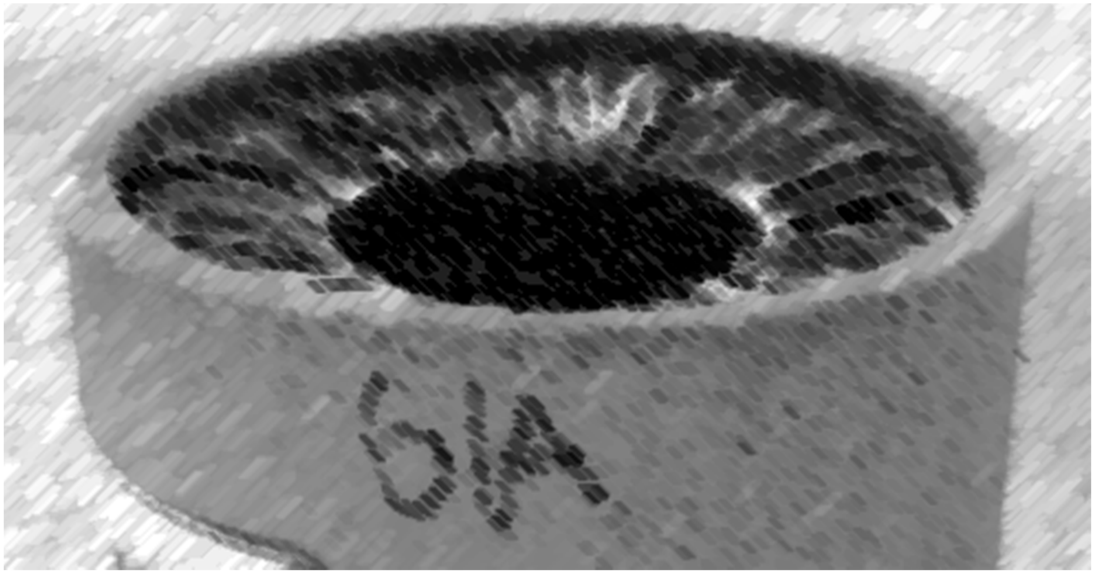

CS61A Lecture 7

Amir Kamil
UC Berkeley
February 6, 2013

Announcements

- HW3 out, due Tuesday at 7pm
- Midterm next Wednesday at 7pm
  - Keep an eye out for your assigned location
  - Old exams posted soon
  - Review sessions
    - Saturday 2-4pm in TBA
    - Extend office hours Sunday 11-3pm in TBA
    - HKN review session Sunday 3-6pm in 145 Dwinelle
- Environment diagram handout on website
- Code review system online
  - See Piazza post for details

How to Draw an Environment Diagram

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3. Bind the <formal parameters> to the arguments in this frame
4. Execute the body of the function in the environment that starts with this frame

Environment for Function Composition

Example: http://goo.gl/5zcug

Environment for Function Composition

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def square(x):
    return x * x

def make_adder(n):
    def adder(k):
        return n + k
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def compose1(f, g):
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compose1(square, make_adder(2))(3)
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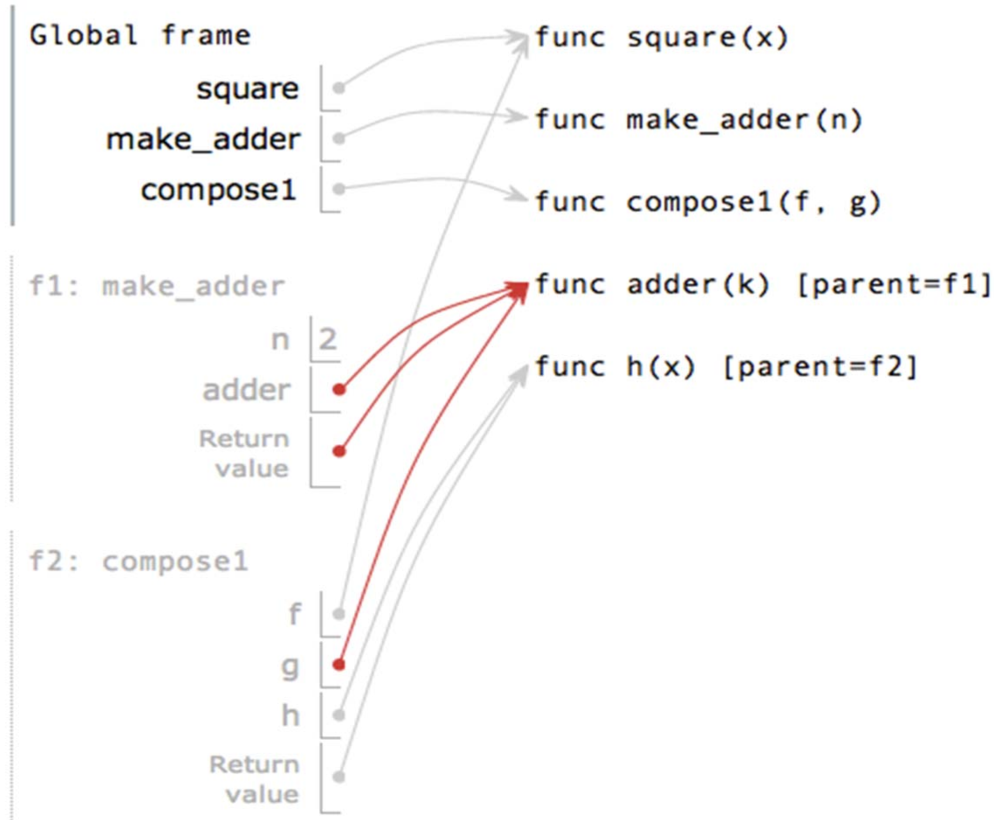

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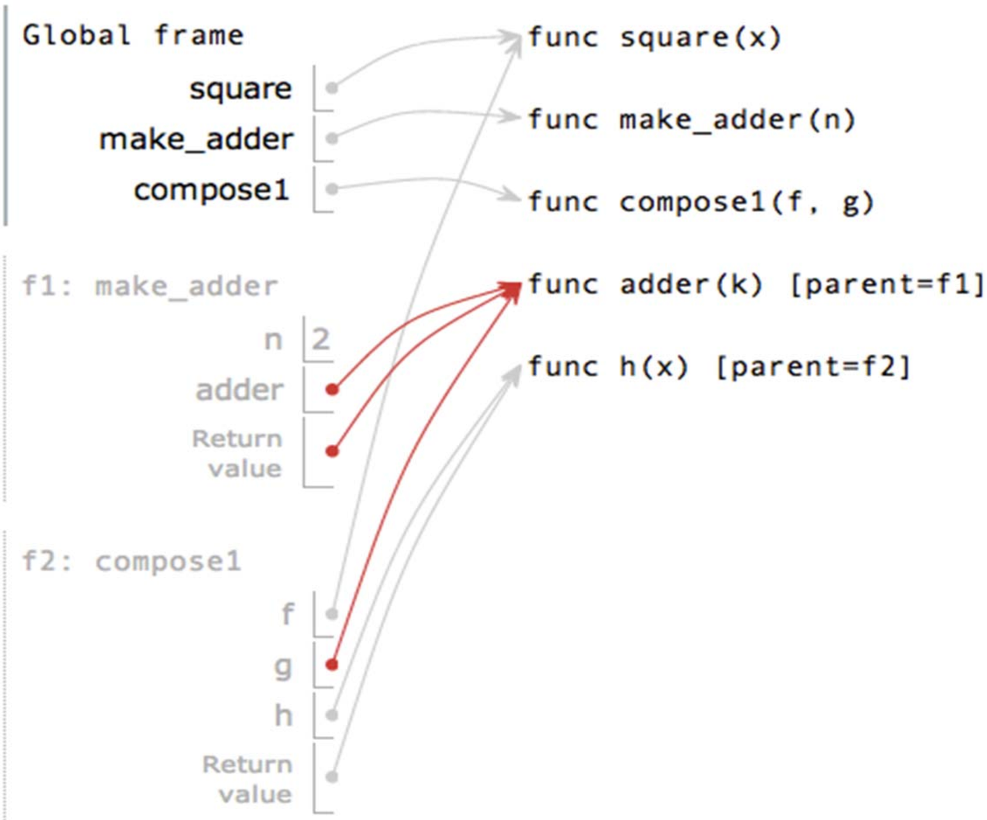

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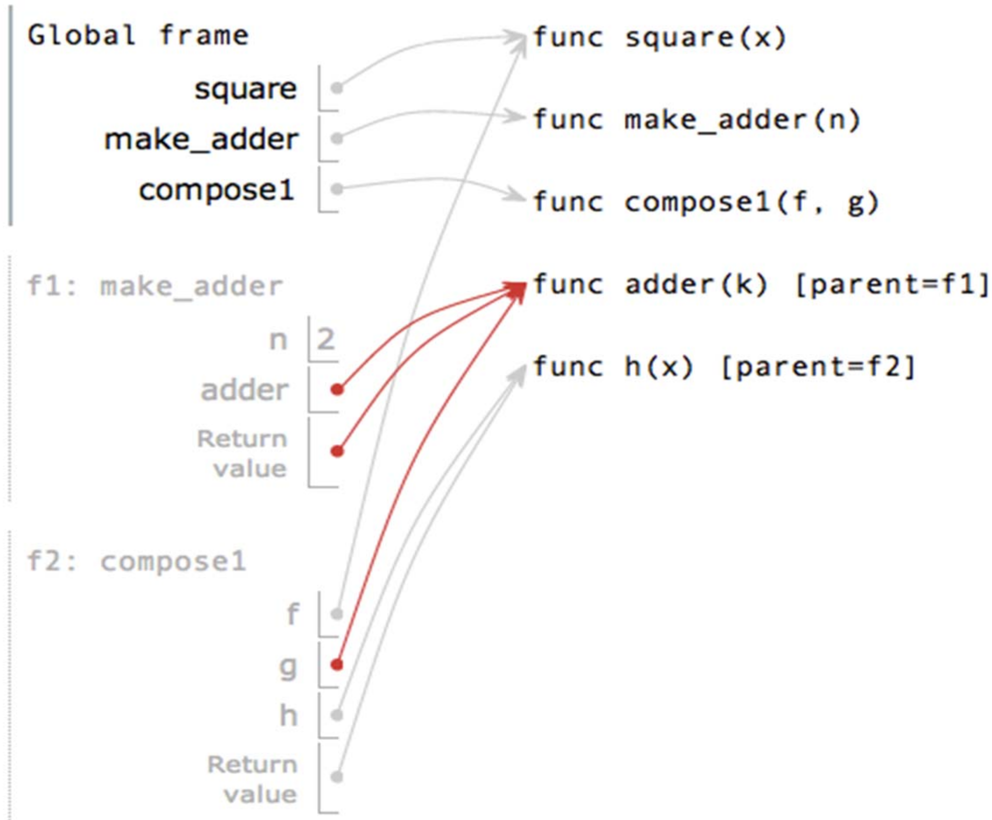

