## CS 61A Spring 2013

# Structure and Interpretation of Computer Programs

Spring 2013 Midterm 2

## INSTRUCTIONS

- You have 2 hours to complete the exam.
- $\bullet$  The exam is closed book, closed notes, closed computer, closed calculator, except one hand-written 8.5"  $\times$  11" crib sheet of your own creation and the official 61A midterm 2 study guide attached to the back of this exam.
- Mark your answers ON THE EXAM ITSELF. If you are not sure of your answer you may wish to provide a brief explanation.

Last name	
First name	
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TA & section time	
Name of the person to your left	
Name of the person to your right	
All the work on this exam is my own. (please sign)	

## For staff use only

Q. 1	Q. 2	Q. 3	Q. 4	Q. 5	Total
/15	/12	/6	/6	/11	/50

## 1. (15 points) You Will Be Baked. And Then There Will Be Cake.

(a) Assume that you have started Python 3 and executed the following statements:

```
the_cake = [1, 2, [3], 4, 5]
a_lie = the_cake[1:4]
the_cake = the_cake[1:4]
great = a_lie
delicious = the_cake
moist = great[:-1]
```

For each of the following expressions, write the value to which it evaluates. If the value is a method value, write Method. If it is a function value, write Function. If evaluation causes an error, write Error. If evaluation would run forever, write Forever. Otherwise, write the resulting value as the interactive interpreter would display it.

Expression	Evaluates to
the_cake	
the_cake is a_lie	
the_cake == great	
the_cake is delicious	
the_cake == moist + 4	
the_cake.append	
the_cake.append == a_lie.append	
the_cake[1] is a_lie[1]	

(b) The following is the recursive list abstract data type from lecture:

```
empty_rlist = None
def rlist(first, rest):
    """Creates an rlist from the element first and the rlist rest."""
    return (first, rest)
def first(s):
    """Returns the first element of the rlist s"""
    return s[0]
def rest(s):
    """Returns the rest (itself an rlist) of s."""
    return s[1]
def len_rlist(s):
    """Returns the length of the rlist s."""
    if s == empty_rlist:
        return 0
    return 1 + len_rlist(rest(s))
def getitem_rlist(s, i):
    """Returns the element at index i in rlist s."""
    if i == 0:
        return first(s)
    return getitem_rlist(rest(s), i - 1)
```

For each of the following pieces of code, circle Y if the code contains at least one data abstraction violation, and N if the code contains no data abstraction violations. Do not guess; leave the answer blank if you do not know it. We will award one point for each correct answer, no points for an incorrect answer, and 0.5 points for each answer left blank.

Y	$\mathbf{N}$	<pre>rlist(4, rlist(5, None))</pre>
Y	N	rlist(1, (2, (3, empty_rlist)))
Y	N	<pre>rlist(rlist(1, empty_rlist), rlist(2, empty_rlist))</pre>
Y	N	first(rest( (1, (2, (3, empty_rlist))) ))
Y	N	<pre>x = rlist(5, rlist( (4, 3, 2), rlist(1, empty_rlist))) first(rest(x))[1]</pre>
Y	N	rlist(empty_rlist, empty_rlist)
Y	N	<pre>len(rlist(3, rlist(4, empty_rlist)))</pre>

## 2. (12 points) Environmental Disaster

(a) (6 pt) Fill in the environment diagram that results from executing the code below until the entire program is finished, an error occurs, or all frames are filled. You need only show the final state of each frame. You may not need to use all of the spaces or frames.

A complete answer will:

- Add all missing names, labels, and parent annotations to all local frames.
- Add all missing values created during execution.
- Show the return value for each local frame.

dof hov():	Global frame	
<pre>def box():     toy = 0     def disp(f, box):         nonlocal toy     if f == 0:         toy += box     elif f == 1:         toy *= box     return toy     return disp</pre>	box	func box()
<pre>toybox = box() toybox(0, 2) toybox(0, 3) toybox(1, 4)</pre>	Return Value	
	Return Value	
	Return Value	
	Return Value	

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(b) (6 pt) Fill in the environment diagram that results from executing the code below until the entire program is finished, an error occurs, or all frames are filled. You need only show the final state of each frame. You may not need to use all of the spaces or frames.

