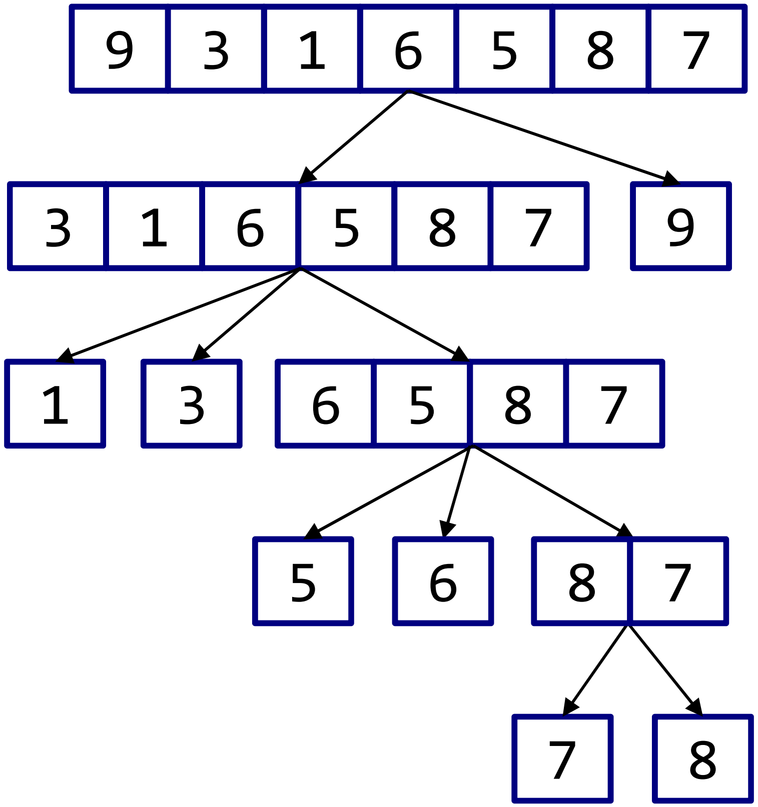
```
(define (filter-comp comp pivot s)
```



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2. Partition into three pieces:
   < pivot, = pivot, > pivot
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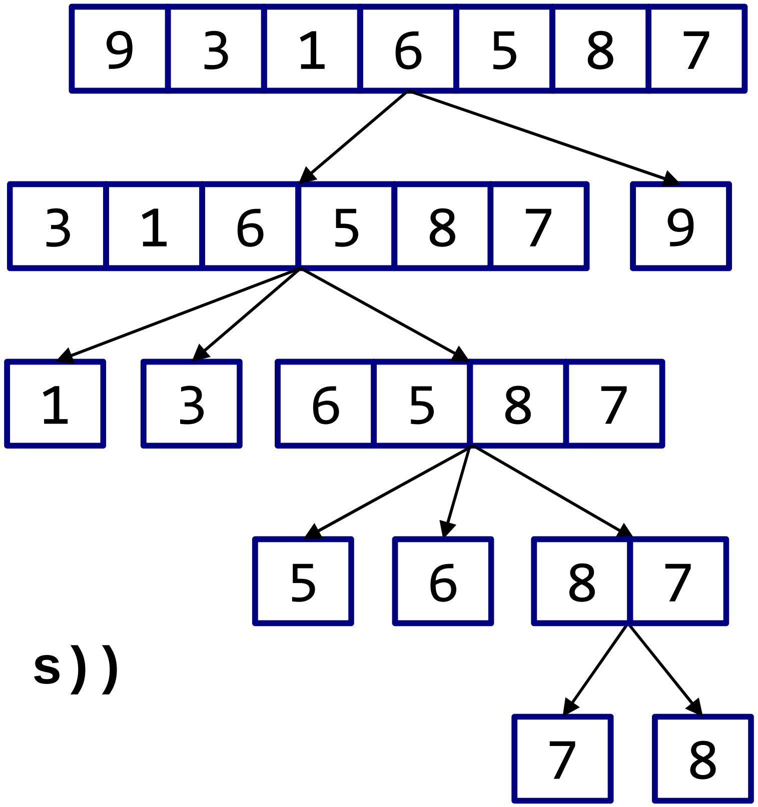
```
(define (filter-comp comp pivot s)
  (filter (lambda (x) (comp x pivot)) s))
```