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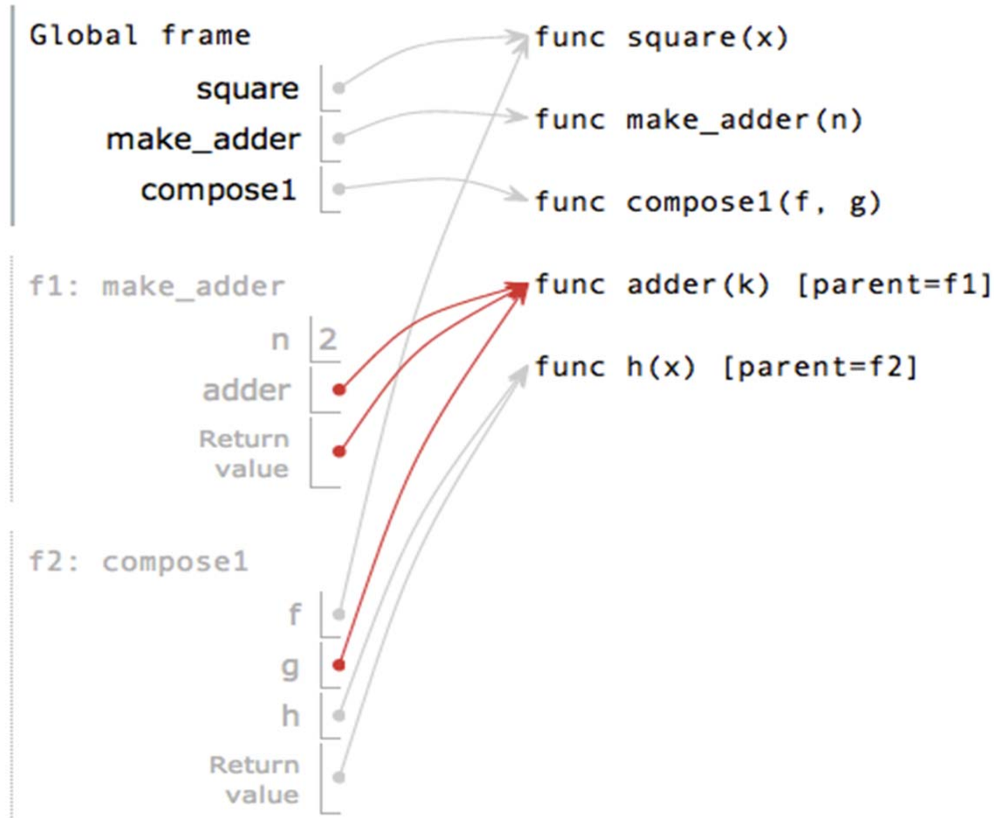

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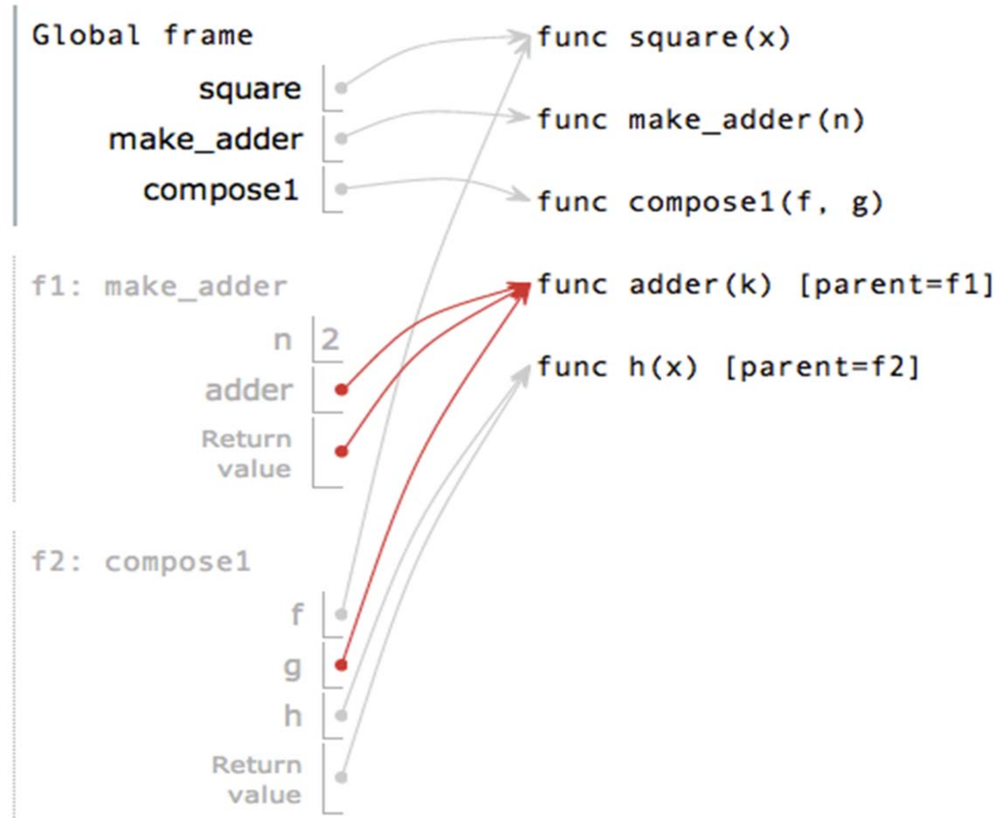

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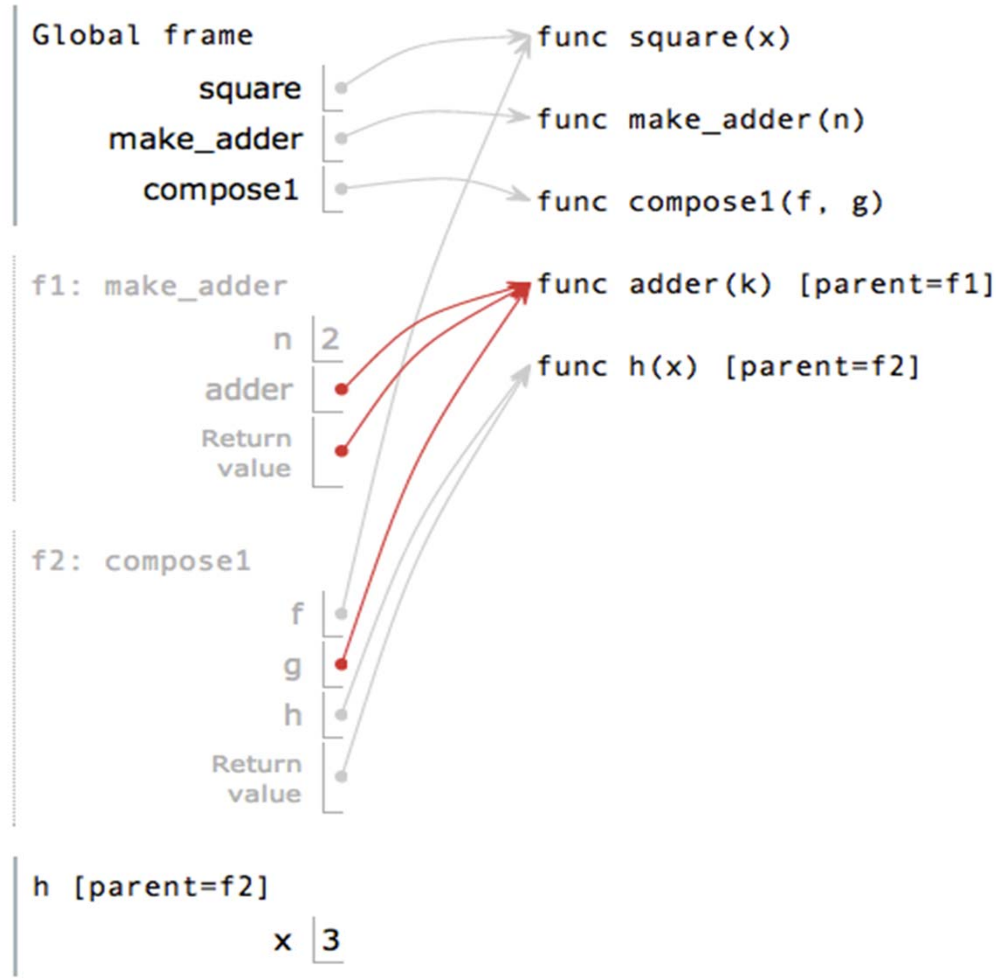