A complete answer will:

- Add all missing names, labels, and parent annotations to all local frames.
- Add all missing values created during execution.
- Show the return value for each local frame.

	Global frame	
<pre>def thrace(x):     def star(y):         print(x, y)         return x(y)     return star</pre>	thrace	func thrace(x)
<pre>def kara(x):     if x &gt; 7:         return x     return kara(x * 2)</pre>		
kara = thrace(kara) buck = kara(5)	Return Value	
	Return Value	
	Return Value	
	Neturn value	
	Return Value	
	Petunn Value	

## 3. (6 points) Cue the Queue that Starts with a Q

For each of the following, cross out any incorrect or unnecessary lines in the following code so that the doctests pass for both classes. Do not cross out class declarations, doctests, or docstrings. You can cross out anything else, including method declarations, and your final code should make as much use of inheritance as possible. Make sure to cross out the entire line for anything you wish to remove.

Note: The pop method of a list removes the item at the given position and returns it.

```
(a) class Queue(object):
       """Creates a Queue, which is like a list that supports 2
       operations: enqueue (adding an item to the back of the queue) and
       dequeue (removing an item from the front of the queue).
       >>> q = Queue()
       >>> q.enqueue(5)
       >>> q.enqueue(3)
       >>> q.enqueue(2)
       >>> q.dequeue()
       11 11 11
       self.items = []
       def __init__(self, items):
       def __init__(self):
           self.enqueue(items)
           self.items = []
       def enqueue(item):
       def enqueue(self, item):
           items.append(self, item)
           self.items.append(item)
           items += item
           self.items += item
       def dequeue():
       def dequeue(self):
           self.items.pop(0)
           self.items.pop(len(self.items) - 1)
           return self.items.pop(0)
```

return self.items.pop(len(self.items) - 1)

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```
(b) class PriorityQueue(Queue):
       """A PriorityQueue is like a sorted list that supports two
       operations: enqueue (adding an item to the PriorityQueue) and
       dequeue (removing the smallest item from the PriorityQueue).
       >>> p = PriorityQueue()
       >>> p.enqueue(5)
       >>> p.enqueue(3)
       >>> p.enqueue(2)
       >>> p.dequeue()
       11 11 11
       self.items = []
       def __init__(self, items):
       def __init__(self):
           Queue.__init__()
           Queue.__init__(self)
           PriorityQueue.__init__()
           PriorityQueue.__init__(self)
           self.items = []
           self.items.sort()
       def enqueue(item):
       def enqueue(self, item):
           self.enqueue(item)
           Queue.enqueue(self, item)
           PriorityQueue.enqueue(self, item)
           items.append(self, item)
           self.items.append(item)
           items += item
           self.items += item
           self.items.sort()
       def dequeue():
       def dequeue(self):
           return self.dequeue()
           return Queue.dequeue(self)
           return PriorityQueue.dequeue(self)
```

## 4. (6 points) Prime RBIs

The Cal Mathletic Club is a group of math enthusiasts who compete in mathematical competitions. Since a sharp mind requires a sharp body, they also field an intramural baseball team. Unfortunately, this team has not been very good in recent years. In fact, they only had 2 runs batted in (RBIs) in all of 2010! The next two years were nearly as dreadful, with 3 RBIs in 2011, and 5 RBIs in 2012.

Being mathletes, they notice that their RBI totals have been consecutive prime numbers in each of the last three years. Being mathletes, they decide they should continue this trend, slowly improving their play each year by batting in the next prime number of runs.

Help the mathletes to determine their long-term goals by writing a higher-order function make\_prime\_generator that returns a function to generate primes. The latter function should return 2 the first time it is called, 3 the next time, then 5, 7, 11, and so on, returning the next prime number each time it is called.

(For the non-mathletic, a prime number can be defined as an integer greater than 1 that is not divisible by any other integer greater than 1. Thus, a prime number p's only positive divisors are 1 and p.)