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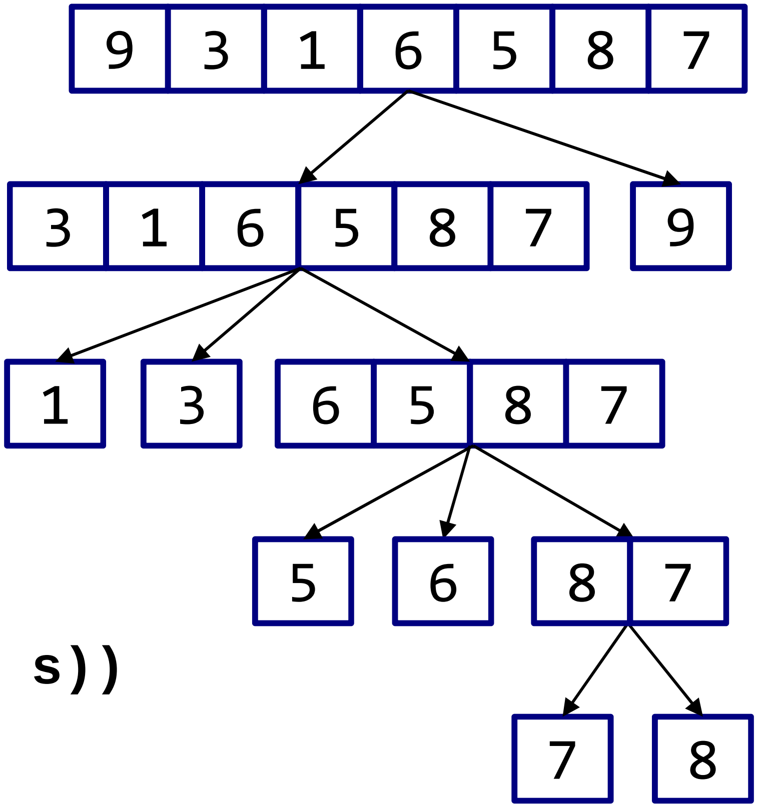

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  (filter (lambda (x) (comp x pivot)) s))

(define (quick-sort s)
```

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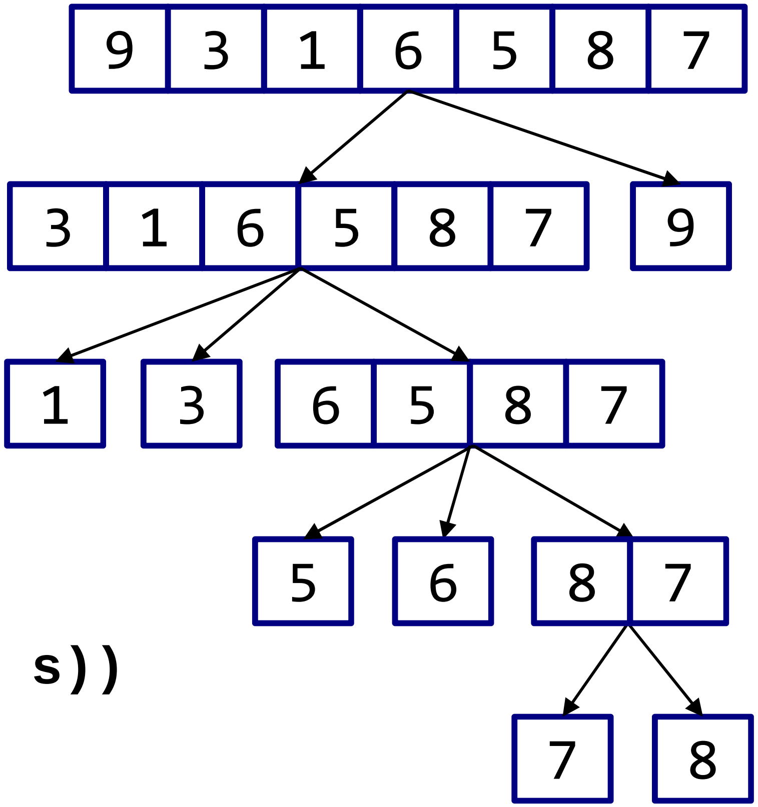