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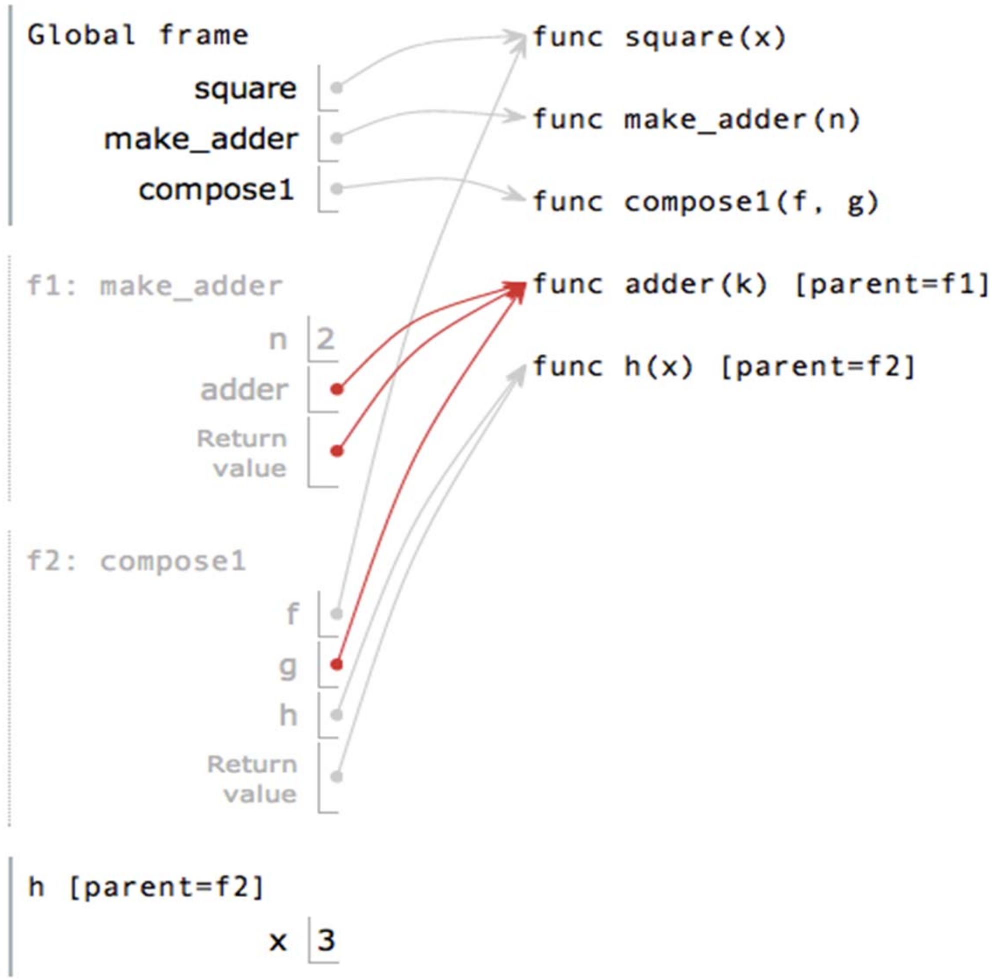

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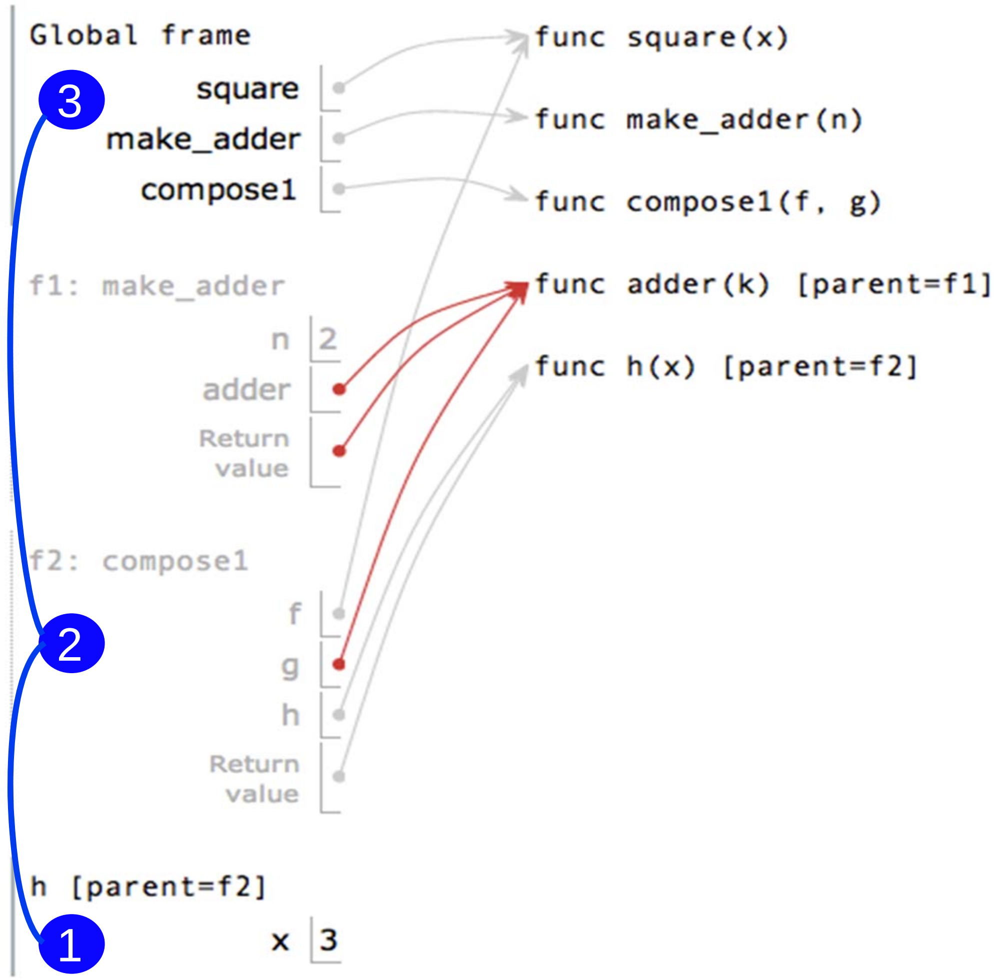

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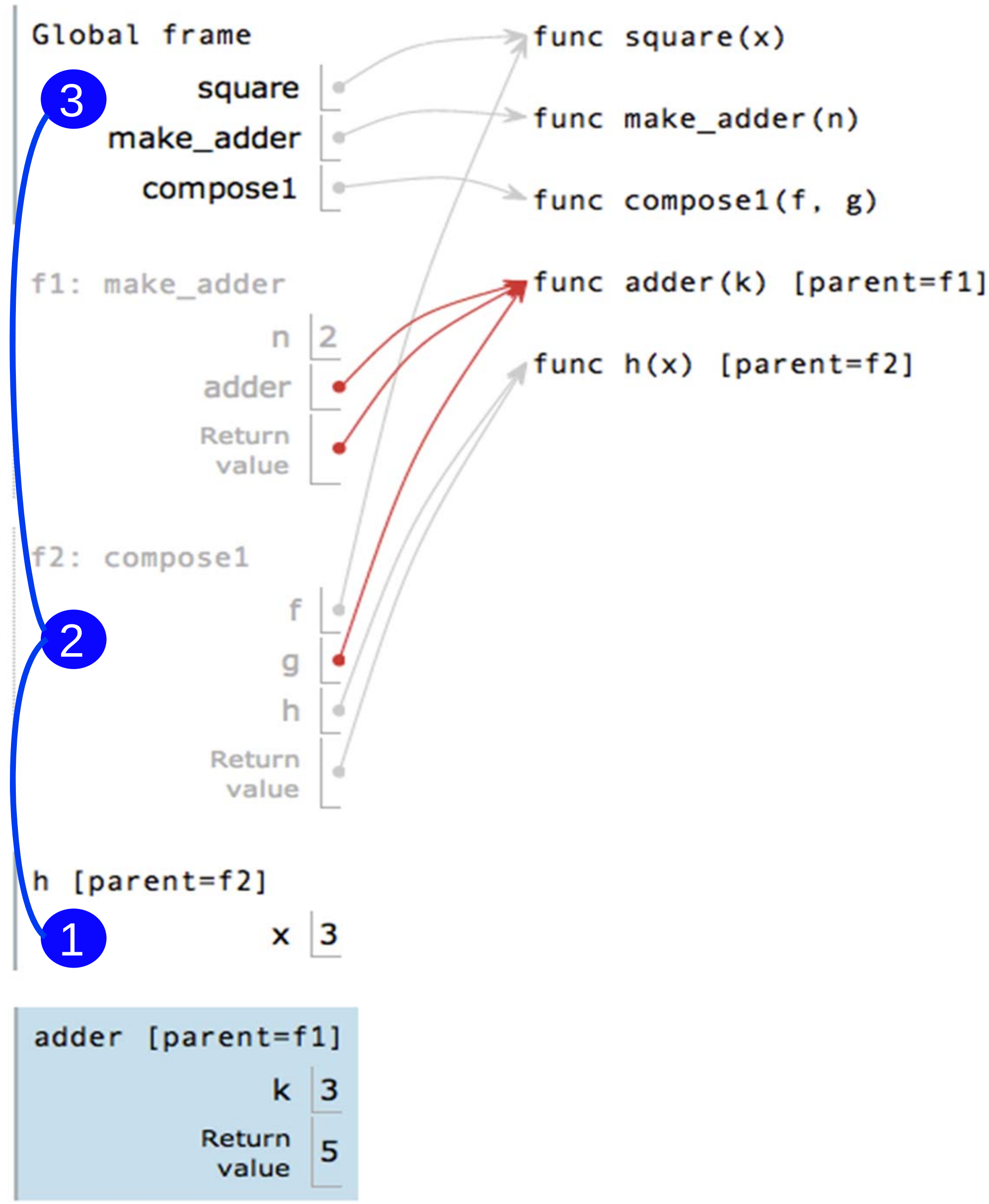