```
def make_prime_generator():
    """Return a function that computes the next prime number each time it
    is called.

>>> gen = make_prime_generator()
    >>> gen(), gen(), gen()
    (2, 3, 5)
    >>> [gen() for _ in range(10)]
    [7, 11, 13, 17, 19, 23, 29, 31, 37, 41]
    """
```

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## 5. (11 points) Mutation: It is the Key to Our Evolution

The following is an object-oriented recursive list implementation:

```
class Rlist(object):
    """A recursive list consisting of a first element and the rest."""
    class EmptyList(object):
        def __len__(self):
            return 0
    empty = EmptyList()
    def __repr__(self):
        f = repr(self.first)
        if self.rest is Rlist.empty:
            return 'Rlist({0})'.format(f)
        else:
            return 'Rlist({0}, {1})'.format(f, repr(self.rest))
    def __init__(self, first, rest=empty):
        self.first = first
        self.rest = rest
    def __len__(self):
        return 1 + len(self.rest)
    def __getitem__(self, i):
        if i == 0:
            return self.first
        return self.rest[i - 1]
```

(a) Implement a mutating\_map method that takes in a function and applies it to each element in an Rlist. This method should mutate the list in place, replacing each element with the result of applying the function to it. Do not create any new objects. You may assume that the input Rlist contains at least one element.

```
def mutating_map(self, fn):
    """Mutate this Rlist by applying fn to each element.

>>> r = Rlist(1, Rlist(2, Rlist(3)))
>>> r.mutating_map(lambda x: x + 1)
>>> r
Rlist(2, Rlist(3, Rlist(4)))
"""
```

(b) The sieve of Eratosthenes is an ancient algorithm for finding prime numbers. It starts with a sequence of numbers between 2 and n, in order. The first number is a prime, and the algorithm removes all larger multiples of that number from the sequence. Then the next remaining number is a prime, and the algorithm removes all larger multiples of that number from the sequence, and so on, until the end of the sequence is reached. At that point, all remaining numbers in the sequence are prime.

Here is a more concrete illustration of this process:

```
Initial sequence: 2, 3, 4, 5, 6, 7, 8, 9, 10
Remove larger multiples of 2: 2, 3, 5, 7, 9
Remove larger multiples of 3: 2, 3, 5, 7
Remove larger multiples of 5: 2, 3, 5, 7
Remove larger multiples of 7: 2, 3, 5, 7
Done.
```

In this problem, you will implement this algorithm on Rlists. Assume that you have map\_rlist and filter\_rlist functions with the following signatures and docstrings:

```
def map_rlist(s, fn):
    """Return an Rlist resulting from mapping fn over the elements of s.

>>> map_rlist(Rlist(1, Rlist(2, Rlist(3))), lambda x: x * x)
Rlist(1, Rlist(4, Rlist(9)))
    """

def filter_rlist(s, fn):
    """Filter the elements of s by predicate fn.

>>> filter_rlist(Rlist(1, Rlist(2, Rlist(3))), lambda x: x % 2 == 1)
Rlist(1, Rlist(3))
    """
```

i. First, write a function sequence\_to\_rlist that converts a Python sequence into an Rlist. Elements in the resulting Rlist should be in the same order as in the original sequence.

```
def sequence_to_rlist(seq):
    """Converts a sequence to an Rlist, preserving order.