```
(define (filter-comp comp pivot s)
  (filter (lambda (x) (comp x pivot)) s))

(define (quick-sort s)
  (if (<= (length s) 1)
```

Quick Sort

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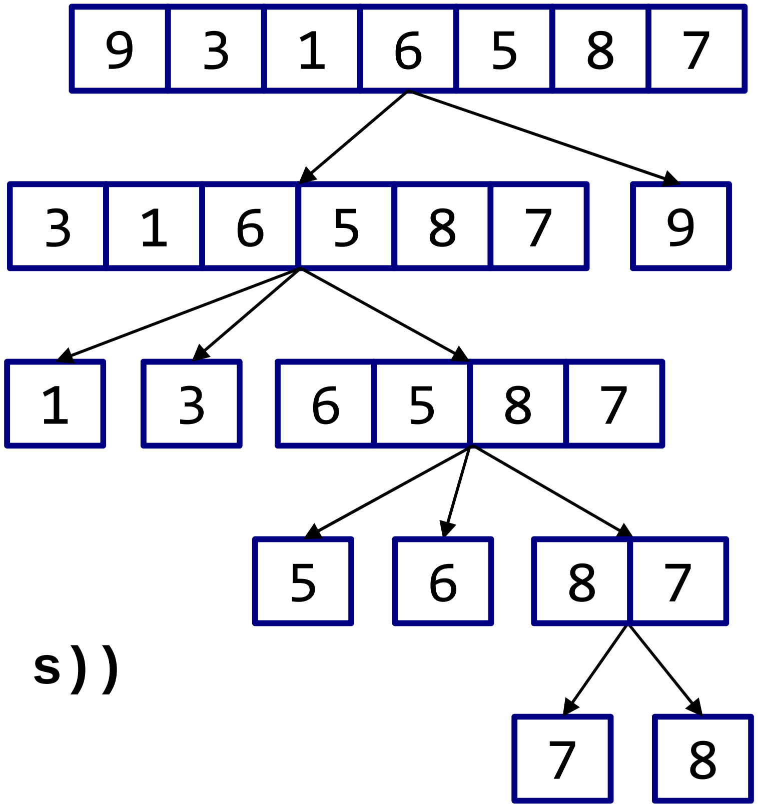

```
(define (filter-comp comp pivot s)
  (filter (lambda (x) (comp x pivot)) s))

(define (quick-sort s)
  (if (<= (length s) 1)
      s
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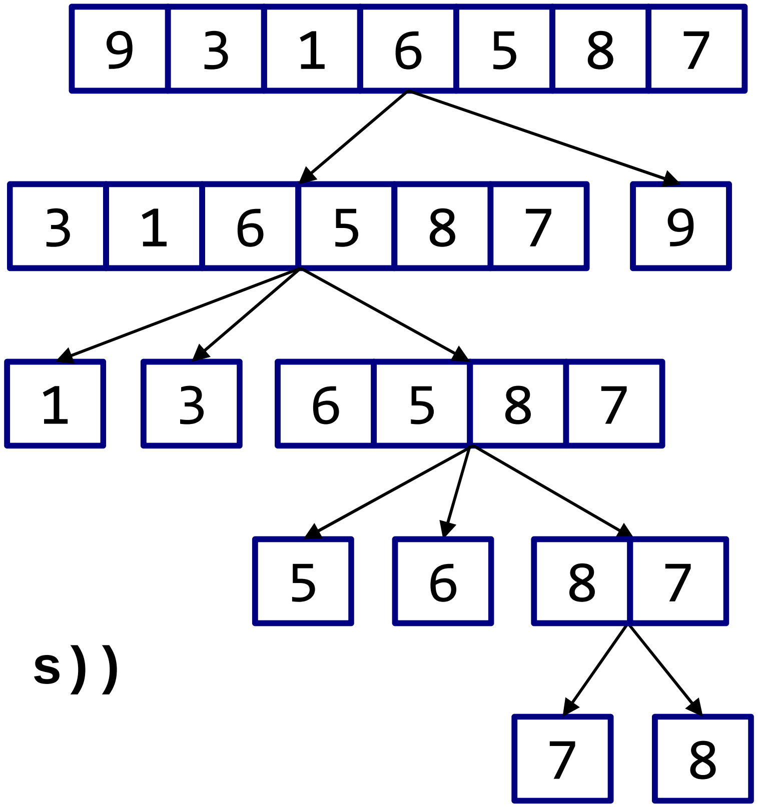

```
(define (filter-comp comp pivot s)
  (filter (lambda (x) (comp x pivot)) s))

(define (quick-sort s)
  (if (<= (length s) 1)
      s
      (let ((pivot (car s)))
```