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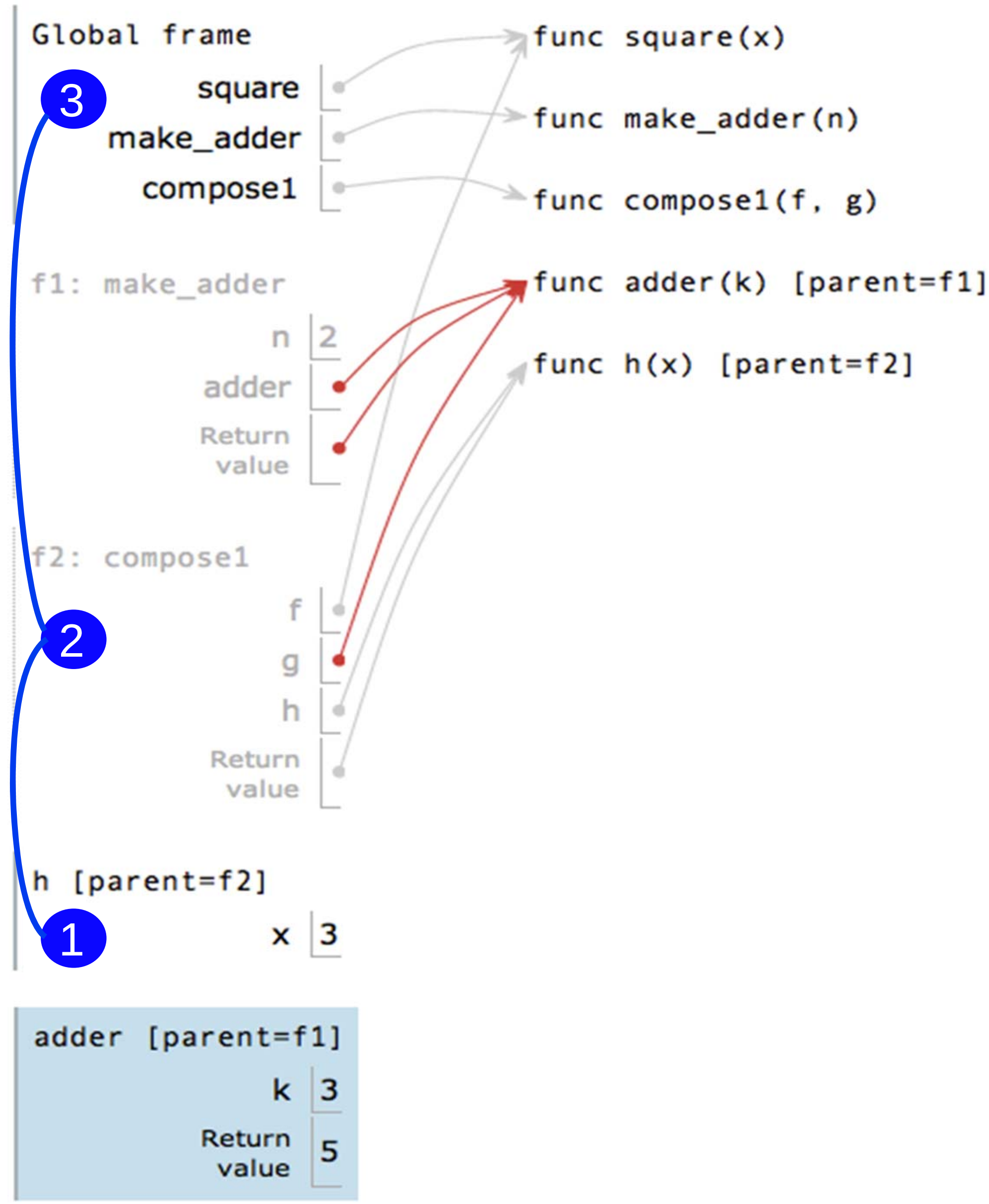

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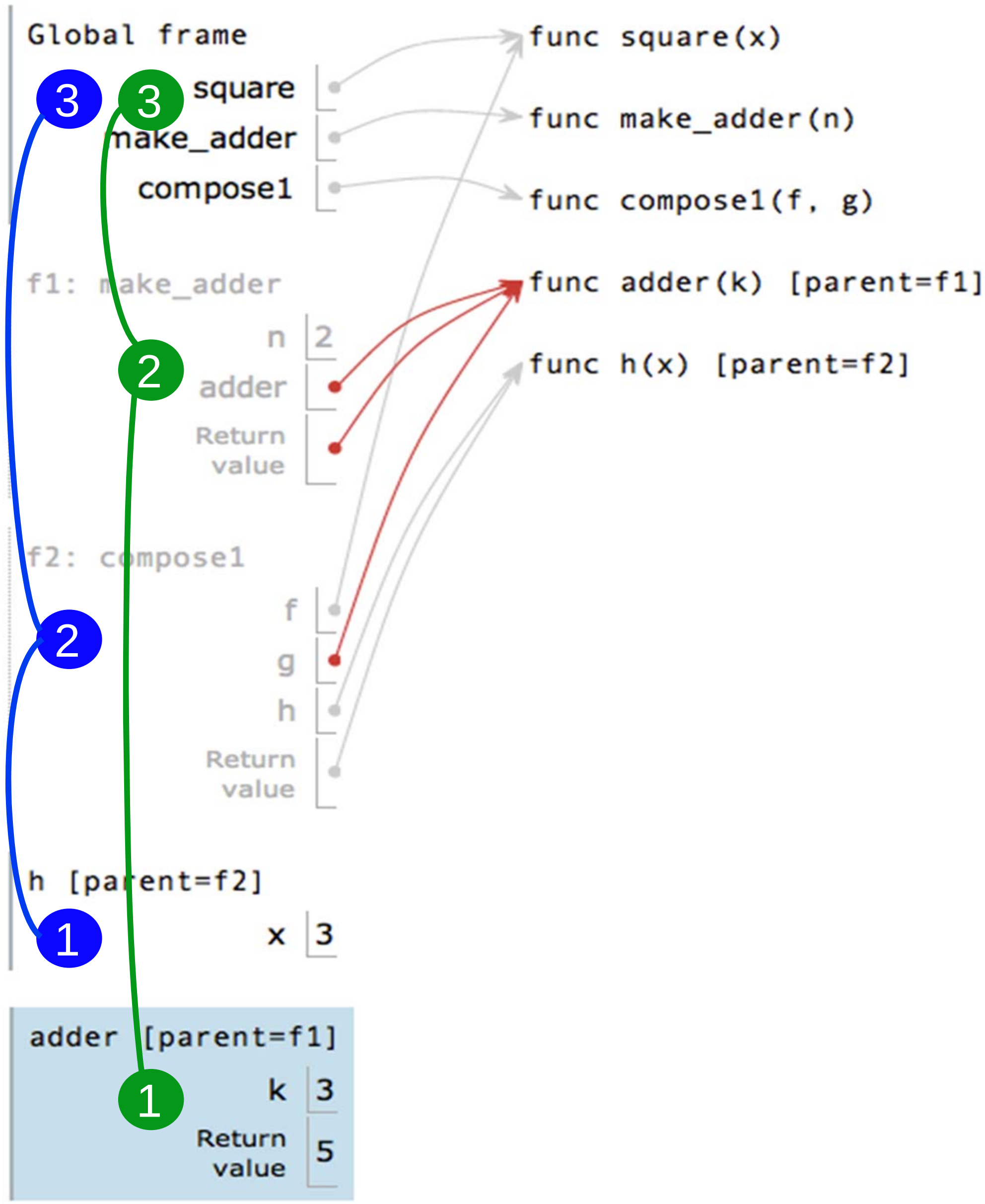

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Lambda expressions are rare in Python, but important in general

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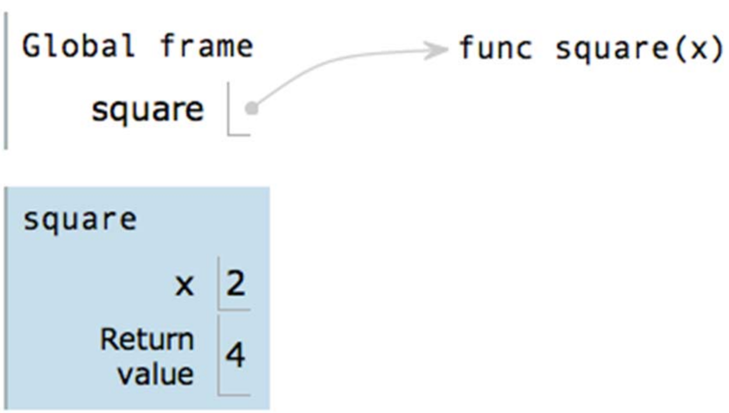

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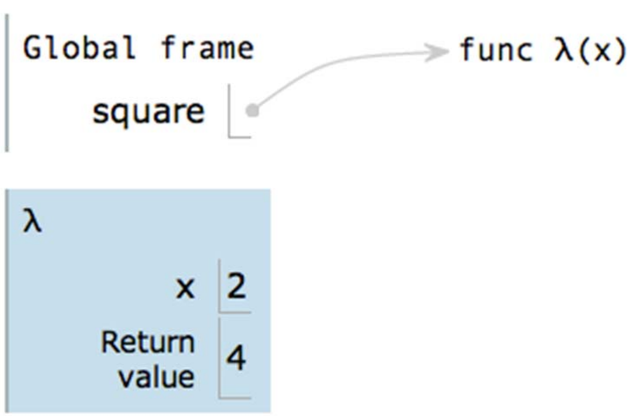