>>> sequence_to_rlist((3, 2, 1))
Rlist(3, Rlist(2, Rlist(1)))
"""
```

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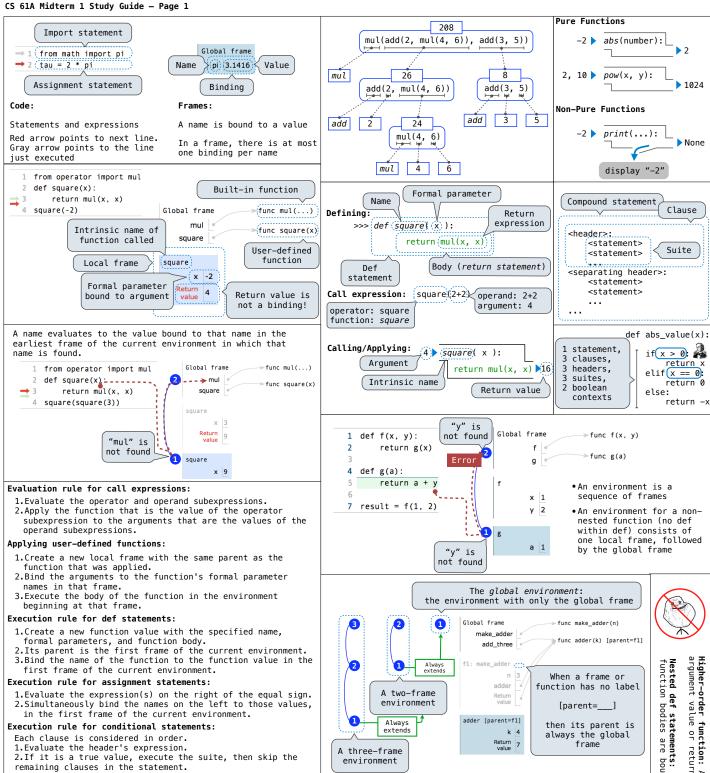
ii. Now fill in the following function prime\_sieve that implements the sieve of Eratosthenes algorithm. This function takes in an Rlist of numbers between 2 and n and removes all composite numbers from the Rlist. You may assume that the input Rlist has at least one element. You may leave the last line blank if you do not need it.

```
def prime_sieve(rlst):
    """Remove all composite numbers from the input Rlist. Assumes
    that the input contains the numbers from 2 to len(rlst), in
    order.

>>> seq = sequence_to_rlist(range(2, 15))
>>> prime_sieve(seq)
>>> seq
    Rlist(2, Rlist(3, Rlist(5, Rlist(7, Rlist(11, Rlist(13))))))
    """

while rlst.rest !=
    func = lambda x:
    rlst.rest =
    rlst = rlst.rest
```

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Evaluation rule for or expressions:

1. Evaluate the subexpression <left>

2.If the result is a true value v, then the expression evaluates to v.

3.Otherwise, the expression evaluates to the value of the subexpression <right>.

Evaluation rule for and expressions:

1. Evaluate the subexpression <left>.

2.If the result is a false value v, then the expression evaluates to v.

 ${\tt 3.0therwise}$ , the expression evaluates to the value of the subexpression <right>.

Evaluation rule for not expressions:

1.Evaluate <exp>; The value is True if the result is a false value, and False otherwise.

Execution rule for while statements:

1. Evaluate the header's expression.

If it is a true value, execute the (whole) suite, then return to step 1.

**Vested def statements:** Functions function bodies are bound to name names es in the l within e local other frame

**>** 2

**1**024

None

Clause

Suite

if(x > 0: 🚜

elif(x == 0)

return x

return 0

return ->

**Higher-or** argument

rder valı

n **function:** A ue or returns

function s a functi

takes a function a return value

as

an

A frame extends the environment that begins with its parent Function of a single def cube(k): argument (not called term) return pow(k, 3) A formal parameter that def summation(n, (term) will be bound to a function """Sum the first n terms of a sequence. >>> summation(5, (cube) 255 The cube function is passed as an argument value total, k = 0, 1while k <= n: total, k = total + (term(k)), k + 1return total The function bound to term  $0 + 1^3 + 2^3 + 3^3 + 4^3 + 5^5$ gets called here

#### CS 61A Midterm 1 Study Guide - Page 2