Quick Sort

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1. Choose a pivot (e.g. first element)
2. Partition into three pieces:
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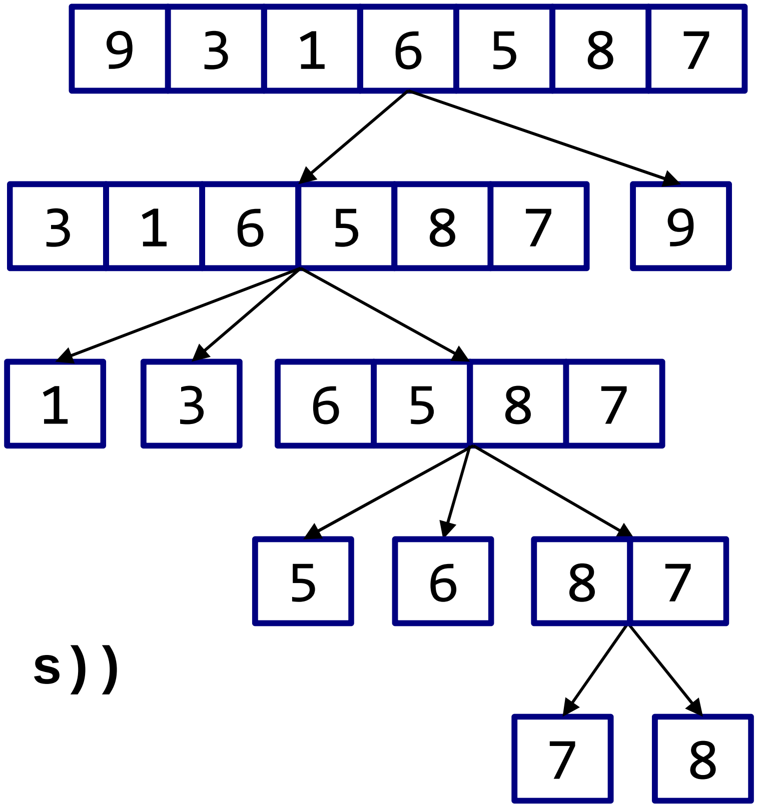

```
(define (filter-comp comp pivot s)
  (filter (lambda (x) (comp x pivot)) s))

(define (quick-sort s)
  (if (<= (length s) 1)
      s
      (let ((pivot (car s)))
        (append (quick-sort (filter-comp < pivot s))
```