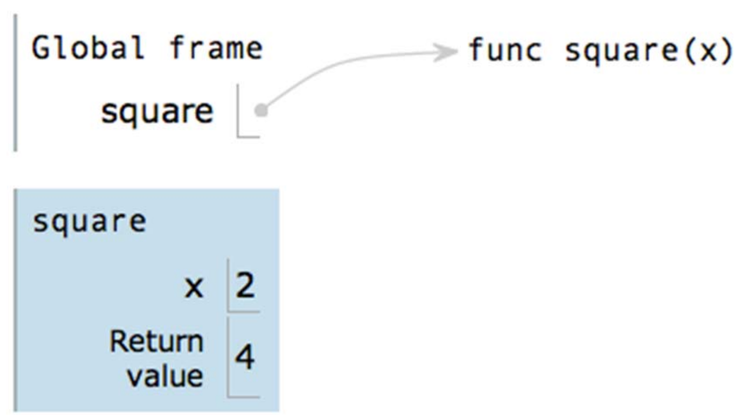

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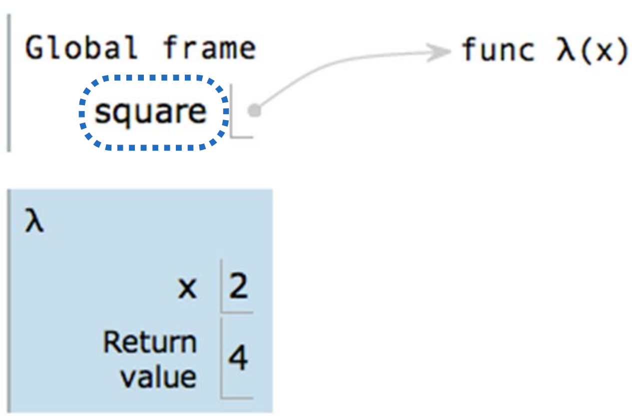

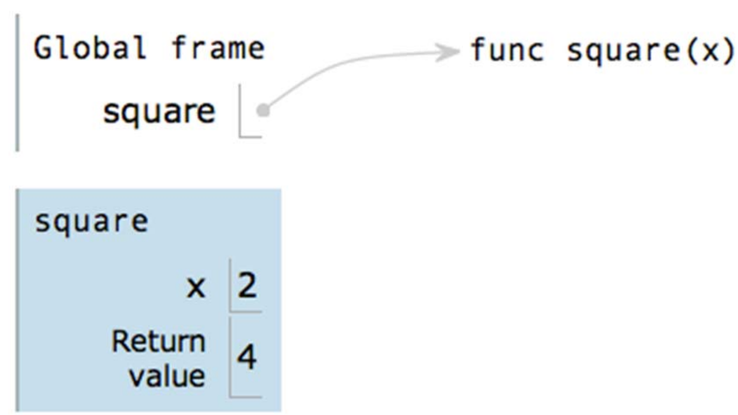

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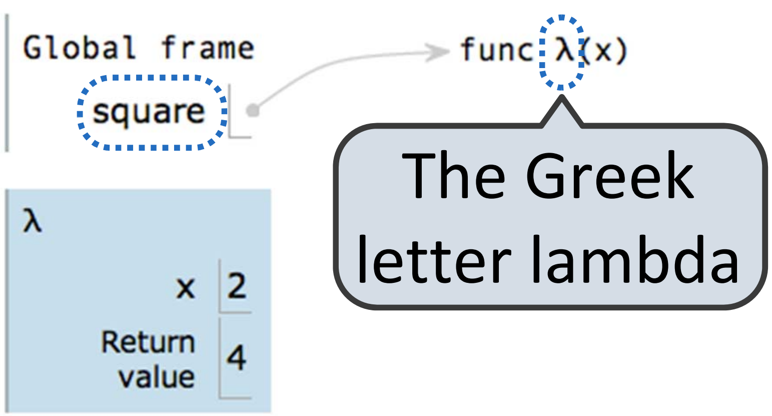

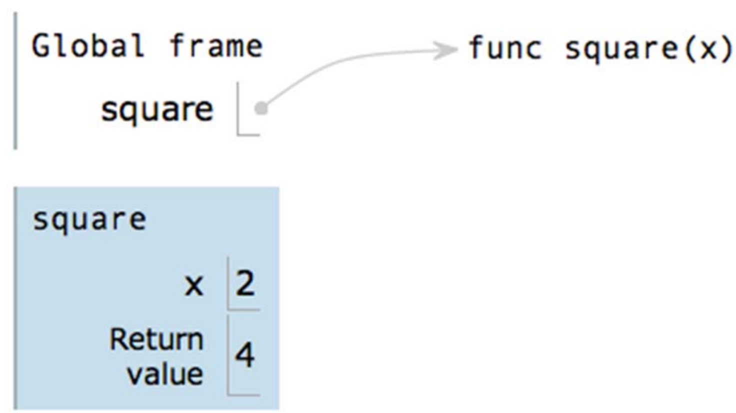

Newton's Method Background

Finds approximations to zeroes of differentiable functions

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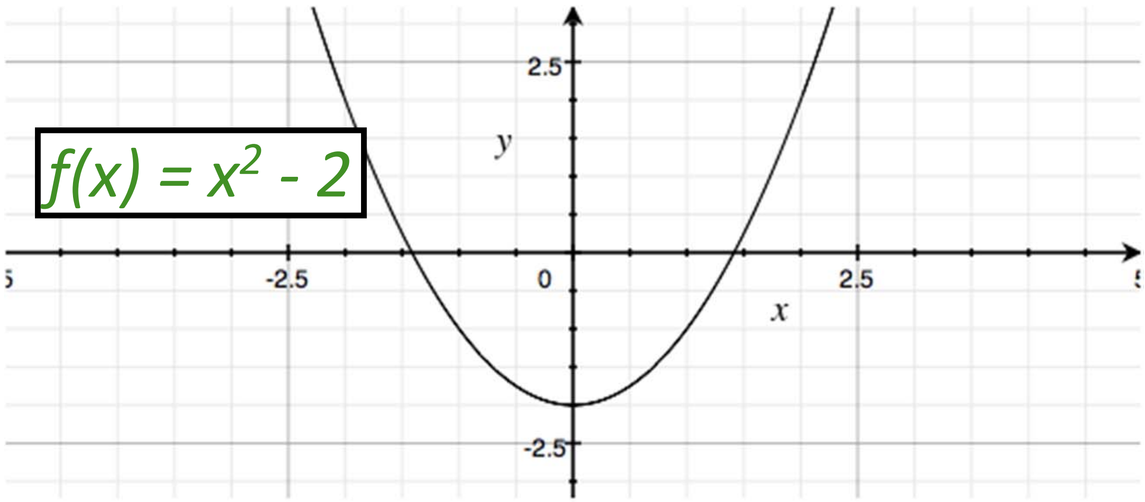

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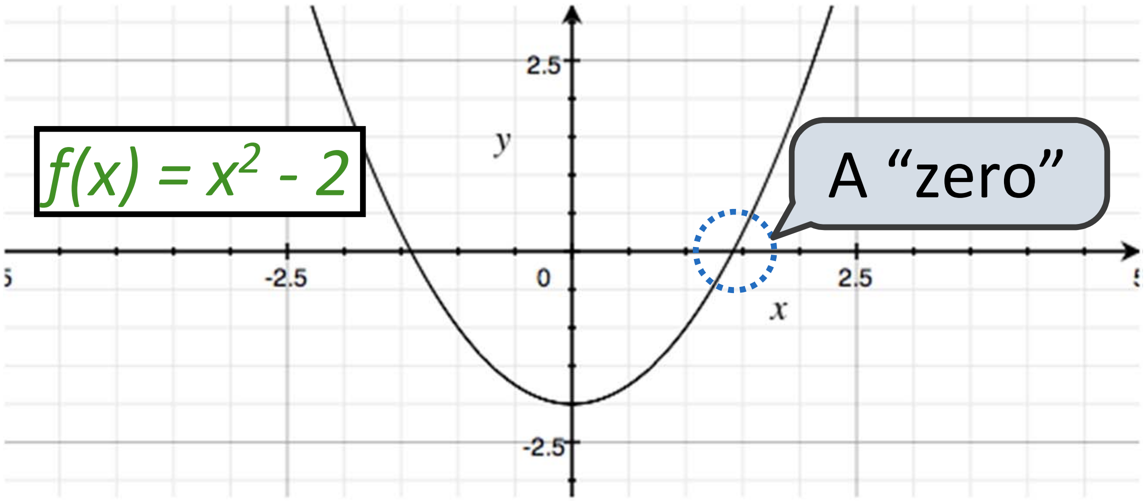