```
square = lambda x, y: x * y
                                             Facts about print
                                              •Non-pure function
    A function
                                              •Returns None
       with formal parameters x and y
                                              •Multiple arguments are
                                              printed with a space between them
            and body "return (x * y)"
                                            >>> print(4, 2)
Must be a single expression
```

```
def make_adder(n): A function that returns a function
       'Return a function that takes one argument k and returns k + n.
                                       The name add three is
    >>> add_three = make_adder(3) 
                                        bound to a function
                            A local
                        def statement
   def adder(k):
    return k +(n)
return adder
                         Can refer to names in
                        the enclosing function
```

```
make_adder(1)(2)
         make_adder(1) (
                                 2
                                         )
            Operator
                             Operand 0
  An expression that
                              An expression that
evaluates to a function
                            evaluates to any value
        value
```

```
def square(x):
                    def sum_squares(x, y):
                       return square(x)+square(y)
    return mul(x, x)
What does sum_squares need to know about square?
```

- Square takes one argument. Yes
- Square has the intrinsic name square. No
- Square computes the square of a number. Yes
- Square computes the square by calling mul. No

```
Global frame
               →func factorial(n)
 factorial
                    def factorial(n):
factorial
                         if n == 0 or n == 1:
    n 4
                             return 1
                         return n * factorial(n - 1)
   Return
value
              → 6 factorial(4)
factorial
     n 3
```

A function is *recursive* if the body calls the function itself, either directly or indirectly Recursive functions have two important components: 1. Base case(s), where the function directly computes an answer without calling itself a. Recursive case(s), where the function calls itself
as part of the computation

factorial n 1

n 2

Return 2

factorial

```
def square(x):
square = lambda x: x * x
                                            return x * x
```

- Both create a function with the same arguments & behavior
- Both of those functions are associated with the environment in which they are defined
- Both bind that function to the name "square"
- Only the def statement gives the function an intrinsic name

```
-f(x)/f'(x)
How to find the square root of 2?
>>> f = lambda x: x*x - 2
>>> find_zero(f, 1)
1.4142135623730951
Begin with a function f and
                                         (x, f(x))
an initial guess x
```

- Compute the value of f at the guess: f(x)Compute the derivative of f at the guess: f'(x)Update guess to be: f(x)3.

return h

• Every user-defined function has a

• The parent of a function is the frame in which it was defined

• Every local frame has a parent

• The parent of a frame is the parent of the function called

14 compose1(square, make\_adder(2))(3)

12

```
Update guess to be: x - \frac{f(x)}{f'(x)}
def iter_improve(update, done, guess=1, max_updates=1000):
    """Iteratively improve guess with update until done returns a true value.
      >>> iter_improve(golden_update, golden_test)
      1.618033988749895
      while not done(guess) and k < max\_updates:
           guess = update(guess)
k = k + 1
      return guess
def newton_update(f):
     """Return an update function for f using Newton's method."""
def update(x):
    return x - f(x) / approx_derivative(f, x)
      return update
def approx_derivative(f, x, delta=1e-5):
"""Return an approximation to the derivative of f at x."""
df = f(x + delta) - f(x)
      return df/delta
def find_root(f, guess=1):
         "Return a guess of a zero of the function f, near guess.
     >>> from math import sin
>>> find_root(lambda y: sin(y), 3)
3.141592653589793
      return iter_improve(newton_update(f), lambda x: f(x) == 0, guess)
                                                  Global frame
    1 def square(x):
                                                  3 square
                                                                            func make_adder(n)
                                                        ake_adder
                                                        compose1 •
     4 def make_adder(n):
                                                                          func compose1(f, g)
           def adder(k):
               return k + n
                                                                            func adder(k) [parent=f1]
           return adde
                                                           n 2
adder 🖸
                                                       2
      def compose1(f, g):
                                                           Return
value
   10
           def h(x):
               return f(g(x))
```

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[par ent=f2] 👍

x 3

A function's signature

has all the information to create a local frame

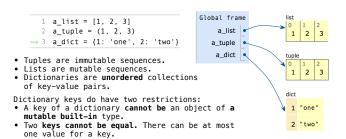
[parent=f1]

k 3

1

adder

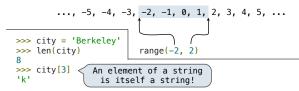
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for <name> in <expression>:
 <suite>

- Evaluate the header <expression>, which must yield an iterable value.
- For each element in that sequence, in order:
   A. Bind <name> to that element in the local environment.
   B. Execute the <suite>.

A range is a sequence of consecutive integers.\*



Length. A sequence has a finite length.

Element selection. A sequence has an element corresponding to any non-negative integer index less than its length, starting at 0 for the first element.

#### **Generator expressions**

(<map exp> for <name> in <iter exp> if <filter exp>)

- Evaluates to an iterable object.
- <iter exp> is evaluated when the generator expression is evaluated.
- Remaining expressions are evaluated when elements are accessed.

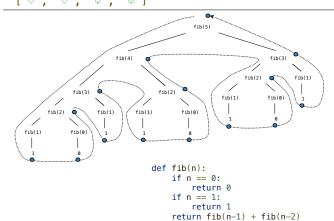
#### List comprehensions

[<map exp> for <name> in <iter exp> if <filter exp>]

Short version: [<map exp> for <name> in <iter exp>]

Unlike generator expressions, the map expression is evaluated when the list comprehension is evaluated.