Quick Sort

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   < pivot, = pivot, > pivot
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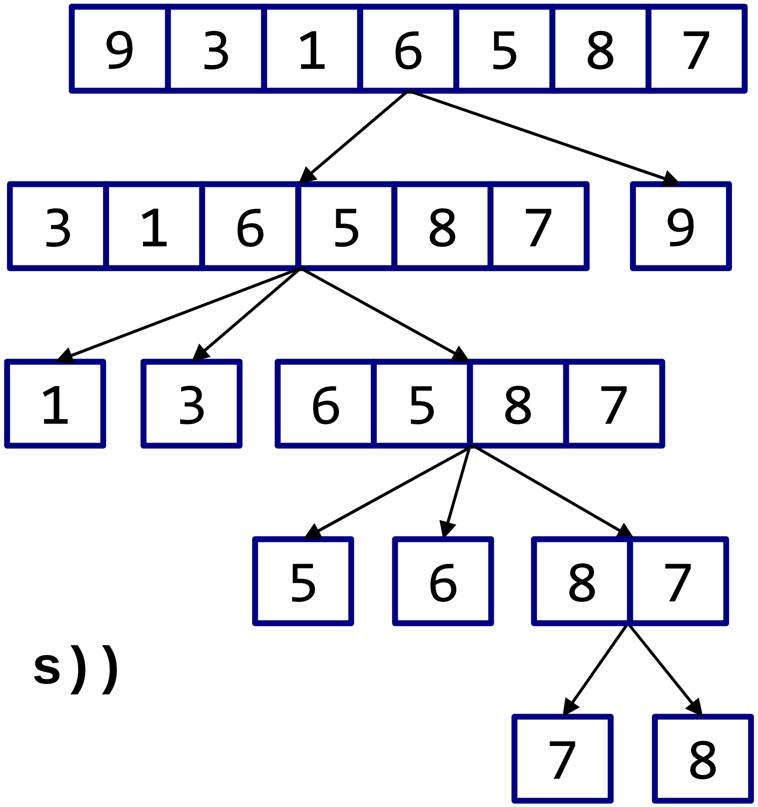

```
(define (filter-comp comp pivot s)
  (filter (lambda (x) (comp x pivot)) s))

(define (quick-sort s)
  (if (<= (length s) 1)
      s
      (let ((pivot (car s)))
        (append (quick-sort (filter-comp < pivot s))
                (filter-comp = pivot s)
```

Quick Sort

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   < pivot, = pivot, > pivot
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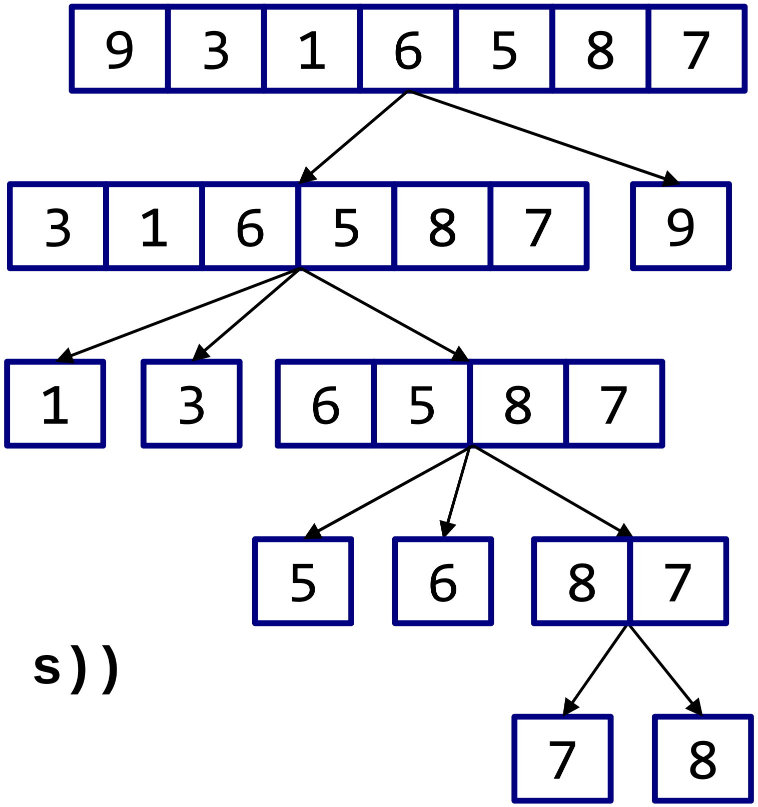

```
(define (filter-comp comp pivot s)
  (filter (lambda (x) (comp x pivot)) s))

(define (quick-sort s)
  (if (<= (length s) 1)
      s
      (let ((pivot (car s)))
        (append (quick-sort (filter-comp < pivot s))
                (filter-comp = pivot s)
                (quick-sort (filter-comp > pivot s))))))
```