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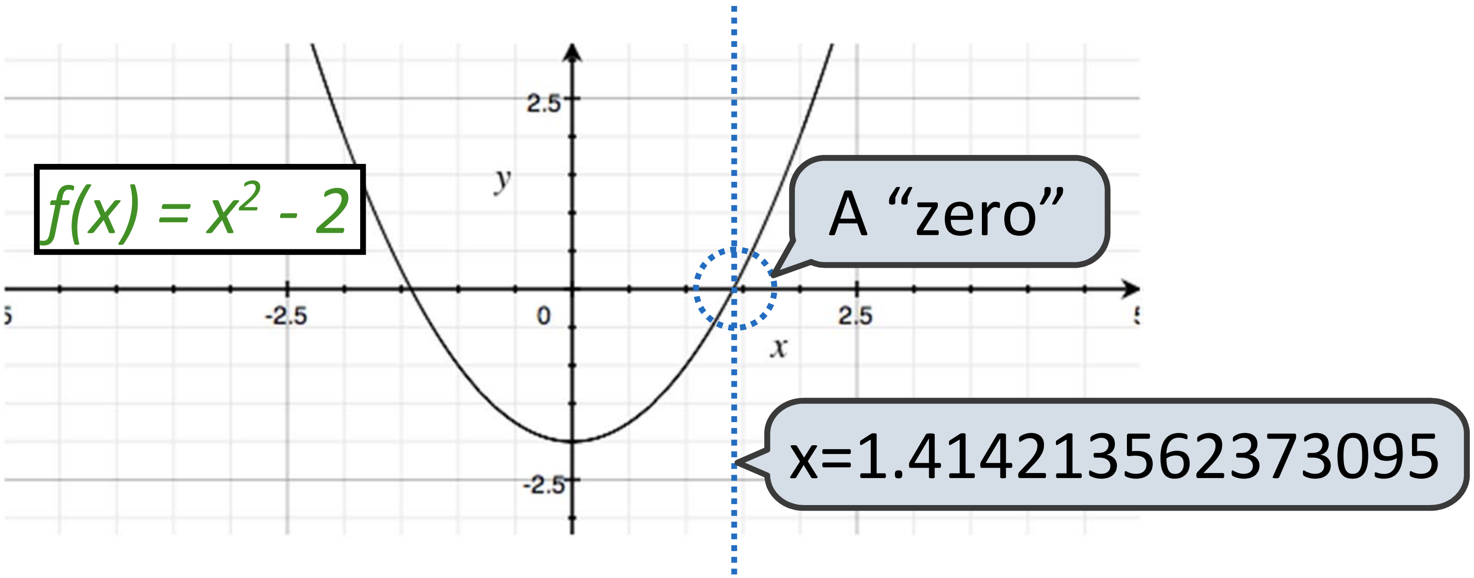

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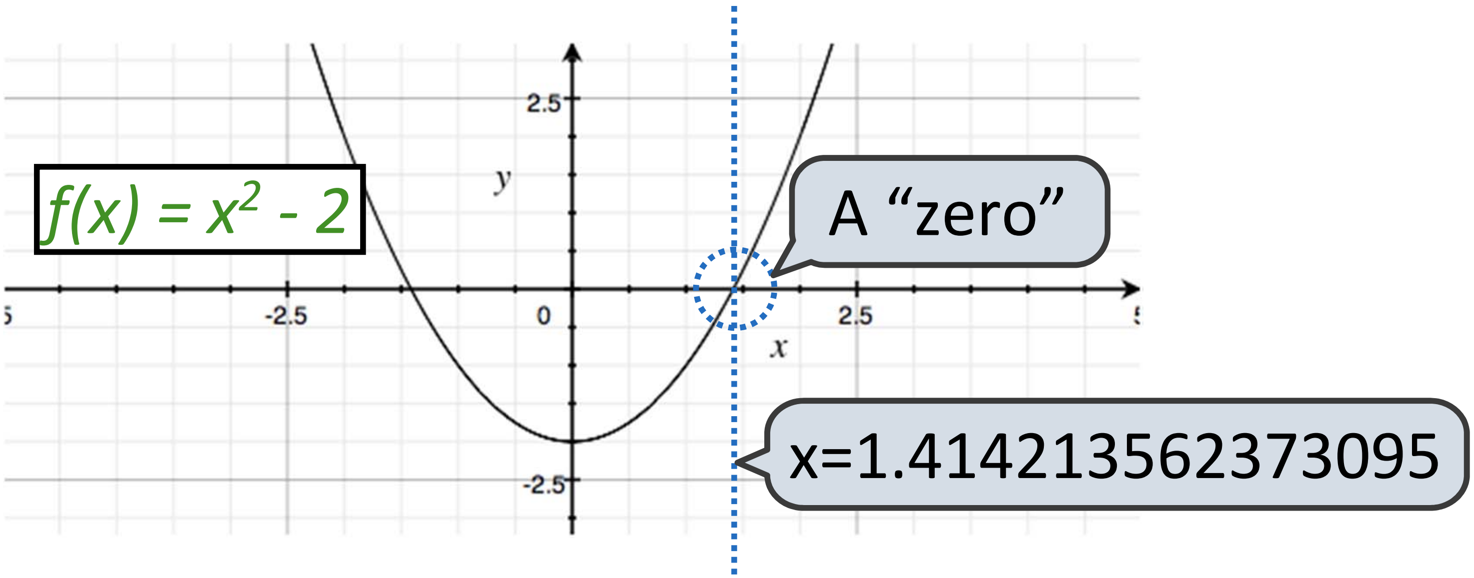

Application: a method for (approximately) computing square roots, using only basic arithmetic.

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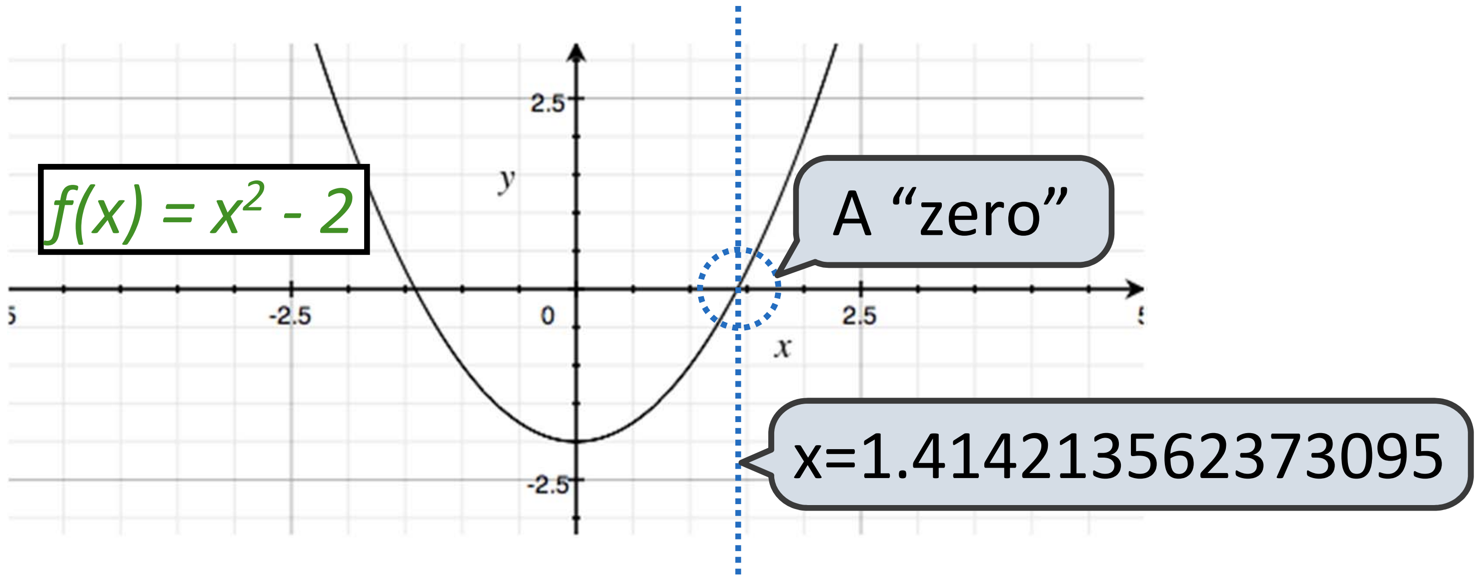

Application: a method for (approximately) computing square roots, using only basic arithmetic.

The positive zero of $f(x) = x^2 - a$ is $\sqrt{a}$

Newton's Method

Begin with a function f and

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Visualization: http://en.wikipedia.org/wiki/File:NewtonIteration_Ani.gif

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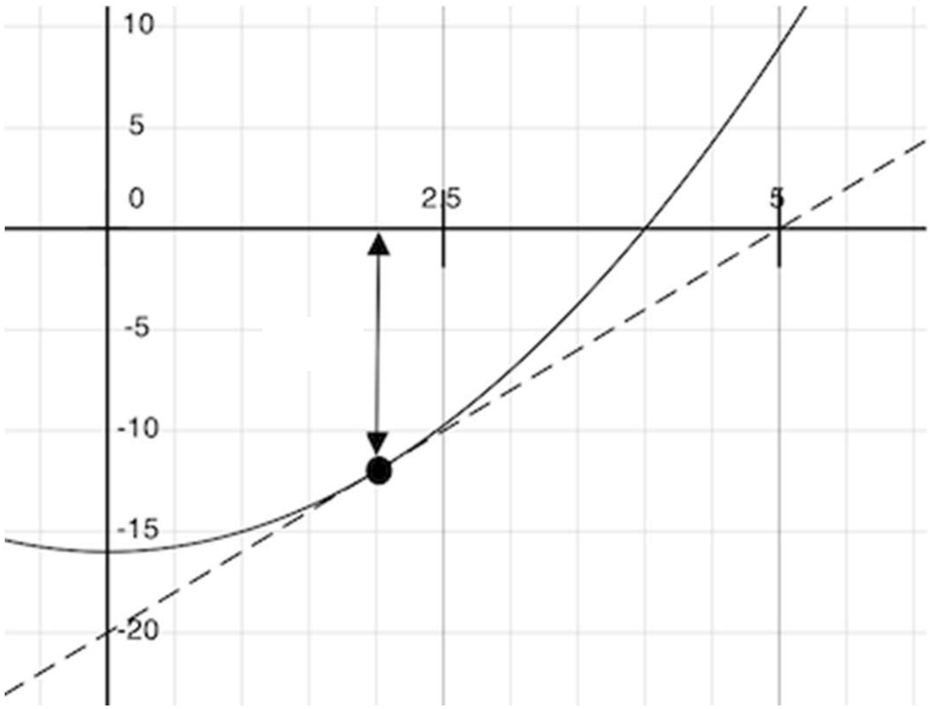