```
>>> suits = ['heart', 'diamond', 'spade', 'club'] >>> from unicodedata import lookup >>> [lookup('WHITE ' + s.upper() + ' SUIT') for s in suits] ['\heartsuit', '\diamondsuit', '\diamondsuit', '\diamondsuit']
```



```
nonlocal <name> . <name 2> . . . .
```

Effect: Future assignments to that name change its pre-existing binding in the first non-local frame of the current environment in which that name is bound.

From the Python 3 language reference:

Python Docs: an "enclosing scope"

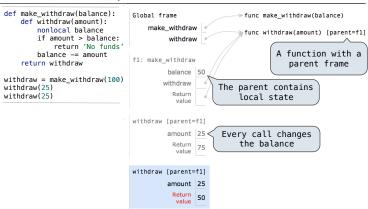
Names listed in a nonlocal statement must refer to pre-existing bindings in an enclosing scope.

Names listed in a nonlocal statement must not collide with pre-existing bindings in the local scope.

```
x = 2
Status
                                         Effect
                                         Create a new binding from name "x to object 2 in the first frame of

    No nonlocal statement

•"x" is not bound locally
                                         the current environment.
•No nonlocal statement
•"x" is bound locally
                                         Re-bind name "x" to object 2 in the
                                         first frame of the current env.
•nonlocal x
                                         Re-bind "x" to 2 in the first non-
•"x" is bound in a non-local frame
                                         local frame of the current
 (but not the global frame)
                                         environment in which it is bound.
• nonlocal x
                                         SyntaxError: no binding for nonlocal
• "x" is not bound in a non-local
 frame
• nonlocal x
                                         SyntaxError: name 'x' is parameter
•"x" is bound in a non-local frame
                                         and nonlocal
•"x" also bound locally
```



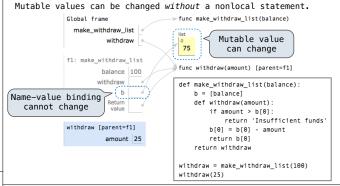
Python pre-computes which frame contains each name before executing the body of a function.

Therefore, within the body of a function, all instances of a name must refer to the same frame.