The Begin Special Form

Begin expressions allow sequencing

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$$(\texttt{begin <exp}_1\texttt{> <exp}_2\texttt{> ... <exp}_n\texttt{>})$$

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(**begin** $<exp_1>$ $<exp_2>$ ... $<exp_n>$)

```
(define (repeat k fn)
```

The Begin Special Form

Begin expressions allow sequencing

$$\texttt{(begin <exp}_1\texttt{> <exp}_2\texttt{> ... <exp}_n\texttt{>)}$$

```
(define (repeat k fn)
  (if (> k 0)
```

The Begin Special Form

Begin expressions allow sequencing

```
(begin <exp₁> <exp₂> ... <expₙ>)
```

```
(define (repeat k fn)
  (if (> k 0)
      (begin (fn) (repeat (- k 1) fn))
```

The Begin Special Form

Begin expressions allow sequencing

$$\texttt{(begin <exp}_1\texttt{> <exp}_2\texttt{> ... <exp}_n\texttt{>)}$$

```
(define (repeat k fn)
  (if (> k 0)
      (begin (fn) (repeat (- k 1) fn))
      'done))
```

The Begin Special Form

Begin expressions allow sequencing

$$\texttt{(begin <exp}_1\texttt{> <exp}_2\texttt{> ... <exp}_n\texttt{>)}$$

```
(define (repeat k fn)
  (if (> k 0)
      (begin (fn) (repeat (- k 1) fn))
      'done))

(define (tri fn)
```

The Begin Special Form

Begin expressions allow sequencing

(begin <exp$_1$> <exp$_2$> ... <exp$_n$>)

```
(define (repeat k fn)
  (if (> k 0)
      (begin (fn) (repeat (- k 1) fn))
      'done))

(define (tri fn)
  (repeat 3 (lambda () (fn) (lt 120))))
```

The Begin Special Form

Begin expressions allow sequencing

$$\texttt{(begin <exp}_1\texttt{> <exp}_2\texttt{> ... <exp}_n\texttt{>)}$$

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(define (repeat k fn)
  (if (> k 0)
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(define (tri fn)
  (repeat 3 (lambda () (fn) (lt 120))))

(define (sier d k)
```

The Begin Special Form

Begin expressions allow sequencing

```
(begin <exp₁> <exp₂> ... <expₙ>)
```

```
(define (repeat k fn)
  (if (> k 0)
      (begin (fn) (repeat (- k 1) fn))
      'done))

(define (tri fn)
  (repeat 3 (lambda () (fn) (lt 120))))

(define (sier d k)
  (tri (lambda () (if (= k 1) (fd d) (leg d k)))))
```

The Begin Special Form

Begin expressions allow sequencing

```
(begin <exp1> <exp2> . . . <expn>)
```

```
(define (repeat k fn)
  (if (> k 0)
      (begin (fn) (repeat (- k 1) fn))
      'done))

(define (tri fn)
  (repeat 3 (lambda () (fn) (lt 120))))

(define (sier d k)
  (tri (lambda () (if (= k 1) (fd d) (leg d k)))))

(define (leg d k)
```

The Begin Special Form

Begin expressions allow sequencing

$$\texttt{(begin <exp}_1\texttt{> <exp}_2\texttt{> ... <exp}_n\texttt{>)}$$

```
(define (repeat k fn)
  (if (> k 0)
      (begin (fn) (repeat (- k 1) fn))
      'done))

(define (tri fn)
  (repeat 3 (lambda () (fn) (lt 120))))

(define (sier d k)
  (tri (lambda () (if (= k 1) (fd d) (leg d k)))))

(define (leg d k)
  (sier (/ d 2) (- k 1)) (penup) (fd d) (pendown))
```