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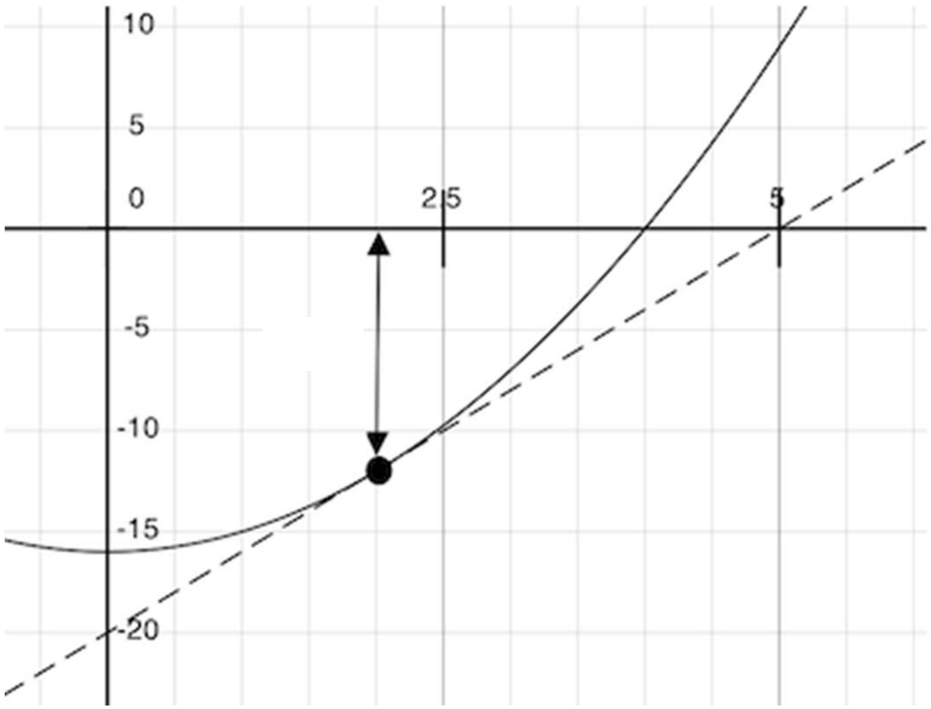

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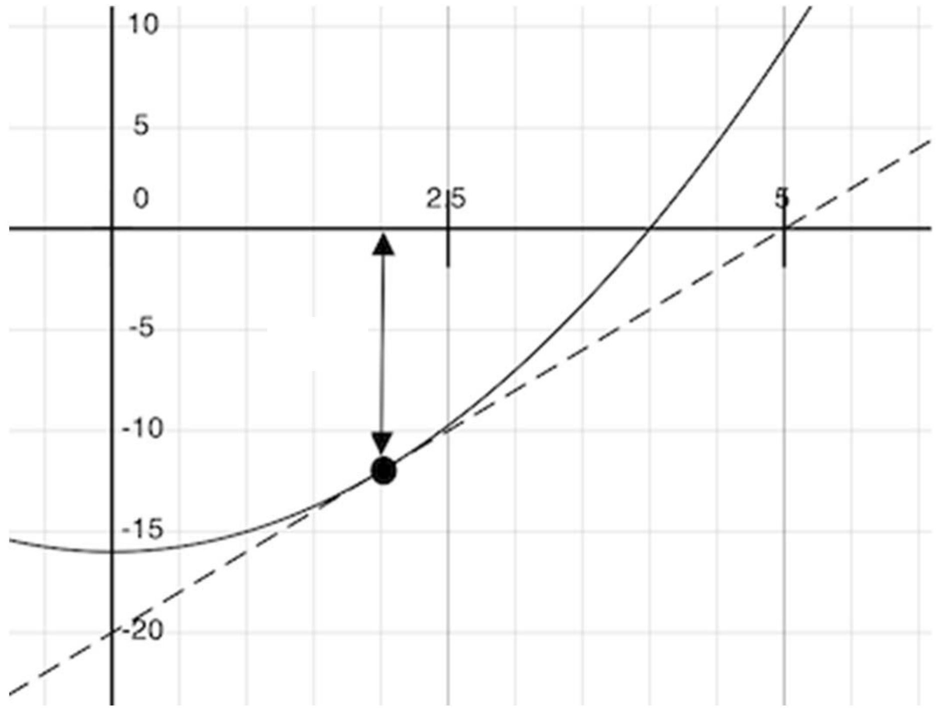

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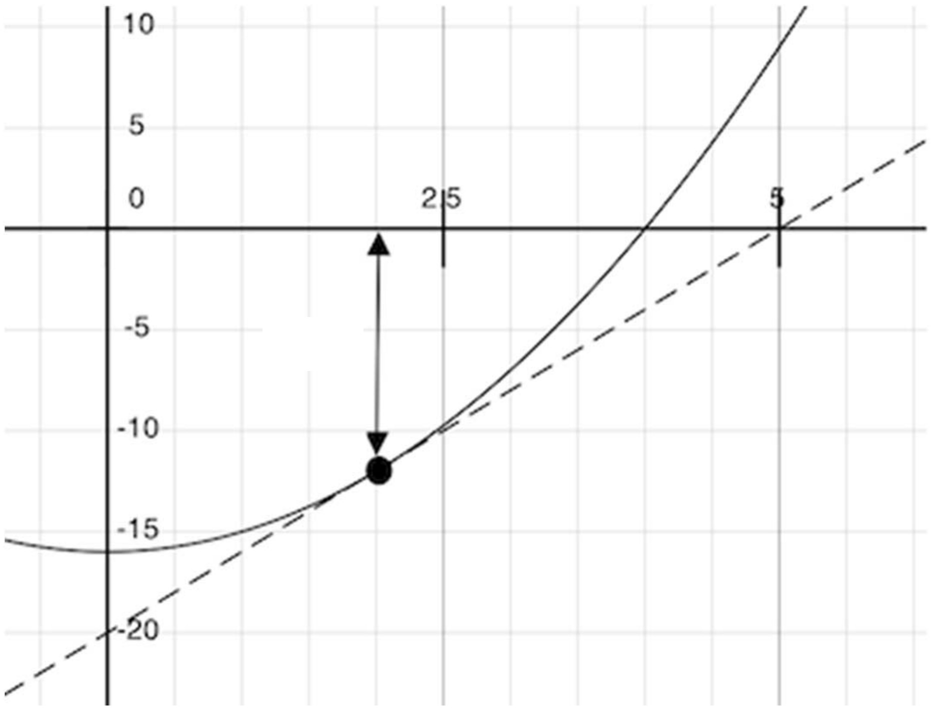
Visualization: http://en.wikipedia.org/wiki/File:NewtonIteration_Ani.gif

Newton’s Method

Begin with a function f and an initial guess x



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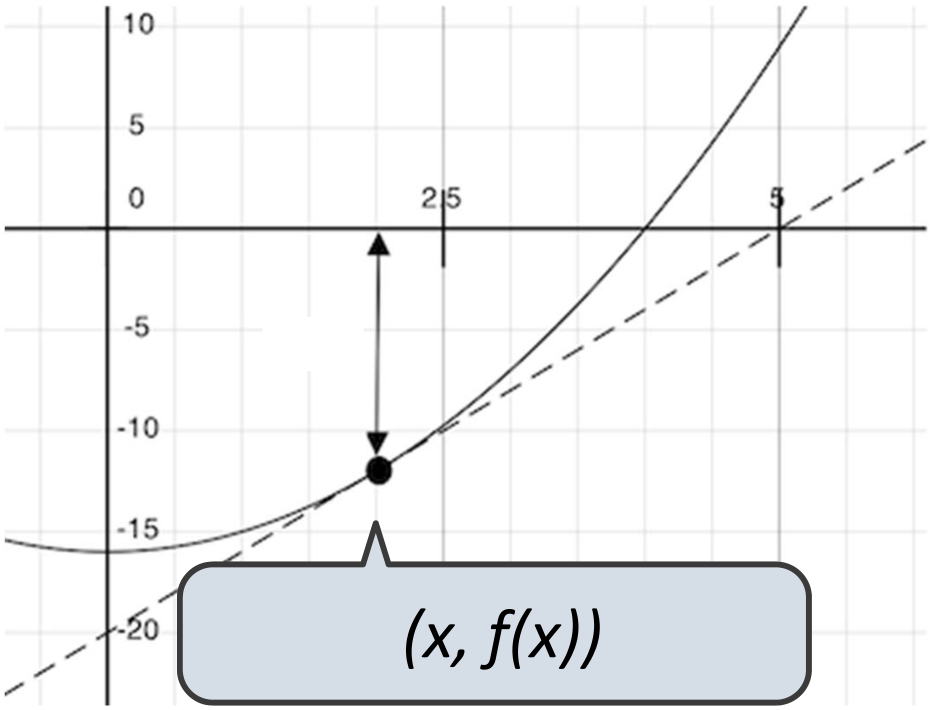