```
def make_withdraw(balance):
    def withdraw(amount):
        if amount > balance:
            return 'Insufficient funds'
        (balance = balance - amount)
        return balance
    return withdraw

wd = make_withdraw(20)
wd(5)
```

UnboundLocalError: local variable 'balance' referenced before assignment



def pig\_latin(w):
 if starts\_with\_a\_vowel(w):
 return w + 'ay'
 return pig\_latin(w[1:] + w[0])

def starts\_with\_a\_vowel(w):
 return w[0].lower() in 'aeiou'

 The def statement header is similar to other functions
 Conditional statements check for base cases
 Base cases are evaluated without recursive calls
 Typically, all other cases are evaluated with recursive calls

```
>>> a = Account('Jim')
     class <name>(<base class>):
          <suite>
                                                                                             When a class is called:
    • A class statement creates a new class and binds that class to

    A new instance of that class is created:

                                                                                             The constructor __init__ of the class is called with the
new object as its first argument (called self), along with
additional arguments provided in the call expression.
       <name> in the first frame of the current environment
    • Statements in the <suite> create attributes of the class.
To evaluate a dot expression: <expression> . <name>
1. Evaluate the <expression> to the left of the dot, which yields the object of the dot expression.
2. <name> is matched against the instance attributes of that object;
                                                                                                              class Account(object);
  def __init__(self, account_holder):
    self.balance = 0
    self.holder = account_holder
    if an attribute with that name exists, its value is returned.
If not, <name> is looked up in the class, which yields a class
attribute value.
                                                                                           A mutable Rlist implementation using message passing
   That value is returned unless it is a function, in which case a
    bound method is returned instead.
                                                                                           def mutable_rlist():
 To look up a name in a class.
                                                                                               contents = empty_rlist
def dispatch(message, value=None):
    If it names an attribute in the class, return the attribute value.
    Otherwise, look up the name in the base class, if there is one.
                                                                                                    nonlocal contents
                                                                                                    if message == 'len':
    return len_rlist(contents)
elif message == 'getitem':
     class Account(object)
          interest = 0.02 Class attribute
                                                            Constructor
                                                                                                          return getitem_rlist(contents, value)
               __init__(self, account_holder)
self.balance = 0
self.holder = account_holder
                                                                                                    elif message == 'push_first':
    contents = make_rlist(value, contents)
                                                              Methods
                                                                                                     elif message == 'pop_first':
                                                                                                          f = first(contents)
         def deposit(self, amount):
    self.balance = self.balance + amount
                                                                                                          contents = rest(contents)
return f
               return self.balance
                                                                                                     elif message == 'str':
          def withdraw(self. amount):
                                                                                                          return str(contents)
               if amount > self.balance:
return 'Insufficient funds'
self.balance = self.balance - amount
                                                                                               return dispatch
               return self.balance
                                                                                           A bank account implemented using dispatch dictionaries
    Assignment statements with a dot expression on their left-hand
                                                                                           def account(balance):
    side affect attributes for the object of that dot expressionIf the object is an instance, then assignment sets an
                                                                                               def withdraw(amount):
                                                                                                     if amount > dispatch['balance']:
        instance attribute
                                                                                                          return 'Insufficient funds
    • If the object is a class, then assignment sets a class
                                                                                                    dispatch['balance'] -= amount
return dispatch['balance']
        attribute
                                                                                                def deposit(amount):
        jim_account = Account('Jim')
                                              >>> jim_account.interest = 0.8
                                                                                               >>> jim_account.interest
   >>> tom_account = Account('Tom')
                                              0.8
       tom_account.interest
   0.02
                                              >>> tom_account.interest
                                              0.04
   >>> jim_account.interest
                                              >>> Account.interest = 0.05
>>> tom_account.interest
   0.02
                                                                                                return dispatch
     >> tom_account.interest
                                              0.05
   0.02
    >>> Account.interest = 0.04
                                              >>> jim_account.interest
                                                                                           A simple container implemented using two accessor methods
   >>> tom_account.interest
                                              0.8
   0.04
                                                                                           def container(contents):
                                                                                                def get():
                      tom_account.interest = 0.08
                                                               Attribute
                                                                                                     return contents
  Attribute:
                                                             assignment
statement adds
                                                                                                def put(value):
 Assignment
                        This expression
                                                                                                     nonlocal contents
                                                             or modifies
the "interest"
attribute of
                    evaluates to an object
                                                                                                     contents = value
                                                                                                return put, get
                                                              tom_account
                    But the name ("interest") is not looked up
                                                                                           class ComplexRI(object):
                                                                                                class CheckingAccount (Account)
                                                                                                                            Special decorator: "Call this
       withdraw_fee = 1
interest = 0.01
                                           Base class
                                                                                                                            function on attribute look-up"
                                                                                               @property
                                                                                                def magnitude(self):
        def withdraw(self, amount)
                                                                                                     return (self.real ** 2 + self.imag ** 2) ** 0.5
             return Account.withdraw(self. amount + self.withdraw fee)
                                                                                            Type dispatching: Define a different function for each
   To look up a name in a class:
                                                   >>> ch = CheckingAccount('T')
                                                                                            possible combination of types for which an operation is valid
                                                   >>> ch.interest
   1.If it names an attribute in the
                                                                                                      def iscomplex(z)
                                                  0.01
      class, return the attribute value.
                                                                                                            return type(z) in (ComplexRI, ComplexMA)
                                                   >>> ch.deposit(20)
   2.Otherwise, look up the name in the
                                                   20
      base class, if there is one.
                                                                                                      def isrational(z):
                                                     >> ch.withdraw(5)
                                                                                                            return type(z) == Rational
                                                                                                                                                         Converted to a
    class SavingsAccount(Account):
                                                   14
                                                                                                                                                     real number (float)
         deposit_fee = 2
                                                                                                      def add_complex_and_rational(z, r):
         def deposit(self, amount):
                                                                                                            return ComplexRI(z.real + (r.numer/r.denom), z.imag)
              return Account.deposit(self, amount - self.deposit_fee)
                                                                                                      def add_by_type_dispatching(z1, z2):
    """Add z1 and z2, which may be complex or rational."""
    if iscomplex(z1) and iscomplex(z2):
    class AsSeenOnTVAccount(CheckingAccount, SavingsAccount):
         def __init__(self, account_holder):
    self.holder = account_holder
                                                                                                           return add_complex(z1, z2)
elif iscomplex(z1) and isrational(z2):
    return add_complex_and_rational(z1, z2)
elif isrational(z1) and iscomplex(z2):
               self.balance = 1
                                                 # A free dollar!
class Rlist(object):
                                                       class Tree(object):
                                                            def __init__(self, entry,
    class EmptyList(object):

def __len__(self):
    return 0
                                                                                                                return add_complex_and_rational(z2, z1)
                                                                            left=None
                                                                                                            else
                                                                right=None):
self.entry = entry
self.left = left
                                                    A valid
                                    The base
                                                                                                                add_rational(z1, z2)
                                                   tree has
                                      case
                                                                                                    1. Attempt to coerce arguments into values of the same type
                                                  no cycles!
    empty = EmptyList()
                                                                 self.right = right
                                                                                                    2. Apply type-specific (not cross-type) operations
         def map_rlist(s, fn):
    if s is Rlist.empty:
                                                                                        Valid if these
                                                                                                              def coerce_apply(operator_name, x, y
                                                                                         are None or a
                                                                                                                   tx, ty = type_tag(x), type_tag(y)
if tx != ty:
                                                                                        Tree instance
                                         call
                                                                                                                       if (tx, ty) in coercions:
    tx, x = ty, coercions[(tx, ty)](x)
elif (ty, tx) in coercions:
         __len__(self):
return 1 + len(self.rest)
                                                            rest = map_rlist(s.rest, fn)
return Rlist(fn(s.first),rest)
    def
    def __getitem__.
    if i == 0:
        return self.first
        rest[i-1]
                                                       def count_entries(tree):
                                                                                                                            ty, y = tx, coercions[(ty, tx)](y)
                                                            if tree is None:
return 0
                                                                                                                        else:
                                                                                                                             return 'No coercion possible.'
                                                            left = count_entries(tree.left)
right = count_entries(tree.right)
return 1 + left + right
                                                                                                                   key = (operator_name, tx)
return coerce_apply.implementations[key](x, y)
```