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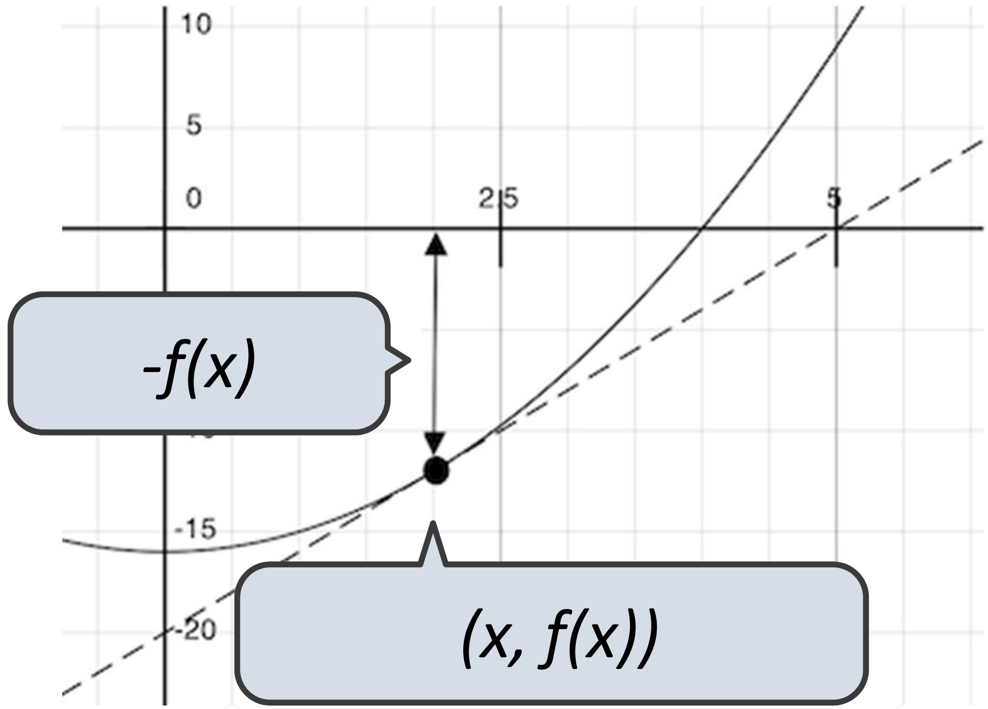

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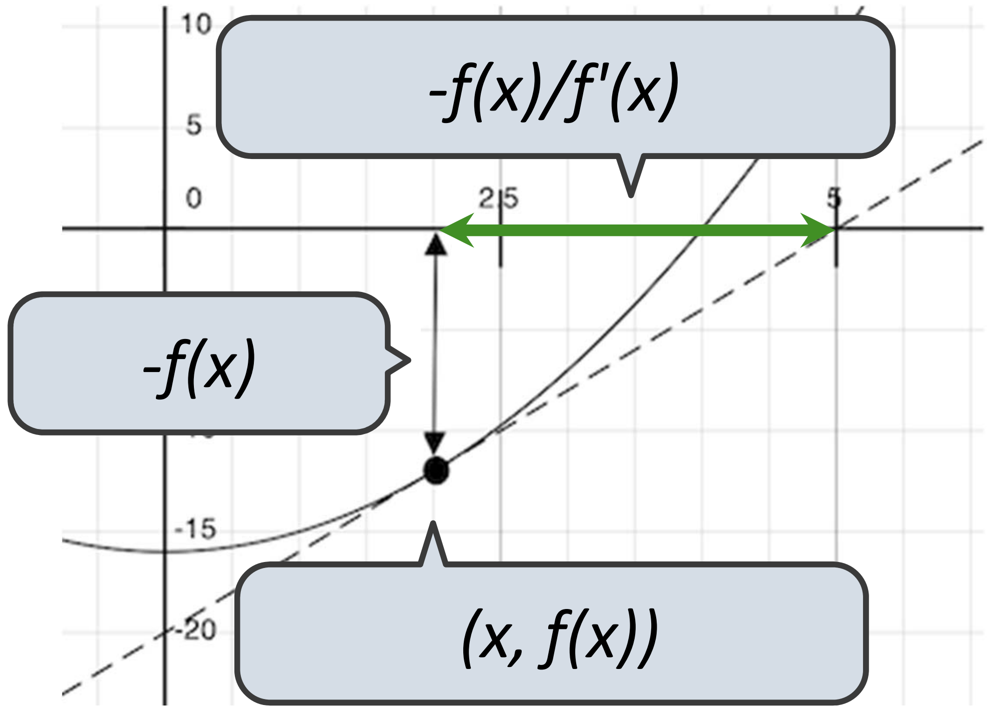

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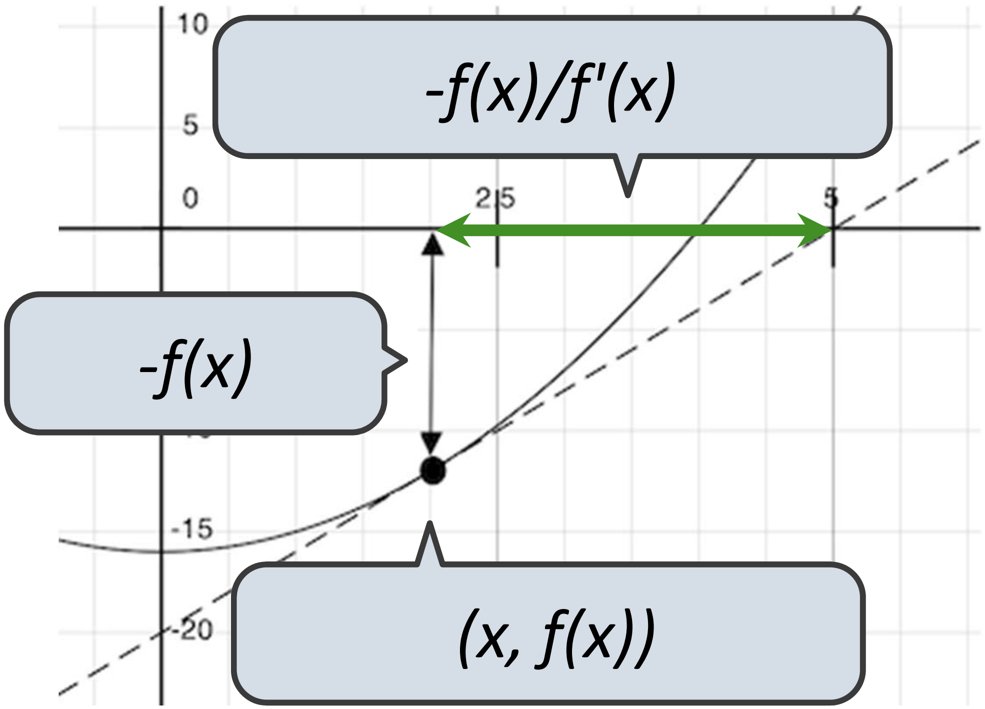

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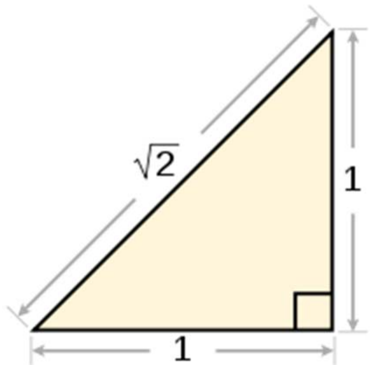


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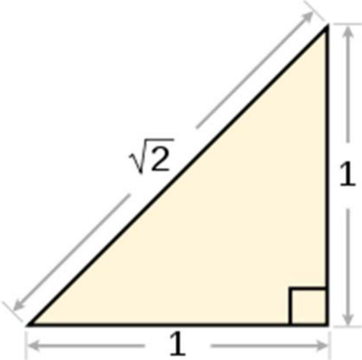


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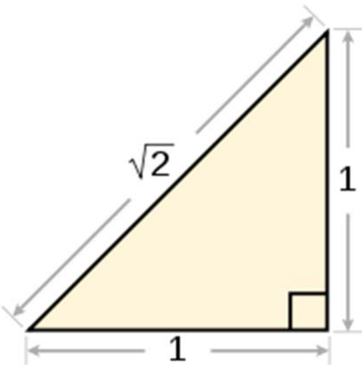


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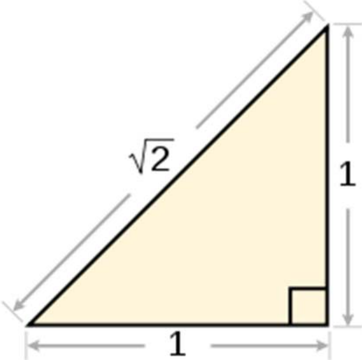


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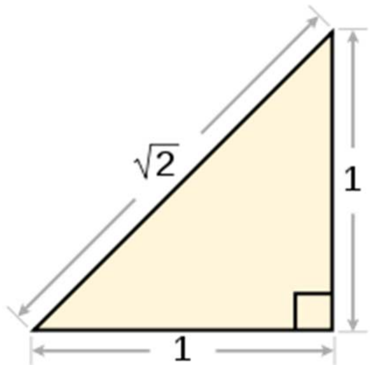


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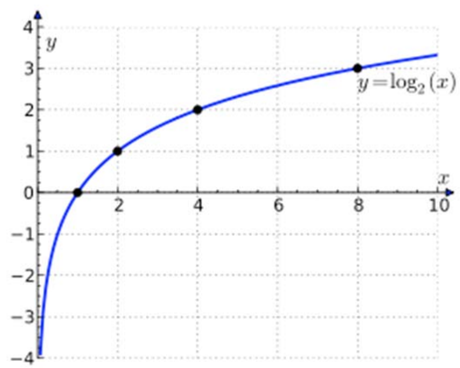


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```
def iter_improve(update, done, guess=1, max_updates=1000):
    """Iteratively improve guess with update until done
    returns a true value.

    >>> iter_improve(golden_update, golden_test)
    1.618033988749895
    """
    k = 0
    while not done(guess) and k < max_updates:
        guess = update(guess)
        k = k + 1
    return guess
```

Newton’s Method for nth Roots

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```
def nth_root_func_and_derivative(n, a):
    def root_func(x):
        return pow(x, n) - a
    def derivative(x):
        return n * pow(x, n-1)
    return root_func, derivative

def nth_root_newton(a, n):
    """Return the nth root of a.

    >>> nth_root_newton(8, 3)
    2.0
    """
    root_func, deriv = nth_root_func_and_derivative(n, a)
    def update(x):
        return x - root_func(x) / deriv(x)
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Definition of a function zero